ASP-pricing: A Black-Scholes option pricing formulation

Chaitanya Singh

Follow this and additional works at: https://digitalcommons.latech.edu/dissertations

Part of the Databases and Information Systems Commons, and the Finance Commons
INFORMATION TO USERS

This manuscript has been reproduced from the microfilm master. UMI films the text directly from the original or copy submitted. Thus, some thesis and dissertation copies are in typewriter face, while others may be from any type of computer printer.

The quality of this reproduction is dependent upon the quality of the copy submitted. Broken or indistinct print, colored or poor quality illustrations and photographs, print bleedthrough, substandard margins, and improper alignment can adversely affect reproduction.

In the unlikely event that the author did not send UMI a complete manuscript and there are missing pages, these will be noted. Also, if unauthorized copyright material had to be removed, a note will indicate the deletion.

Oversize materials (e.g., maps, drawings, charts) are reproduced by sectioning the original, beginning at the upper left-hand corner and continuing from left to right in equal sections with small overlaps.

Photographs included in the original manuscript have been reproduced xerographically in this copy. Higher quality 6" x 9" black and white photographic prints are available for any photographs or illustrations appearing in this copy for an additional charge. Contact UMI directly to order.

ProQuest Information and Learning
300 North Zeeb Road, Ann Arbor, MI 48106-1346 USA
800-521-0600

UMI®

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.
ASP-PRICING: A BLACK-SCHOLES
OPTION PRICING FORMULATION

by
Chaitanya Singh, BCOM, M.B.A

A Dissertation Presented in Partial Fulfillment
of the Requirements for the Degree
Doctor of Business Administration

COLLEGE OF ADMINISTRATION AND BUSINESS
LOUISIANA TECH UNIVERSITY

March 2002
December 20, 2001

We hereby recommend that the dissertation prepared under our supervision by Chaitanya Singh entitled ASP-Pricing: A Black-Scholes Option-Pricing Formulation be accepted in partial fulfillment of the requirements for the Degree of Doctor of Business Administration.

Supervisor of Dissertation Research

Head of Department

Finance

Department

Committee

Approved:

Director of Graduate Studies

Dean of the College

Approved:

Director of the Graduate School

GS Form 13
(1:00)
ABSTRACT

The Applications Service Provider (ASP) arrangement has engendered a revolution in the area of corporate information technology (IT) by transforming software from a packaged off-the-shelf product to an on-line virtual service. The outsourcing mechanism comprising software renting is intrinsic to applications hosting, and offers a viable alternative to purchasing the shrink-wrapped or retail counterpart of the ASP-deployed software. The major advantages to renting are affordable access to high-priced key business applications, reduced total cost of ownership, opportunity to implement improved IT solutions in the future, and decreased time to market. The fact that renting software is a potentially valuable IT alternative is manifest in favorable demand forecasts for ASP offerings with total revenues from outsourcing expected to be between $8 – $25 billion in 2004. This study extends the scope of real-options applications by conceptualizing and evaluating a software outsourcing or ASP mechanism that offers decision flexibility to the end user.

Although the ASP arrangement has surfaced as a feasible IT investment alternative, its continuance as a business solution depends on the outsourcing firm’s operative life. The rental services are likely to be terminated if the ASP’s operations are disrupted due to bankruptcy or consolidation. Moreover, different ASPs charge disparate subscription fees for hosting the same software applications. One
explanation for the variability in subcontractor pricing seems to be the dynamic nature of applications software development. Since the IT landscape is characterized by rapid and sustained introduction of software innovations, there exists a veritable risk of obsolescence. As a result of the inability to integrate current systems with software innovations that promote operational efficiency, technology-intensive firms (comprising those that employ as well as deploy applications software solutions) are unlikely to ensure maximization of overall value. Therefore, outsourcing mechanisms that offer flexibility in the face of technological changes represent an attractive alternative to investment in rigid ASP arrangements. An obvious example of contract flexibility is the embedded ‘exit clause’ that grants the end user a right to terminate the service arrangement prior to its expiration. Due to the fact that numerous ASP contracts compete with each other on the basis of pricing in a dynamically charged IT environment, the problem of evaluating a flexible outsourcing investment becomes quite challenging.

The focus of this study then is to establish a sound mathematical foundation for evaluating software rental agreements (embedding exit flexibility) by incorporating a real options framework (based upon the Black-Scholes approach) into the traditional capital budgeting technique. The static discounted cash flow or net present value analysis may not adequately serve as a ‘barometer’ of outsourcing value due to its inherent weaknesses. On the other hand, the options approach to valuing real investments appropriately prices the state-contingent opportunity risk of outsourcing flexibility in the model’s variance parameter.
ASP or outsourcing mechanisms embedding the exit (or, deferral) option are developed and examined from the viewpoint of the renter as well as the subcontractor. From the renter's perspective, the value of the flexible outsourcing contract is modeled as a combination of tangible and intangible payoffs. A numerical illustration is used to demonstrate the applicability of the proposed model. The intangible payoff (given applications software alternatives), which is evaluated within the Margrabe's simple exchange option model, is found to increase at higher volatility levels, with the highest option prices (and investment values) tending to occur where the technological divergence between underlying applications environments is the greatest. Therefore, while evaluating rental software alternatives, IT managers should also consider the underlying applications technology in terms of the directional impact of new information.

From the subcontractor's perspective, the value of the flexible outsourcing contract is modeled as a combination of a continuing ASP arrangement and the 'aggregate' option premium. A numerical analysis is conducted using actual data to examine model outcomes in the light of some results gleaned from related financial and real options literature. The value of exit flexibility, calculated as a 'truncated' nested call within a modified version of Carr's compound exchange option model, is less than the commonly designated upper bound. The analysis also reveals that the intermediate exit options can be expressed in terms of the terminal exit opportunity. Hence, one may obtain the outsourcing value by easily 'weighing' the simple option premium for the final decision implementation point with the appropriate 'probability-discount' factor. Further, consecutive options in the nested series exhibit a decreasing
price trend as is observed under other multi-stage options scenarios. Finally, the study develops a theory of optimal exit times for outsourcing contracts that are designed to continue indefinitely into the future.
# TABLE OF CONTENTS

ABSTRACT ........................................................................................................................... iii

LIST OF TABLES ................................................................................................................ ix

LIST OF FIGURES ............................................................................................................ xi

ACKNOWLEDGEMENTS ................................................................................................ xii

CHAPTER 1 INTRODUCTION ........................................................................................... 1

Background ....................................................................................................................... 1
Outsourcing Advantages ................................................................................................. 4
Factors Facilitating ASP Growth .................................................................................... 5
Rent Inconsistency ............................................................................................................ 7
Pricing Paradox ................................................................................................................. 10
Capital Budgeting Imbroglio ......................................................................................... 12
Options Methodology .................................................................................................... 15

CHAPTER 2 THEORY DEVELOPMENT ....................................................................... 17

Flexibility ......................................................................................................................... 17
Simple Option – Fixed Exercise Price ........................................................................... 18
Simple Option – Stochastic Exercise Price ................................................................... 19
Compound Option ........................................................................................................... 22
Outsourcing ..................................................................................................................... 23
Option Pricing Advantages and Assumptions ............................................................... 26
  Lognormal Distribution ............................................................................................... 28
  Arbitrage Free Valuation ............................................................................................ 30
  Fixed Contract Maturity .............................................................................................. 32
  Constant Volatility ..................................................................................................... 32
  Fixed Exercise Price .................................................................................................. 33
<table>
<thead>
<tr>
<th>Chapter</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>SUMMARY OF EMPIRICAL WORK</td>
<td>34</td>
</tr>
<tr>
<td></td>
<td>Capital Budgeting Practice</td>
<td>34</td>
</tr>
<tr>
<td></td>
<td>Real Investment Valuation</td>
<td>35</td>
</tr>
<tr>
<td></td>
<td>Option Valuation Procedures</td>
<td>36</td>
</tr>
<tr>
<td>4</td>
<td>HYPOTHESES</td>
<td>39</td>
</tr>
<tr>
<td></td>
<td>Outsourcing Scenario I – Renter Perspective</td>
<td>39</td>
</tr>
<tr>
<td></td>
<td>Outsourcing Scenario II – Subcontractor Perspective</td>
<td>40</td>
</tr>
<tr>
<td></td>
<td>Outsourcing Scenario III – Exit Times</td>
<td>41</td>
</tr>
<tr>
<td>5</td>
<td>METHODOLOGY</td>
<td>43</td>
</tr>
<tr>
<td></td>
<td>Renter Perspective</td>
<td>43</td>
</tr>
<tr>
<td></td>
<td>Simple Exchange Offer Model</td>
<td>44</td>
</tr>
<tr>
<td></td>
<td>Outsourcing Scenario I Illustration</td>
<td>48</td>
</tr>
<tr>
<td></td>
<td>Subcontractor Perspective</td>
<td>49</td>
</tr>
<tr>
<td></td>
<td>Compound Option Model</td>
<td>51</td>
</tr>
<tr>
<td></td>
<td>Volatility</td>
<td>58</td>
</tr>
<tr>
<td></td>
<td>Volatility Simulation Procedure</td>
<td>61</td>
</tr>
<tr>
<td></td>
<td>Outsourcing Scenario II Illustration</td>
<td>63</td>
</tr>
<tr>
<td></td>
<td>Exit Times</td>
<td>66</td>
</tr>
<tr>
<td>6</td>
<td>RESULTS</td>
<td>70</td>
</tr>
<tr>
<td></td>
<td>Renter Perspective</td>
<td>70</td>
</tr>
<tr>
<td></td>
<td>Subcontractor Perspective</td>
<td>75</td>
</tr>
<tr>
<td></td>
<td>Exit Times</td>
<td>80</td>
</tr>
<tr>
<td>7</td>
<td>CONCLUSIONS</td>
<td>82</td>
</tr>
<tr>
<td></td>
<td>Renter Perspective</td>
<td>82</td>
</tr>
<tr>
<td></td>
<td>Subcontractor Perspective</td>
<td>85</td>
</tr>
<tr>
<td></td>
<td>Exit Times</td>
<td>90</td>
</tr>
<tr>
<td></td>
<td>BIBLIOGRAPHY</td>
<td>91</td>
</tr>
</tbody>
</table>
LIST OF TABLES

TABLE 1 ................................................................................................................................. 46
Financial Option – Outsourcing Scenario Mapping

TABLE 2 ................................................................................................................................. 49
Outsourcing Scenario I Data

TABLE 3 ................................................................................................................................. 51
Ongoing and Flexible SRA Comparison

TABLE 4 ................................................................................................................................. 55
Decision Probability Matrix for a Four-year SRA

TABLE 5 ................................................................................................................................. 64
Parameter Values for Single User Software Prices

TABLE 6 ................................................................................................................................. 64
Continuing SRA Data and Rent Calculation ($T = 4$)

TABLE 7 ................................................................................................................................. 65
Forgone Revenues/ Exercise Price for $t$-year Options

TABLE 8 ................................................................................................................................. 67
Software Development Firm or Option Writer Payoff

TABLE 9 ................................................................................................................................. 68
Subcontractor or Call Option Writer and Put Option Holder Payoff

TABLE 10 ............................................................................................................................... 70
Preliminary SRA Comparison

TABLE 11 ............................................................................................................................... 71
Net Present Value Calculations (a): Valuation of cash flows expected over $T$ (Year 1)
LIST OF FIGURES

FIGURE 1 .............................................................................................................................. 10
   Software Price Scenarios

FIGURE 2 .............................................................................................................................. 75
   Relationship between $V$ and $p_{12}$ for Different $(\sigma_1, \sigma_2)$ Combinations

FIGURE 3 .............................................................................................................................. 80
   Cumulative DCF Profiles – Backsourcing vs. Outsourcing

FIGURE 4 .............................................................................................................................. 81
   Cumulative DCF Profiles – Backsourcing vs. Switching

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.
ACKNOWLEDGEMENTS

I am eternally indebted to my heavenly Father for providing me with patience and direction. His mercy and blessing are instrumental in bringing this project to fruition.

I am grateful to Dr. Roger Shelor, the chairman of my dissertation committee, for his leadership and support in consummating this study. Dr. Shelor established a viable framework for this study, and patiently ironed out related quirks. My sincere thanks to the other committee members: Dr. James Jiang, Dr. Sylwan Ellis, and Dr. Gary Klein for their consistent encouragement and invaluable contribution.

I am very appreciative of my wife, Mandie, for supporting me in this endeavor. To my son, Chaten Dev, thank you for your smile that acted as an ‘anti-depressant’ on many occasions. I thank my parents, Mrs. and Justice Anil Dev Singh, my grandparents, Mrs. and Justice Jaswant Singh, my Godfather, Raja Balwant Singh, my sisters, and other paternal/ maternal family members who provided unwavering support from thousands of miles away. I also thank my wife’s family for their understanding. Finally, I am grateful to those who petitioned God in their prayers for the successful completion of this project.
CHAPTER 1

INTRODUCTION

Background

Software can be construed as a necessary mechanism to generate an information good. As in the case of digitized output, software development requires substantial initial outlay without entailing significant marginal costs for reproduction and distribution. Therefore, the appropriate method of pricing software is assessing the dollar net benefits expected by end users from utilization – a value-based as opposed to a cost-based approach (Shapiro and Varian, 1999).

The value-to-ultimate-consumer method of pricing is evident in outsourcing contracts that provide for applications software renting. In an Internet-powered economy, subcontracting has emerged as an alternative to purchasing software applications outright because it enables renting the shrink-wrapped applications as well as related services from the developing firm or a licensed third party (Carter, 2000). Renting has been likened to outsourcing the company’s information technology (IT) department or service so that systems, applications, and subsequent support reside with an independent entity organized to provide such services (Fox, 2000). ‘Hosting’ is sometimes used in reference to ‘renting,’ although the former entails a higher level
of investment in systems and personnel on the part of the subcontractor or service provider. Under regular renting, a firm may actually hire the applications software under some lease-type agreement, such that the firm accepts responsibility for software installation and subsequent upgrades. By virtue of such an agreement, the firm must also support a staff of highly skilled personnel trained to troubleshoot computer-networking, systems-administration, and applications-deployment issues.

Renting applications as opposed to purchasing expensive shrink-wrapped or packaged software solutions is fast becoming an attractive proposition (Vellotti, 2001). Business firms are increasingly looking to a fairly recent form of subcontracting, namely, the Applications Service Provider (ASP) mechanism for managing copious information and streamlining complex processes (Maselli, 2000a; Torode and Follett, 2000). An entity that “offers an outsourcing mechanism whereby it develops, supplies, and manages application software as well as hardware for its customers,” is known as the ASP (Holohan, 2000). According to the International Data Corporation, global spending for ASP offerings in 2004 is expected to reach $7.8 billion, a fifty-two percent compound annual growth rate over five years (Carter, 2000). A more optimistic Gartner Group predicts that the worldwide revenues in the ASP market will grow by approximately $24 billion by the end of 2004 (Hall, 2000).

The integration of the Internet into enterprise resource planning strategies has fundamentally redefined the IT landscape (Violino, 2000). The opportunities that the Internet affords for electronic commerce, information exchange, supply-chain management, and economies of scale and scope have grown at an astounding pace in the
last decade. The business world is recognizing the ASP approach as a practicable means of capitalizing on these opportunities by delivering related solutions to end users (Wittmann, 2000). For example, IT solution providers, with intent to tap into the profit potential generated by electronic commerce, are rushing to provide Internet-managed and delivered services, from web-based application hosting to custom application development and management.

The fact that the ASP arrangement has engendered a revolution in corporate IT development by transforming software from a packaged off-the-shelf product to an on-line virtual service, investment in such software outsourcing mechanisms continues to grow in importance. This study serves to establish a sound mathematical and theoretical foundation for evaluating flexible software rental agreements by incorporating a real options framework into the traditional capital budgeting technique. Both subcontractor and renter perspectives are examined to develop and illustrate valuation of option-embedding outsourcing mechanisms. Results of the analyses provide decision implications for IT managers.

The remaining sections of this chapter describe outsourcing-related issues. Chapter 2 provides an overview of the relevant financial and real options theories, models, and assumptions. Chapter 3 discusses empirical work in real options. Chapter 4 develops the hypothetical scenarios under software outsourcing or renting. Chapter 5 explains model development procedures, and presents numerical illustrations under two hypothetical outsourcing scenarios. Chapter 6 contains the empirical results of the analyses. Chapter 7 concludes the study.

1 Since an ASP is, in general, a subcontractor, the terms, ‘ASP’ and ‘subcontractor’ has been interchangeably used throughout this study. The firm receiving contractual service from an ASP is referred,
Outsourcing Advantages

Application hosting offers a convenient way of renting otherwise commercially available or shrink-wrapped applications software over the Internet or via leased lines to business organizations that aspire to be on the cutting edge of technology but lack the core IT competencies (Violino, 2000; Wittmann, 2000). With centralized application hosting, small and mid-sized firms acquire an affordable means of providing employees scattered across the enterprise with the same premier ERP tools that were once exclusively available to Fortune 500 companies (Keegan, 1999; King and Cole-Gomolski, 1998). Larger companies facing IT staff scarcity and time pressure are also beginning to view subcontracting as a suitable means of implementing and maintaining complex IT projects (Maselli, 2000b; Violino, 2000; Whiting, 1999).

Another significant benefit from outsourcing IT functions and adopting a rental model is the savings that manifest in reduced capital spending. The advent of applications hosting shifts IT resource costs (including software/hardware installation and technical know how) to the subcontractor. The renter need not absorb substantial implementation costs, such as hardware, training, and installation expenses, which would ordinarily accrue to an organization with an internal IT department (Schmerken, 2000; Torode and Follett, 2000). The rental alternative allows the renter to spread out its ‘total investment’ (including ongoing maintenance expenses) toward outsourcing IT into manageable (usually, monthly) payments (Holohan, 2000; Torode and Follett, 2000). Since the outsourcing alternative is directly and easily scaleable (Maselli, 2000b), economies are also observed by eliminating the need for continual expansion of the renter’s internal IT
infrastructure. As the average cost of operations diminishes, the renter can allocate the available savings to leverage its own service quality (King and Cole-Gomolski, 1998) and gain a competitive advantage in the market (Torode and Follett, 2000).

A changing economic climate spearheaded by technological innovation demands that a business maintain a flexible IT infrastructure with an option to switch application environments in the future, if necessary. An outsourcing mechanism, taking on the deployment, hosting, managing, and application access needs of the end user, can capture flexibility within its service offering (King and Cole-Gomolski, 1998). Centralized application deployment facilitates client migration from one software version to another without risking downtime or obsolescence (Schmerken, 2000). New functionality, which is easily introduced in a matter of days with each software update of a rental application, may prove cost prohibitive and slow-to-emerge for a comparable off-the-shelf application. Outsourcing not only affords the end user time and opportunity to focus constrained resources on its core business, but also decreases time to market by providing rapid delivery of IT solutions (Laberis, 2000).

**Factors Facilitating ASP Growth**

Although subcontracting, as an ASP service offering, is fairly new, the market for rental applications is expected to mature at an increasing pace (Hall, 2000; White Paper, 2000). There exist several incentives primarily within the business-to-business IT marketplace for the ASP to provide a flexible outsourcing mechanism. One, the demand for IT professionals has outpaced supply prompting businesses to implement systems by taking advantage of strategies that essentially rule out the traditional practice of hiring high-priced skilled workmanship (Laberis, 2000; Maselli, 2000b; White Paper, 2000).
Two, a significant customer base of small and mid-sized businesses, which was previously excluded from sales calls by large software vendors, now serves as a viable recipient for IT solutions (Fox, 2000; Keegan, 1999; Schmerken, 2000). Moreover, new and upcoming companies intending to deliver integrated IT solutions may encounter some difficulty in pushing their products through either channel or direct sales to reach a certain class of customers. Obscure startup companies are neither privy to the market reach, nor known for the credibility that larger, more mature software firms seem to enjoy (Mateyaschuk, 2000; Schmerken, 2000; Torode and Follett, 2000).

Three, Fortune 500 companies are pushing to augment their respective IT operations since a rapid deployment of technology is indispensable to business success in an Internet economy (Hall, 2000; White Paper, 2000). Moreover, a growing number of financially entrenched organizations are experiencing increased margin pressure from a precipitous rise in competition in their respective market segments (Wittmann, 2000). Therefore, reducing the total ownership cost of software applications while enhancing the speed to market is becoming crucial to bottom-line sensitive businesses (Ulfelder, 2000; White Paper, 2000). Four, the software industry is experiencing a shift from one-time 'buy-it and forget-it' philosophy to time-based billing of software and services because of the increased affordability (ability to amortize costs of implementing IT solutions) offered by the ASP model (Violino, 2000; Whiting, 1999). Further, the software release cycles are following a continual downward shift, which implies that new upgrades replace 'obsolete' software at an increasingly faster rate (White Paper, 2000). An upgrade may incorporate not only additional monetary investment in new software and hardware, but also skilled human capital for software/hardware installation and troubleshooting.
related problems. *Five, organizations frequently desire operationally simple and easy-to-implement business solutions with the least amount of time lag. Knowing customer preferences is integral to the success of any business, but compiling comprehensive information about customers within one data source and making it readily accessible to Sales, Marketing, and Customer Service is an all-too-familiar problem. In deploying mission-critical enterprise-wide applications, the ASP not only provides economical access to core services, but also forms a bridge between the recipient and its customers without significant downtime (White Paper, 2000; Wittmann, 2000).*

**Rent Inconsistency**

Although outsourcing renders major benefits (such as cost savings, attractive cash flow, extensive knowledge base, rapid deployment of cutting edge application environments, and flexibility), it suffers from a significant drawback -- valuation inconsistencies. The evolution of the ASP from an application-centric service provider to an aggregator of multiple services (including systems integration) that can be customized to meet the needs of the renter has been an ongoing phenomenon within the dynamic ASP industry (Holohan, 2000; White Paper, 2000). The by-product of this process of growth and change is a steady emergence of sundry ASP types, each catering to the specific IT needs of the end user. Further, the ASP industry is in its infancy (Ulfelder, 2000; Whiting, 1999) with numerous firms entering the market (Hall, 2000) expecting to reap profits by satisfying a niche or market segment (Wittmann, 2000). One downside to the consistent influx of firms into the ASP market is the increased difficulty in differentiating the services of one firm from those of its rivals since competing firms tend to mimic the market behavior of industry leaders. Price relative to the degree of
customization of rental software applications and quality of services offered provides the ASP with the ultimate means of differentiation in a fiercely competitive market (Violino, 2000). Naturally, establishing appropriate software rental values is crucial to the survival of a value-maximizing ASP².

With the preponderance of ASP offerings, the valuation of IT projects dealing with acquisition and development (of, especially, software applications) is teeming with new challenges. Since the explosion of ASPs on the IT horizon, the primary focus of the software outsourcing industry has been on consumer-defined concepts of value. In the past five years, a medley of ASP types that attempt to deliver outsourced applications based upon certain end-user activity levels have emerged. The end-user measurement of value is a consequence of the selected ASP type that is likely to vary from one hosting firm to another. As a result of disparate end-user risk and benefit assessments regarding software outsourcing alternatives, pricing discrepancies exist among the variants of the same ASP type (Torode, 2000). Moreover, pricing inconsistencies are heightened due to the likelihood of increased fluctuations in end-user cost structures over the long-term investment horizon (required by most outsourcing contracts) for at least three reasons.

First, the rental mechanism involves a legally binding agreement between the ASP and the end user, whereby the end user is locked into paying one price (in general) for an extended period of time. In addition to the possibility of being charged a disproportionately higher price than that established in a perfectly-competitive market for

---

² A traded firm must maximize the net market value of its shares if unanimity exists together with perfect competition in the market for products characterized by spanning (Grossman and Stiglitz, 1977). In contrast, a sole proprietor maximizes personal satisfaction by making operating decisions that equate marginal utility derived from consumption with that obtained from fractional reduction in general purchasing power (Jensen and Meckling, 1976). Based on equilibrium unanimity theorems, firm-value maximization is the globally optimal decision because it simultaneously maximizes personal wealth regardless of individual preferences (DeAngelo, 1981).
a given level of technology, the renter may be subjected to dissatisfactory service month after month until contract expiration. Thus, a rental model that obligates an end user into a long-term partnership with an ‘unreliable’ ASP is likely to cost the price-conscious and technologically-dependent outsourcing organization in terms of operational inefficiency and forgone revenues (Whiting, 1999). The opportunity cost of remaining with a single ASP that provides a rigid applications software configuration is compounded especially given a market characterized by rapid technological obsolescence.

Second, continuing fragmentation and specialization of services in the ASP market poses a predicament to the organizations that seek to outsource IT operations. Due to a growing number of vertical software vendors adopting the ASP model, the IT-deficient end user is left to either coordinate the activities of multiple ASPs or select certain applications for outsourcing while ‘insourcing’ the remainder with the view to implementing several different IT functions (Hall, 1999). In both the single- and the multiple-ASP scenarios, the cost to a cash-strapped renter for deploying IT solutions becomes increasingly variable, especially over the long haul.

Third, since the number of ASPs is expected to shrink in the future through shutdowns and consolidations (DeBellis, 2000; Ulfelder, 2000), the risk generated by the anticipated mergers-and-acquisitions environment can present the recipient with increased opportunity costs of endorsing long-term rental contracts. Successful ASP startups tend to become attractive takeover targets, which further intensifies consolidation risk as well as discourages crucial long-term investment in rental applications (Flanagan, 2000; Ulfelder, 2000).
**Pricing Paradox**

The problem of resolving rent inconsistencies observed in the ASP market may be exacerbated by the failure of practitioners to discern 'hidden' variability in applications software prices. Software pricing is somewhat of a paradox. In general, technological improvement renders products and services based upon the current body of knowledge conveniently obsolete and relatively inexpensive. A cursory observation of the software industry may induce one to succumb to the notion of continually declining prices in the face of increased software innovation. The observation is syllogistically flawed because one tends to view the output (i.e., applications) resulting from software-development undertakings as isolated and independent rather than sequential. Thus, prices of applications seem to exhibit a consistent downward bias as the corresponding original software versions age over time. In fact, if latest versions entering the market are not touted as "new" but instead recognized as augmented products or services, software prices would probably demonstrate the converse, at least, in the short run. Figure 1 illustrates that both rising and falling software prices are probable.

![Figure 1-a](image1-a.png) ![Figure 1-b](image1-b.png)

**Figure 1. Software Price Scenarios**

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.
The demand for and supply of a certain applications software package determine the market price $P_E$ and the corresponding equilibrium quantity $Q_E$ (Figure 1-a). The software development firm plans to consolidate its presence in the market by launching an augmented version of the original package once $Q_0$ units of the previous version are sold. Since the newer package requires additional research and developmental, administrative, and promotional expenditures, the associated supply curve is established to the left of the original curve, thereby fixing a higher price $P_F$ for the augmented package. Moreover, the augmented version is relatively cheap, given that the previous version is likely to produce declining marginal benefits (of service, such as, troubleshooting assistance and actual functionality) per additional dollar spent. As a result of added innovations and improved functionality, the augmented version is successful in not only effecting old-customer conversions, but also attracting new clients. Increased demand causes the software price to rise further to $P_G$ (Figure 1-b). However, once the market absorbs excess demand and economies of scale result, the software price decreases to $P_H$.

Figure 1 delineates one possible scenario for the existence of volatile software prices that may, otherwise, appear to follow a continually declining and predictable path. Since IT market demand and supply conditions are governed by future technological innovations and market pressures to a large extent, software prices are likely to be unpredictable. The volatility in software prices, as observed in other asset prices, is associated with uncertainty or risk. As with Fama’s theory of financial market efficiency, information (primarily, pertaining to technological innovations) is a significant input in
the determination of software price movements\textsuperscript{3}. Therefore, if the IT market (characterized by the servicization of software) is informationally efficient, only random innovations may lead to software price fluctuations. If an innovation is anticipated, rival software firms will incorporate the same in their respective applications design, thereby, losing any competitive advantage. The demand and the supply responses of expected innovations precisely cancel each other, and no price change is observed, ceteris paribus. Unless the market fails to anticipate technological improvements in design and deployment, no uncertainty in either the provision or the consumption of software services exists. Clearly, with a completely predictable information set, the risk of obsolescence (the predominant source of uncertainty affecting software applications) is neutralized.

**Capital Budgeting Imbroglio**

Although the ASP mechanism has surfaced as a feasible IT investment alternative (Maselli, 2000b), its continuance as a business solution rests upon the operation of the firm deploying the model. The rental services are liable to be terminated if the ASP’s operations are disrupted due to bankruptcy or consolidation. Due to the fact that numerous ASP contracts compete on the basis of pricing in a dynamically charged IT environment, the problem of selecting an appropriate outsourcing solution poses a veritable challenge to the decision maker. Evidently, at the core of pricing inconsistencies observed in the market for rental applications software lies the more fundamental issue of investment valuation.

\textsuperscript{3} See Fama (1970) for an explanation of market efficiency.
One obvious method for assessing rent on outsourced software is the traditional capital budgeting technique akin to the static Net Present Value (NPV) or Discount Cash Flow (DCF) analysis. The standard NPV approach amortizes the software cost together with the annual maintenance (including management, monitoring, and customer support) over a three-to-five-year period using an interest rate that varies from one ASP to another (Cole-Gomolski, 1999; Maselli, 2000a; Turek, 2000). Any rate variation (possibly, manifest in product customization and service quality differentials) has the potential to introduce a large margin of 'error' in software rental values.

The standard NPV analysis does not capture the price for bearing all relevant risk associated with flexible projects undertaken in dynamic settings (Ross, 1995; Trigeorgis, 1993b). An estimated risk-adjusted discount rate used in common NPV scenarios to express expected cash flows on a present value basis awards no attention to changing future valuations of either the project payoff or the embedded option(s). Since technological innovations in rental applications development and deployment are likely to occur over time, the static hurdle or discount rate does not appropriately capture the uncertainty risk inherent in the opportunities underlying ASP contracts. Understandably, the use of static NPV approach to value rental software fails to quantify the worth of embedded opportunities (or potential benefits) derived by the recipient from outsourcing. As a result, the ASP is likely to understate software rents, and inadvertently sub-optimize firm value. Moreover, in the case of option-laden investments, where appropriate discount rates are practically impossible to assess (hindering application of the traditional capital budgeting technique) over different planning horizons, IT practitioners are known
to make decisions based on "gut-feeling" and qualitative rules as opposed to sound quantitative reasoning (Taudes, 1998).

A well-known fact in the capital budgeting field is that every investment project competes with itself delayed in time (Ross, 1995). For example, the decision to outsource IT applications at the present time vies with the same outsourcing alternative that may be exercisable at a more favorable time in the future. The fact that both the decision to outsource today and the opportunity to invest in the same rental mechanism at a later date are mutually exclusive conflicts with an essential requirement of the NPV rule. Specifically, the caveat fundamental to the standard capital budgeting approach to investment selection requires that a project (upon meeting the positive NPV criterion) be undertaken only if its implementation does not preclude any other investment (Ross, 1995). A current positive NPV assessment does not negate the likelihood that the project's future payoff may change in the aftermath of adverse information, thereby necessitating a possible negative NPV at some time \( t \) in the future. Similarly, a negative NPV at the present time may eventually become positive at \( t \). No appropriate measure seems to be available within the standard DCF approach to reflect the impact of new information on project payoff (Benaroch and Kauffman, 1999).

When traditional capital budgeting evaluation techniques are employed to facilitate IT decision making, practitioners tend to make the erroneous assumptions of project rigidity and reversibility (Taudes, 1998). The evaluation results of a passive DCF technique are "cleaner" if investment policy is taken as fixed from the onset of the planning horizon and capital expenditures are presumed recoverable in worse-than-expected scenarios. Kumar (1996) contends that the NPV technique is inflexible because
it does not allow changing the investment pace or terminating the investment to escape unfavorable economic conditions.

‘Mispricing’ attributable to the use of the standard NPV model in outsourcing evaluations may be validated by the existence of variability in rental values assessed for the same software applications (Fonseca, 2000; Torode, 2000). The differential pricing that emerged in the software rental market perhaps resulted in suboptimization of shareholder wealth contributing to the demise of financially vulnerable ASPs. The search for a viable means of pricing outsourced applications software has acquired new significance with the bursting of the ‘dot-com bubble.’

**Options Methodology**

Given the drawbacks of the standard NPV analysis, particularly, in the area of IT project evaluation, the problem of sub-optimal valuation may be circumvented by evaluating rental software within a real-options pricing framework (Dos Santos, 1991; Taudes, 1998). Although recent IT literature (Benaroch and Kauffmann, 1999 and 2000; McGrath, 1997; Panayi and Trigeorgis, 1998) presents a number of scenarios where opportunities embedded in IT investments are mapped onto financial options theory, none of these cases encompass the evaluation of flexibility inherent in ASP offerings. This study presents the first practical application of the option-enhanced traditional NPV model to valuing a software rental mechanism (embedding exit flexibility) from the perspective of the renter as well as the subcontractor. The problem of valuing an appropriate outsourcing mechanism is recognized and addressed by exploring the nature of risk associated with applications software development and deployment. A dynamic option valuation measure that appropriately compensates for risk relevant to pay-for-
service or rental software contracts is introduced. In the course of formulating an appropriate options approach, this study attempts to make several contributions to the existing investment-valuation literature. One, it conceptualizes outsourcing flexibility as simple and sequential options written on the underlying applications software. Two, it develops numerical procedures to value single and multi-stage optionality. Three, it develops a theory of optimal exit times for outsourced software when related service contracts are designed to continue indefinitely into the future.
CHAPTER 2

THEORY DEVELOPMENT

Flexibility

The information systems (IS) literature proposes the modeling of embedded flexibility underlying investments, especially those pertaining to IT, within a real-options framework (for example, Dos Santos, 1991, and Taudes, 1998). The work on real options in the IT decision-making field is based upon financial options research. A brief overview of the financial options theory is provided in the following section.

Financial Options Paradigm

Options pricing (contrary to general financial pricing) theory investigates the issues concerning the valuation of derivative assets. Cox and Ross (1976) define a derivative asset as “a security whose value is explicitly dependent on the exogenously given value of some underlying primitive asset.” Call and put options are two commonly traded derivatives written on an underlying share of stock. Black-Scholes (1973) first expressed the call option value \( c \) as a function of five parameters – the underlying stock price \( S \), the fixed cost of purchasing the underlying stock (i.e., the option’s exercise price) \( X \), the instantaneous variance of the stock price returns \( \sigma^2 \), the option’s time to maturity \( T \), and the risk-free interest rate \( r \). The call value can be
written in a general functional notation:  

\[ c = f(S^+, X^-, \sigma^2^+, T^+, r^+) \]  

(1)

The sign following each parameter in the above functional form indicates the direction of relationship between the call option value and the underlying parameter. At maturity, the call option yields \( \text{max}[S^*-X, 0] \), where \( S^* \) is the terminal date stock price. Black-Scholes assume that the underlying stock price returns follow a lognormal diffusion process, i.e., a continuous process characterized by rapid changes in direction (Cox and Ross, 1976):

\[ \frac{dS}{S} = \alpha_s \, dt + \sigma_s \, dz_s \]  

(2)

such that \( dS/S \sim \Phi(\alpha_s, \sigma_s) \), where \( dS/S \) is the percentage change in the stock price, \( \alpha_s \) is the instantaneous expected rate of return that measures the drift in the random walk of the stock price through time, \( \sigma_s \) is the instantaneous standard deviation of the rate of return (i.e., stock price volatility, assumed constant in percentage terms or invariant with respect to time), \( dt \) is a small increment in time, and \( dz \) is the increment to a Wiener process (Hull, 1997).

**Simple Option – Fixed Exercise Price**

Given the return distribution assumption, Black-Scholes derive a closed-form option valuation solution by essentially constructing a hedge portfolio (comprising a purchased position in the stock and a sold position in the call on the stock) that is continuously re-balanced to return the risk-free interest rate (Black, 1989). The Black-Scholes option pricing model (OPM) in final form is given by:

\[ c = S \, N_1(d_1) - e^{-rt} \, X \, N_1(d_2) \]  

(3)

Where,
call option premium (value of the right to purchase the underlying stock in the future), or option contract price that the option holder (buyer) pays to the option writer (seller) irrespective of the terminal payoff

N_1(\cdot) : cumulative univariate normal distribution function

N_1(d_i) : probability that a normally distributed random variable is less than \( d_i \), where \( d_i \) is expressed by:

\[
N_1(d_i) = \frac{\ln(S/X) + \tau (r + \sigma_s^2/2)}{\sigma_s \sqrt{\tau}}
\]  \hspace{1cm} (4)

N_1(d_2) : probability that a normally distributed random variable is less than \( d_2 \), where \( d_2 \) is expressed by:

\[
d_1 = \sigma_s \sqrt{\tau}
\]  \hspace{1cm} (5)

\( \tau \) : option’s time to maturity expressed as the difference between the pre-designated expiration date \( T \) and the analysis date \( t \)

\( X \) : stated strike price, or the cost incurred by the call option holder to purchase the underlying stock and exercise the option at expiration

\( r \) : riskless expected rate of return, or the annualized continuously compounded interest rate on a US treasury bill that matures at the same time as the call option

\( \sigma_s^2 \) : instantaneous variance of return on the stock

\( S \) : current stock price

**Simple Option – Stochastic Exercise Price**

Black and Scholes OPM, in its most rudimentary form, facilitates the option premium calculation by holding \( T, X, r, \) and \( \sigma_s^2 \) fixed over the planning horizon. Fischer (1978) relaxes the constancy requirement for the exercise price \( X \), and derives a modified
OPM by creating a riskless investment position consisting of the call option $c$, the underlying asset $S$, and some hedge security $H$ with return characteristics that exactly mirror changes in the option's exercise price $X$. Fischer provides the following options formula given a stochastic exercise price.

$$c = S N_1 \left\{ \frac{\ln(S/X) + \tau (\eta + \sigma^2/2)}{\sigma \sqrt{\tau}} \right\} - e^{-\delta T} X N_1 \left\{ \frac{\ln(S/X) + \tau (\eta - \sigma^2/2)}{\sigma \sqrt{\tau}} \right\}$$

(6)

Where, all variables are as previously defined except:

- $\sigma^2$: instantaneous variance of the proportional change in $S/X$, which is expressed by:

$$\sigma_s^2 = \rho_{sx} \sigma_s \sigma_x + \sigma_x^2$$

(7)

with

$$\rho_{sx} dt = dz_s dz_x$$

(8)

- $\rho_{sx}$: instantaneous correlation coefficient that relates stochastic changes in $S$ and $X$

- $\eta$: expected rate of return shortfall expressed as the difference between $r_h$, the rate of return on $H$, and $\alpha_x$, the expected rate of change in $X$

- $X$: exercise price (although given at $t$, is unknown at $T$) that follows the diffusion process:

$$dX/X = \alpha_x dt + \sigma_x dz_x$$

(9)

Whereas the Black and Scholes simple call option value is a function of absolute changes in the underlying asset price, the payoff on the derivative purchase opportunity with an uncertain strike depends on proportional changes in the underlying asset price relative to the given hedge security return (Fischer, 1978). In Margrabe (1978), the return differential $\eta$ is eliminated since the call option comprises an opportunity to exchange
one asset for another. Margrabe develops a variant of the Black-Scholes formula for calculating the value of an option to exchange one asset for another within a predefined time interval. The option comprises a call written on the first asset with the second asset's price as the exercise price, and a put written on the second asset with the first asset's price as the strike price. To derive the valuation equation, Margrabe defines the exponential of the risk-free rate as the factor by which the price of a riskless discount bond increases, and further assumes that the value of a discount bond follows a stochastic process until its maturity. The option price depends on the current value of the individual assets being exchanged, the variance-covariance matrix for the rates of return on the two assets, and the length of time before the option is exercised. If \( X_1 \) and \( X_2 \) respectively denote asset values, the option to exchange \( X_2 \) for \( X_1 \) at maturity will yield:

\[
c = X_1 N_1(d_1) - X_2 N_1(d_2)
\]

and,

\[
d_1 = \frac{\ln(X_1/X_2) + \frac{\sigma_2^2 T}{2}}{\sigma_2 \sqrt{T}}
\]

\[
d_2 = \frac{\ln(X_1/X_2) + \frac{\sigma_2^2 T}{2}}{\sigma_2 \sqrt{T}}
\]

\[
\sigma_2^2 = \sigma_1^2 - 2 \rho_{12} \sigma_1 \sigma_2 + \sigma_2^2
\]

where, \( \rho_{12} \) is the instantaneous coefficient of correlation between \( X_1 \) and \( X_2 \), \( \sigma_1^2 \) is the instantaneous return variance of \( X_1 \), \( \sigma_2^2 \) is the instantaneous variance of the proportional change in \( X_2 \), and \( \sigma^2 \) is the instantaneous variance of the proportional change in the ratio of \( X_1 \) and \( X_2 \).

---

4 Under the exchange option scenario, the expected rate of return on the hedge security is equilibrium compensation for risk from appreciation in the exercise price.
**Compound Option**

Carr (1988) combines the models of Margrabe (1978) and Geske (1979) to value a finite series of nested or sequential opportunities as a compound exchange option.

Carr’s pricing equation for two possible exercise dates is defined by:

\[
\text{CEO} \{ \text{SEO}(S, X, \tau_1), qX, \tau_c \} = S N_2 \left\{ \frac{\ln(P/P^*) + \frac{1}{2} \sigma^2 \tau_c, \ln(S/X) + \frac{1}{2} \sigma^2 \tau_c, \sqrt{\tau_c/\tau_1}}{\sigma \sqrt{\tau_1}}, \frac{\ln(S/X) + \frac{1}{2} \sigma^2 \tau_c, \sqrt{\tau_c/\tau_1}}{\sigma \sqrt{\tau_1}} \right\}
\]

\[
- X N_2 \left\{ \frac{\ln(P/P^*) - \frac{1}{2} \sigma^2 \tau_c, \ln(S/X) - \frac{1}{2} \sigma^2 \tau_c, \sqrt{\tau_c/\tau_1}}{\sigma \sqrt{\tau_1}}, \frac{\ln(S/X) - \frac{1}{2} \sigma^2 \tau_c, \sqrt{\tau_c/\tau_1}}{\sigma \sqrt{\tau_1}} \right\}
\]

\[
- qX N_1 \left\{ \frac{\ln(P/P^*) - \frac{1}{2} \sigma^2 \tau_c}{\sigma \sqrt{\tau_1}} \right\}
\]

(14)

Where, all variables are as previously defined except,

**CEO** : compound exchange option with time to maturity \( \tau_c \)

**SEO** : simple exchange option with time to maturity \( \tau_1 \)

**N_2(\cdot)** : cumulative bivariate normal probability distribution function

**P^*** : critical ratio of the optioned asset price \( S \) to the delivery asset price \( X \), which is given by Margrabe’s (1978) exchange option formula with \( X \) as the numeraire:

\[
P^* N_1(d_1) - N_1(d_2) = q
\]

(15)

**q** : quantity of the delivery asset exchanged for one SEO

---

5 A compound exchange option comprises the right to exchange an asset with another option.

6 The bivariate probability evaluated at \( a \) and \( b \) with correlation coefficient \( \rho \), or \( N_2(a, b, \rho) \), can be approximated by a technique described in Drezner (1978).
Outsourcing

Adopting the ASP alternative in lieu of purchasing a shrink-wrapped application (license and media) may be viewed as a first stage IT investment with embedded growth options\(^7\). For example, an opportunity embedded in ASP contracts is manifest in the potential to augment base applications software configurations at a later date. Moreover, an ASP agreement essentially grants the end user the flexibility to repudiate the original contract at expiration and implement an augmented rental applications solution through a rival ASP. This countervailing facet to making the outsourcing decision may also be viewed as a deferral opportunity that originates in the flexibility to postpone deployment of an alternative IT solution until payoff contingencies dictate otherwise.

An outsourcing mechanism may be formulated as a software rental agreement (SRA) that entitles, but not obligates, an end user to terminate the original ASP service on or before contract maturity\(^8\). The renter, essentially, enjoys the flexibility to exit the original SRA under three possible scenarios. One, a competing ASP may lure the end user to migrate IT application requirements to benefit from a technologically advanced offering (i.e., the switching scenario). Two, the end user may employ the exit opportunity by purchasing the retail equivalent of the rental applications software underlying the outsourcing contract with a view to implementing an internal IT solution (i.e., the backsourcing scenario). Three, the renter may terminate original SRA usage altogether, and proceed to develop the necessary software applications indigenously (i.e., the insourcing scenario).

\(^7\) See Kumar (1996) for multi-stage investments, and Taudes (1998) for investments with growth options.
\(^8\) The industry term that is commonly used to describe a service contract is “SLA” – service level agreement. See Vellotti (2001) for a description of performance-level guarantees.
Whereas insourcing or backsourcing may require extensive capital investment in applications software and trained personnel, outsourcing offers an affordable service-based alternative. The compensation for service comprises periodic payments (rent) in lieu of one large lump-sum disbursement (purchase price). A firm will subcontract its IT requirements if the benefit derived from outsourcing outweighs the value of outright ownership. Under the following scenario, outsourcing emerges as the value-maximizing alternative.

\[ V_t - S_t > 0 \]  \hspace{1cm} (16)

Where, \( V_t \) is the value of the outsourcing contract at time \( t \), and \( S_t \) is the market price paid at \( t \) to purchase the applications software package\(^9\).

Or,

\[ V_t > S_t \]  \hspace{1cm} (17)

When a firm decides to subcontract, in essence, it agrees to pay an amount in addition to the base ownership cost over the outsourcing horizon. If the outsourcing premium is \( B_t \), and \( \phi \) is a set of all relevant variables except \( V_t \) and \( S_t \), then:

\[ B_t = f(V_t, S_t, \phi) \]  \hspace{1cm} (18)

Where, \( f(\cdot) \) is a functional notation.

\( B_t \) can also be considered as the renter’s opportunity cost of electing to outsource. Since the benefit expected from outsourcing is a function of the applications software technology impacting system compatibility and functionality, \( B_t \) accounts for the risk of obsolescence. As the obsolescence risk associated with the currently employed applications software increases, the renter generally incurs a greater opportunity cost for its decision to persist with the present outsourcing contract. The market measure of \( B_t \) is

---

\(^9\) Although not directly observed in the market, the outsourcing contract value may be estimated by quantifying the potential benefits that flow to the renter from subcontractor service.
the value of the next best alternative (namely, option to exit the present SRA with the objective of migrating) that is forsaken\textsuperscript{10}.

From the subcontractor's perspective, $B_t$ is the amount of premium that is commensurate with the measure of risk manifest in the renter's propensity to migrate. The direct relationship between outsourcing premium and exit risk is realized through the impact of innovation on software price. Given that the software market is Fama efficient, the absence of software augmentation surprises (zero supply effect) is likely to cause reduced demand and lower price. With increased probability of obsolescence, the exit opportunity becomes more attractive to the renter. Consequently, the subcontractor would require a higher premium to render continued services and compensate for the increased exit risk.

However, if the price paid for uncertainty due to increased probability of obsolescence (the value of flexibility derived from persisting with the current outsourcing contract) exceeds (trails) the next best alternative value, the renter may default on the premium amount\textsuperscript{11}. In such a scenario, the renter would perceive that the outsourcing contract is no more valuable than the ownership position, and would migrate IT applications. Otherwise, the renter will pay the outsourcing premium and defer exit. The value of such outsourcing mechanism is derived within a simple as well as a multi-stage option-pricing approach.

\textsuperscript{10} Migrating is used in a general sense to represent insourcing, backsourcing, or switching.

\textsuperscript{11} $B_t$ is the outsourcing premium (net option price) paid in advance for time $t+\Delta t$. 

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.
**Option Pricing Advantages and Assumptions**

Investment in an outsourcing mechanism, such as an SRA, yields both direct (constant) and sequential (variable) net benefits. Since the opportunity cost of capital of an option embedded in the SRA is a function of time, its dynamics cannot be easily specified by estimating one or more discount rates for different points in time. The options approach appropriately values the state-contingent opportunity risk of SRA-embedded flexibility in the proportional variance of the underlying applications software price. Moreover, a key characteristic of the options model is that the option price can be calculated independent of the expected return on the underlying asset or the investors' attitude towards risk (Black, 1975). If investor risk-return tradeoff has no bearing on in-project optionality, the assumption of risk neutrality can be made to evaluate investments embedding flexibility. For risk-neutral investors, at equilibrium, the expected returns on all assets in the economy must equal the risk-free rate (Abken and Nandi, 1996) to preclude the existence of riskless arbitrage opportunities. Consequently, an SRA may be evaluated by discounting the expected terminal payoff from exercising the embedded option at the risk-free rate. Further, by substituting the risk-free interest for the hurdle rate, the options method serves to prevent the subjective risk preferences of IS managers from entering the SRA evaluation process.

Since the sequential net benefits derived from implementing a flexible applications software configuration have an unlimited growth potential but a zero lower bound, the outsourcing opportunity value seems to depend on the asymmetric distribution of returns that span the investment evaluation period. The options approach determines the value of embedded flexibility by specifically mapping such asymmetric return
distribution. Moreover, a closed-form option-valuation solution (where possible) helps to examine the option's sensitivity to changes in model parameters in a way that serves to test IS managers' instincts about the effect of changing exogenous and endogenous conditions on expected SRA payoff. In addition to facilitating sensitivity analysis, partial derivatives of the closed-form equation of option value can be employed to create hedges against adverse movements in risk-generating model parameters so that the risk-free rate obtains on the final investment position.

Another useful property of the options approach is that it can provide a quantitative estimate of uncertainty characterizing the asymmetric SRA return distribution. Such uncertainty is manifest in the volatility implied by or consistent with a given valuation of embedded flexibility. Although an iterative process, the implied volatility calculation from one investment opportunity can be employed to price another in-project option underlying the same SRA. In summary, the options approach to evaluating an SRA is likely to bring conceptual superiority to the decision-making process by appropriately quantifying embedded flexibility, and establish a systematic framework to identify viable opportunities for migrating software applications.

Given an SRA project embedding flexibility necessary to yield sequential business opportunities, the options pricing approach seems to be the most pragmatic investment evaluation technique. In attempting to assess the worth of an option-laden investment in rental applications software, the end user can optimize its decision by drawing on the conceptual and computational strengths of a tailor-made option-valuation technique.
To realize the advantages from employing a closed-form (or, numerical) options valuation solution in a real-options scenario, the decision maker must first examine the original Black-Scholes presumptions about model parameters and market dynamics. Below, the study, not only explores some assumptions that are the bedrock of Black-Scholes OPM, but also shows that these assumptions may not necessarily prove restrictive in an SRA-investment setting.

**Lognormal Distribution**

Black-Scholes (1973) derive the value of a call option written on the underlying shares of common stock by presuming that stock price behavior can be expressed as a continuous-parameter, continuous-state-space Markov process. Further, the probability distribution of terminal stock prices, given a finite time period, is lognormal. The Markov process suggests that the distribution of possible stock prices at the end of a determinable planning horizon is contingent only on the current value of the stock. The terminal stock price distribution is lognormally skewed implying that the stock can assume any value in the zero-to-infinity range. Both the option and the underlying stock are subject to the same fundamental source of uncertainty regarding the logarithm of the terminal stock price (Hull, 1997).

In the outsourcing case, the assumption that the natural logarithm of the underlying applications software price is normally distributed allows the price to drop (rise) from infinity (zero) to zero (infinity) without becoming negative. If the lognormality assumption fails to hold, pricing biases will emerge causing the embedded flexibility to be under- or over-valued relative to the corresponding Black-Scholes option calculation (Benaroch and Kauffman, 1999). As in Hull and White (1987), such biases...
may be quantified by exactly modeling the application software returns distribution. The calculated price biases can then be used to adjust Black-Scholes option prices to express unbiased values of in-project flexibility.

The presumption that applications software price follows a random walk in continuous time has two important implications for the outsourcing scenario. One, as in the case of common stock (Fama, 1970), only the current price of any shrink-wrapped applications software is relevant for predicting the future. The fierce competition in the IT solutions market (as observed in Whiting, 1999 and Wittmann, 2000) tends to ensure that the present applications software prices impute all past, contemporaneous, and expected developments in technology. Prices tend to fluctuate at random only in response to technological innovation surprises. Two, applications software is a continuously traded asset (Benaroch and Kauffman, 1999). Current software-industry practice dictates that a commercially available applications package be sold (or rented) as a license that is site- or user-specific and may not be offered by the ultimate consumer for resale (Holohan, 2000).

Further, stock prices continuously change with exogenous demand and supply conditions. However, the continuous trading assumption may be unnecessary to obtain the Black-Scholes OPM given that investor behavior is characterized by logarithmic utility, and underlying asset returns distribution is lognormal (Cox and Ross, 1976). Following an observation in Benaroch and Kauffman (1999) that a firm should be interested in market objectivity as opposed to human subjectivity with a view to evaluating IT investments, the study seeks to assess the contribution that SRA-embedded
flexibility would make to firm value as if software applications were in fact continuously traded.

**Arbitrage Free Valuation**

The Black-Scholes OPM generates the value of an option on the underlying asset with known returns distribution function, provided that there exist no arbitrage possibilities (Cox and Ross, 1976). The absence of arbitrage potential within the Black-Scholes framework implies that the weighted sum of the option value and the underlying stock price is dynamically equated to the hedge portfolio value so that no riskless returns arbitrarily flow to the investor at option maturity as a result of the replicating process. Alternatively, without arbitrage possibilities, the investor cannot borrow at the risk-free rate and invest in a risky asset to obtain a riskless profit (Benaroch and Kauffman, 1999). By creating a risk-free hedge comprising the underlying stock and the call option, Black-Scholes finessed the problem of estimating a required return on the derivative asset. For any risk-neutral investment in equilibrium, the expected return is the risk-free interest rate.

Further, there invariably exists an efficient portfolio of assets (and/or options) such that any complex contract (or hedge) may be created as a combination of calls and puts on this portfolio. If such a contract is formulated, given that the underlying asset obeys some stochastic process with a known distribution function, the composition of the hedge portfolio becomes a trivial consideration in the option valuation procedure. The single matter of crucial importance is that whether a hedge can be constructed by combining the underlying asset and the derivative instrument (spanned with or without other assets or options) in offsetting long and short positions. Once the hedge is formed,
the risk-neutral equilibrium model may be applied to show that the option value is simply the expected value of $\max [S^* - X, 0]$ discounted at the risk-free interest rate (Cox and Ross, 1976). If the underlying stochastic process is not clearly delineated, but the embedded option is spanned by or rendered equivalent to a portfolio of other assets or options, only the minimum bound on the option value can be obtained as follows:

$$c \geq \max [S - Xe^{rT}, 0]$$  \hspace{1cm} (19)

Moreover, given that the underlying asset obeys some stochastic process with a known distribution function, and the risk-neutrality assumption holds, the formation of a perfect hedge is no longer an imperative condition to evaluating the option (Merton, 1976). The fact that the constitution of a riskless hedge need not be known (given an efficient fund of assets and underlying stochastic process), or an exact hedge need not be formed (given risk-neutrality and underlying stochastic process) to derive option value extends the scope of derivative pricing to include valuation of flexible options embedded in IT projects. Taudes (1998) as well as Benaroch and Kauffman (1999) use the options pricing approach to value in-project optionality by assuming a risk-neutral world, although certain risk characteristics (such as, managerial acceptability) of the IT investment can not be completely diversified by holding the underlying asset-option portfolio. In the outsourcing scenario, the assumption of risk neutrality (given a lognormal distribution of the applications software price) serves to eliminate the IT managerial subjectivity (i.e., project-specific discount rate) from the evaluation process thereby precluding arbitrage possibilities.
**Fixed Contract Maturity**

In the standard Black-Scholes model, the option on the underlying stock can be exercised only on a given date that corresponds with the option’s maturity. Within the context of a time-bound SRA, the original outsourcing contract may be designed to expire in $t$ monthly periods (or, $T$ years) from its inception date. Therefore, the option inherent in such an SRA also has a fixed life of $T$ years. As renter of software applications and holder of embedded opportunity, the recipient avoids sacrificing any net benefits of outsourcing because the option cannot be exercised before its expiration. In terms of financial options, this situation is similar to dividend protection on the underlying stock. An exception to fixed maturity can be found in the case of a ‘perpetual’ SRA (embedding an American-style option) wherein the option automatically matures with the shut down of either the subcontractor’s or the recipient’s business.

Further, the uncertainty underlying the embedded option is proportional to the square root of $T$. The longer the time to option maturity, the greater is the uncertainty about future technological changes that are reflected as random variability in software prices. Information regarding potential technological improvements in applications software development and delivery will naturally contribute to the underlying price volatility as value-maximizing firms chase after emerging innovation surprises. Intuitively, as the expiration date for the SRA-embedded option tends to coincide with future conversion opportunities, the value of in-project flexibility is likely to increase.

**Constant Volatility**

Under the basic Black-Scholes OPM, volatility is instantaneously uncorrelated with the underlying stock price, and does not change over time. However, the
instantaneous variance of stock returns not only seems to vary across time in a random fashion (Johnson and Shanno, 1987), but also tends to be correlated with the stock-price level (Beckers, 1980). In such a case, (3) generates prices that are higher or lower than the corresponding Black-Scholes OPM values under constant variance. Clearly, as expected outcomes become more certain and volatility diminishes with the length of the planning horizon, the embedded option becomes less valuable to the holder of the IT investment (Benaroch and Kauffman, 1999). The assumption that software price changes have a non-random and deterministic variance allows simple numerical illustrations regarding SRA flexibility value without undertaking tedious calculations.

**Fixed Exercise Price**

The holder of a call in the Black-Scholes scenario knows with certainty at zero time to maturity the price required to be paid to the option writer on the expiration date for the underlying asset. Thus, only the variance of the underlying stock’s return is relevant to value the call. As noted in Kumar (1996), however, evaluation of IT investments that offer the right to make subsequent investments contingent upon the preceding projects requires the assumption of a stochastic exercise price. In such a case, the option value is a joint function of the variance parameters that define the first-stage and second-stage investment payoff distributions. An application of the OPT to outsourcing mechanisms with a stochastic exercise price can then be made by using Margrabe’s (1978) variant of the basic Black-Scholes OPM.
CHAPTER 3

SUMMARY OF EMPIRICAL WORK

Capital Budgeting Practice

The value of an investment project is derived from its “in-the-money” (or net) worth as well as the value of explicit and implicit options embedded in the project (Ross, 1995). The DCF or NPV analysis, measures value attributable to cash flows expected from undertaking the project today without regard to embedded optionality. Therefore, the traditional capital budgeting technique may not adequately serve as a ‘barometer’ of investment value. The weaknesses of the standard NPV model are noted among others in Dos Santos (1991), Ross (1995), Kumar (1996), Taudes (1998, 2000), and Banaroch and Kaufmann (2000).

The traditional capital budgeting technique performs well in the IT field so long as the project payoffs are taken as fixed without the possibility of either accelerating expected cash flows, or effecting premature termination once the investment is deployed (Kumar, 1996; Taudes, 1998). Common NPV precepts tend to give inconclusive results especially since all investment instruments in an uncertain economy exhibit option-like characteristics. As such, managerial decisions that are incumbent upon the associated project payoffs affect the residual set of accessible investment opportunities (Ingersoll and Ross, 1992).
Options analysis, unlike the static NPV approach, helps to measure the value added due to discretionary rights or opportunities (such as, abandonment, deferral, or expansion options) embedded in a project by quantifying the amount of uncertainty associated with underlying asset prices (Leuhrman, 1998). Moreover, Benaroch and Kauffman (2000) illustrate that a real options approach permits consistency in IT investment timing option valuation despite the nonexistence of any trade for the underlying investment opportunity, and conclude that the application of traditional capital budgeting methods instead would lead to erroneous implications. This study establishes the basis for employing a real-options framework to evaluate flexibility inherent in outsourcing IT applications under the terms of an SRA.

**Real Investment Valuation**

In recent years, a plethora of studies have attempted to value IT investments as real options to overcome the limitations of the passive NPV model. Benaroch and Kauffman (1999) present an options approach to analyzing a real-world investment opportunity manifest in the deployment of point-of-sale debit services by a shared electronic banking network. Triantis and Hodder (1990) develop an analytical valuation formula to price multiple European options, each delineating the opportunity to manufacture an optimal product mix, as part of the procedure to evaluate an investment in flexible production facilities. Taudes (2000) employs options analysis to value opportunities offered by a software platform for implementing applications that incorporate novel functionality. Such applications-enabled functionality has the potential to generate a stream of future benefits in the form of productivity improvements and logistical savings. Since no prior investigative study attempts to assess the multi-stage
flexibility embedded in the emerging practice of outsourcing or renting applications software, the proposed research serves to complement the real options literature.

Recent research in finance and decision sciences suggests that in-project flexibility has value especially if uncertainty persists over the investment horizon (Kumar, 1999; Panayi and Trigeorgis, 1998; Smith and Nau, 1995). Pindyck (1993) values the right to continue building a partially constructed nuclear power plant as a function of the source and the level of cost uncertainty. Kumar (1999) conceptualizes the increased flexibility in uncertain decision scenarios resulting from IT investment in decision support and executive information systems as a change in the value of an options portfolio. Further, the value of a decision scenario tends to increase as the range of possible decision alternatives that facilitate revision of managerial strategy in response to new information expands. In this study, two disparate decision-making circumstances with varying degrees of uncertainty are developed – one corresponding to the perspective of the outsourcing firm, and the other representing the viewpoint of the renter.

Option Valuation Procedures

While pricing of financial options involves valuation of contingent claims to an underlying asset, real-options analysis seeks to quantify the opportunities embedded in IT investment projects (Taudes, 1998). Benaroch and Kauffman (1999) use the traditional Black-Scholes formula with a “floating” expiration period to value a deferral option inherent in the timing of an IT project’s implementation. Option-price calculations for expiration dates specified in semi-annual increments revealed that the deferral-decision value first increased with time to option maturity and then decreased beyond a certain implementation point. Dos Santos (1991) proposes the application of Margrabe’s option
pricing model (1978) to evaluate investment in innovative technology that provides the opportunity to incorporate the novel technique in prospective IT projects. Taudes (1998) employs a number of option pricing methods (specifically, Carr’s sequential exchange options model, Margrabe’s exchange offer formulation, Geske’s compound options approach, and Black-Scholes OPM to examine the value of a flexible IT platform that offers several software growth options in the form of embedded IS functions. This study represents an original attempt to evaluate an investment in outsourced software as a combination of the traditional NPV technique and Margrabe’s simple exchange offer approach as well as Carr’s compound exchange option model.

The ability of a firm to prepare itself for strategic expansion frequently depends on the nature of initial investment decisions. The flexibility inherent in IT projects facilitates “conversion” of base IS functions in response to uncertain business conditions as a means of fostering sequential IT investments. Such flexibility is often characterized by an intangible payoff that cannot be specified in terms of a properly delineated expected cash flow stream (Taudes, 1998). Further, the net benefits expected from a project with an embedded option tend to be contingent upon economic outcomes following the implementation of the project, and therefore, are likely to vary with time (Benaroch and Kauffman, 1999). In contrast, the direct net benefits of deploying the base project can be specified as invariant net cash flows that are expected to occur with project utilization. Taudes (1998) notes that with IT projects embedding novel software growth options intangible benefits may exceed tangible ones. The proposed research incorporates any such easily overlooked intangible value by expressing decision-implementation
opportunities intrinsic to outsourced software as a function of the state-contingent option payoffs.
CHAPTER 4

HYPOTHESES

Outsourcing Scenario I – Renter Perspective

A firm is considering deployment of a completely integrated accounting applications software system. The firm lacks adequate cash to make a one-time purchase decision regarding canned software\textsuperscript{12} that may not meet future managerial expectations or acceptance standards. Moreover, the firm does not possess core IT competencies to develop the requisite accounting software solution internally. However, there exist two distinct outsourcing alternatives that could cut costs. Both outsourcing alternatives require four-year contracts. The software applications underlying both outsourcing contracts are also commercially available.

The decision to rent software applications now is, essentially, an investment in the underlying commercially available package with an option to terminate enjoyment of outsourcing benefits on designated future dates. At each contract anniversary date, the firm has the right to exit the original SRA and undertake an alternative that is technologically efficient. For example, the potential business solution may take the form of deploying an alternative ASP offering, or developing applications by

---

\textsuperscript{12} Standardized shrink-wrapped or pre-packaged software that is sold as-is, without being customized to suit end-user operation requirements.
establishing an internal IT department. The firm, thus, seeks to evaluate an outsourcing mechanism that allows it to take advantage of higher productivity gains from deploying a technologically superior outsourcing solution at the end of one through four years from the original SRA’s implementation.

**Outsourcing Scenario II – Subcontractor Perspective**

A firm that develops software applications for the petroleum distribution industry is considering entry into the ASP market. The firm does not market via the retail channel, but has an established sales network that relies mainly on referrals obtained at national trade shows. Although successful in generating sales for its flagship accounting package in the market comprising mid-sized oil jobbers, the firm has experienced some difficulty in luring small businesses (such as, convenience stores). A web-enabled rental version of the applications package is in the offing primarily with the objective of alleviating the current sales predicament. The outsourcing mechanism will enable a small-to-medium business prospect to spread its investment in applications software over a number of months in lieu of making one significant capital expenditure at acquisition.

The firm’s management intends to amortize the current market price of the shrink-wrapped accounting package over forty-eight months to obtain a fixed monthly subscription fee for the corresponding rental applications solution\(^{13}\). The forty-eight month ‘planning’ horizon is selected as the payback period, i.e., the time in which the firm expects to recoup the market price of software as well as make a return on investment. The problem, however, lies in structuring and pricing an appropriate

\(^{13}\) Hereafter, the firm is referred to as the ASP.
The firm is, in essence, faced with two alternatives in designing the rental contract. One design alternative is the static SRA that obligates the end user to rent the accounting applications package for four years at a fixed monthly subscription rate. The other is the flexible or option-laden SRA that charges a 'variable' subscription rate that depends upon the value of embedded opportunity.

**Outsourcing Scenario III – Exit Times**

An ASP introduces a 'perpetual' SRA that offers the opportunity to defer exit indefinitely, or, conversely, exit at any point in time. Such an outsourcing mechanism incorporates flexibility in the form of multiple decision implementation opportunities uniformly dispersed over the service horizon. These decision implementation opportunities may be considered as discrete points (in time) that enable the renter to take the value-maximizing course of action.

For time-bound outsourcing contracts embedding an American option, one must determine the decisive exercise times that yield a payoff greater than that expected upon exit at expiration. The terminal consideration is an important one even for an outsourcing contract that has no stipulated expiration date. As a result of competition from rival software innovations, the value of outsourcing IT applications with the original subcontractor tends to depreciate with time. The rate of decline in contract value depends on whether the subcontractor is proactive and preempts the market with its own software augmentations, or, otherwise, is quick to respond to the challenge imposed by competing technological innovations. In the long term, retail software prices also tend to fall as the rate of obsolescence increases. Therefore, given diminishing outsourcing contract values and applications software prices, the renter is
predisposed to an early exit. The problem is one of identifying optimal exit times within the outsourcing context.
CHAPTER 5

METHODOLOGY

Renter Perspective

A typical SRA establishes a protocol under which an end user outsources its applications software requirements to an ASP that designates a monthly subscription fee for software usage and applications management. Once the ASP enters into an agreement with the end user to host applications software, the end user establishes an implicit long (rented) position in the underlying commercially available applications software package. Simultaneously, the ASP establishes a short (sold) position in a European call option ‘written’ on the applications software package, since the renter reserves the right (but, is not obligated) to terminate the original contract at a designated date and seek an alternative investment. The pace of technological obsolescence and expected ASP consolidations necessitate that the renter maintain a flexible IT infrastructure that enables future augmentation and replacement of business applications to remain competitive. A four-year-term SRA with successive anniversary-date exit opportunities offers the renter the flexibility to exercise its right to deploy an ‘efficient’ applications environment without being relegated to the use of unproductive technology over extended periods of time. An efficient applications environment is expected to yield the most sequential net benefits from potential
technological innovations. Thus, as business applications tend to become more technologically advanced, the recipient is likely to benefit from rental contracts that offer an early exit policy.

To enable the renter to evaluate the exit flexibility embedded in SRAs, the exchange offer method developed by Margrabe (1978) is employed. A decision implementation point \( t_i \) exists for each SRA anniversary date such that at some positive integer \( i \leq 4 \) the renter may opt for exchanging the net benefit stream from renting with that from the alternative investment. The choice to continue with the current SRA or switch to the competing project at \( t_i \) depends on the magnitude of sequential net benefits expected from each investment. The respective project (applications software) values assessed at \( t_i \) then simply reflect the expected individual sequential net benefits discounted back to the implementation decision point. Given that project values impounding the net benefits of sequential projects follow a random walk in continuous time, the investment with the higher expected value will be implemented at \( t_i \). To maintain simplicity, the outsourcing investment is evaluated at \( i=1 \) so that the deferral exit opportunity may be represented by a simple call option.

**Simple Exchange Offer Model**

A shrink-wrapped applications software package (underlying SRA one) is available on the market for \( X_1 \) dollars. A competing package (underlying SRA two), developed using an alternative applications environment, sells on the market for \( X_2 \) dollars. The stochastic process that delineates the software-price dynamics for underlying packaged applications is given according to Margrabe (1978) as follows:

\[
\frac{dX_i}{X_i} = \alpha_i \, dt + \sigma_i \, dz_i
\]

\( (20) \)
Where, the proportional change in $X_i$ has instantaneous expected drift rate $\alpha_i$ and instantaneous variance rate $\sigma_i^2$, and $dz_i$ is a standard Gauss-Wiener process (for $i = 1, 2$). The instantaneous correlation coefficient between $dz_1$ and $dz_2$ is $\rho_{12}$, or

$$dz_1 dz_2 = \rho_{12} \, dt$$

(21)

If $X_1$ and $X_2$ represent software prices for two alternative packaged applications, such that respective price fluctuations resulting from new technological information are equal in magnitude but opposite in direction to each other, $\rho_{12}$ is exactly -1. For example, in a competitive and efficient rental software market, technological innovations in applications development, ceteris paribus, are likely to have a disparate impact on rival Oracle and Microsoft offerings. The converse is true if the two rental software solutions depend on a single source of technology (such as, Visual Basic source code) for creating base applications and future enhancements. Since new information is likely to have a uniform effect on SRA-investment values for two underlying applications that are homogeneous with respect to development environment and functionality, $\rho_{12}$ lies between 0 and 1.

The problem is to value a European option to exchange SRA two for SRA one at the end of the finite planning horizon $T = t_i$, or evaluate $c(X_1, X_2, T)$. The call (rent) option on SRA one with exercise price $X_2$ is simultaneously a put (terminate) option on SRA two with exercise price $X_1$. Table 1, below, shows a mapping of financial option parameters onto an option-embedded outsourcing scenario.
Table 1. Financial Option – Outsourcing Scenario Mapping

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Outsourcing Variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>European call (inherit) option exercise price</td>
<td>( X_1 )</td>
<td>SRA I applications software price</td>
</tr>
<tr>
<td>European put (abandon) option exercise price</td>
<td>( X_2 )</td>
<td>SRA II applications software price</td>
</tr>
<tr>
<td>Time to exercise</td>
<td>( T )</td>
<td>SRA term</td>
</tr>
<tr>
<td>Risk-free rate of return</td>
<td>( r )</td>
<td>Discount rate</td>
</tr>
<tr>
<td>Variance of the proportional change in ( \frac{X_1}{X_2} )</td>
<td>( \nu^2 )</td>
<td>Uncertainty regarding future technological changes</td>
</tr>
</tbody>
</table>

Upon completing one year of the current contract (SRA two) at \( T \), the option returns \( X_1^* - X_2^* \) if the renter exercises the option or zero otherwise. Clearly, the renter will exercise the exchange option if and only if the terminal return is greater than zero. Thus, the boundary condition at the inception of SRA one is given by:

\[
c(X_1, X_2, T) = \max [X_1^* - X_2^*, 0]
\]  

(22)

Where, the LHS of (22) implies that the option value is a function of the joint distributional characteristics of \( X_1 \) and \( X_2 \) as well as the time to maturity, and RHS suggests that the option is worth at least zero but cannot exceed \( X_1 \) given \((X_1, X_2) \geq 0\). Mathematically, the RHS of (22) describes the condition:

\[
X_1 \geq c(X_1, X_2, T) \geq 0
\]  

(23)

Margrabe (1978) obtains the closed-form solution in (10) by evaluating the differential process that describes the function \( c(X_1, X_2, T) \) with respect to the above boundary condition. Equation (10), thus, quantifies the otherwise intangible payoff inherent in the flexibility (to exit one SRA and migrate to the alternative contract) that is contingent upon the manner in which the technological environment evolves over a
definite planning horizon. To determine the value of outsourcing IT given that there exist two competing rental software applications, the option price from (10) must be adjusted by the fixed net benefits of each SRA. If $B_1$ and $B_2$ represent the respective present values of tangible net benefits (i.e., savings less subscription fees) expected from SRA one (inheriting) and SRA two (forsaking), and $K$ denotes the fixed cost (such as, invariant implementation and/or training expenses) of outsourcing IT applications, the SRA-investment (or, outsourcing) value at zero time to maturity is given by

$$V = c + [ B_1 - B_2 - K ]$$

Equation (24) requires three noteworthy clarification arguments. One, savings capture the value of operational efficiency and increased productivity from renting. For example, economies in applications deployment, software licensing, and IT personnel are likely to result from outsourcing (Schmerken, 2000; Torode and Follett, 2000). Two, in a risk-neutral world, since investors need not be paid a risk premium as compensation for bearing uncertain outcomes, the appropriate discount rate for calculating the present value is the risk-free rate. Three, $K$ is assumed fixed as a matter of convenience because the predominant objective of our approach is to demonstrate that SRA valuation encompasses both the direct and the sequential facets of the investment. If $K$ enters the evaluation process as a stochastic variable with an underlying distribution function, only direct and sequential attributes of (24) change while its basic composition remains intact.
Outsourcing Scenario I Illustration

To implement a total accounting package that will serve as an integrated applications software system from order procurement to customer invoicing, an end-user firm is willing to evaluate two outsourcing solutions (SRA-I and SRA-II) that are offered by two distinct ASPs. The respective monthly subscription fees (including software maintenance) for SRA-I and SRA-II are $1650 and $1400, and each contract offers four-year terms to maturity with the flexibility to terminate service and exit at any one of four anniversary dates. Irrespective of the SRA alternative chosen, the firm must make an initial investment of $2000 in basic hardware and software to enable access to ASP-hosted applications. Moreover, the applications software constituting both SRA-I and SRA-II are commercially available for $23,000 and $18,000 respectively. The expected rate of change in these software prices as well as the variance of proportional price changes is assumed constant over the one-year planning horizon.

Preliminary research by the recipient firm reveals that although the source code for each completely integrated rental applications solution is generated using disparate development environments, the respective functional features are comparable. In addition, both SRAs are expected to yield the same amount of cost savings of $800 per month for the first six months of applications software use. As end-user learning improves and expected technological innovations are incorporated into base software configurations, SRA-I and SRA-II are expected to yield monthly cost savings of $2000 and $1600 respectively. The firm expects technological upgrades to base applications under both SRAs in six months from today. The effect of the
enhancements is to increase cost savings by a factor of 2 under SRA-I and 2.5 under SRA-II. These additional productivity gains fail to accrue in the absence of learning. The firm estimates that its staff will become proficient in applications usage in six months from inception regardless of which SRA alternative is selected. Table 2 summarizes the above data.

Table 2. Outsourcing Scenario I Data

<table>
<thead>
<tr>
<th>Description</th>
<th>SRA-I</th>
<th>SRA-II</th>
</tr>
</thead>
<tbody>
<tr>
<td>Retail software price</td>
<td>$23,000</td>
<td>$18,000</td>
</tr>
<tr>
<td>Monthly rent</td>
<td>$1,650</td>
<td>$1,400</td>
</tr>
<tr>
<td>Initial monthly cost savings</td>
<td>$800</td>
<td>$800</td>
</tr>
<tr>
<td>Subsequent monthly cost savings</td>
<td>$2,000</td>
<td>$1,600</td>
</tr>
<tr>
<td>Initial Outlay</td>
<td>$2,000</td>
<td>$2,000</td>
</tr>
</tbody>
</table>

**Subcontractor Perspective**

Confronted with the knowledge of a rapidly changing IT landscape, the ASP adds value to outsourcing by providing the renter with the flexibility to exit the SRA at each anniversary date over the forty-eight month period. Nevertheless, the ASP identifies key advantages of the continuing SRA as being the invariant four-year revenue stream, and the protection from competitive pressures in the software market. On the other hand, the ASP recognizes that such a contract may serve as a deterrent to a prospective renter given the dynamic technological environment. Once the renter is locked in for four years under the continuing SRA, it is (prior to contract maturity) precluded from migrating to rival software innovations that offer the potential to lower total ownership cost and/or augment operational efficiency. Even if the ASP incorporates competing innovations within its applications solution, it is likely to be
confronted with a pricing dilemma. If rent under the continuing SRA is fixed at inception, the cost of augmenting the basic applications package cannot be simply passed on to the renter. To avoid making a loss in such a case, the ASP would restrict end-user deployment of the augmented applications software. Conversely, the ASP would increase the monthly subscription fees (while the original SRA is in effect) to render major software enhancements accessible to the recipient.

The flexible SRA is able to mitigate the valuation problem that results from fixing rental contracts over a lengthy planning horizon characterized by a constantly evolving technological landscape. The renter is afforded annual exit opportunities (under the flexible SRA) so that it is free to explore alternative accounting software solutions. Such benefit to the renter is perceived as a competitive disadvantage for the outsourcing firm. The ASP bears the risk that it may not be able to recoup the market price of the accounting package if the renter were to exit in the first couple of years of the flexible SRA’s inception. Moreover, a high attrition rate within the ASP’s customer base due to exit is likely to send a negative signal to the market. The ASP evidently seeks adequate compensation for assuming the additional ‘opportunity cost.’

From the perspective of the ASP, the SRA embedding flexibility may be analogous to writing a call option on the packaged applications software with the renter ‘purchasing’ the right to exit the contract at designated time intervals that coincide with contract anniversary dates. If the exit option expires without exercise on any anniversary date, the original SRA is automatically enforced in the subsequent year. With the flexible SRA, however, the ASP faces the risk of relinquishing a valuable resource, i.e., future expected cash flows from renting applications software.
Further, since the magnitude of the forgone rental revenues may tend to increase with random technological improvements in the base applications software, the ASP’s opportunity cost may actually be higher than that perceived a priori. Clearly, the value of the option-laden outsourcing agreement is primarily derived from the underlying applications software price that determines software rental revenues. Table 3 presents a comparison between the ongoing SRA and the option-embedded SRA.

Table 3. Ongoing and Flexible SRA Comparison

<table>
<thead>
<tr>
<th>Contract Terms</th>
<th>Ongoing SRA</th>
<th>Flexible SRA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rent</td>
<td>Fixed</td>
<td>Variable\textsuperscript{14}</td>
</tr>
<tr>
<td>Exercise time(s)</td>
<td>On expiration</td>
<td>On or before expiration</td>
</tr>
<tr>
<td>Exercise date(s)</td>
<td>One</td>
<td>Four</td>
</tr>
<tr>
<td>Upgrades and minor enhancements</td>
<td>Included</td>
<td>Included</td>
</tr>
<tr>
<td>Major augmentations</td>
<td>Offered at increased rent</td>
<td>Included</td>
</tr>
</tbody>
</table>

\textbf{Compound Option Model} \newline

To determine the value of renting IT applications that are otherwise available on the market as identical or comparable off-the-shelf products, this section begins by expressing the axiom (propagated by the options-pricing literature) that an asset embedding an option is worth more than its passive NPV estimate (Ross, 1995; Taudes, 1998) within the context of flexible SRAs. Redefined, this axiom states that if \( V_o \) and \( V_c \) respectively denote the values of two SRA types, the ongoing (or, continuing) and the call option-laden rental contracts, then \( V_c > V_o \). The continuing SRA delineates a rental contract that is set for termination at the end of \( T \) years with
no possibility of an early exit. The flexibility inherent in the option-laden SRA is one of terminating the original contract at any of the anniversary dates comprising the $T$-year planning horizon. By accepting the SRA embedding the exit option, the renter effectively purchases the right to terminate the contract at specific dates in the future. The ASP, naturally, expects 'adequate compensation' for offering SRA flexibility possibly with a view to differentiating its service. Given that the present values for the ongoing and the call option-laden SRAs are $V_o$ and $V_c$ respectively, the call option value at the end of the $T$-year planning horizon can be expressed by:

$$c(T) = K (V_c - V_o)$$ (25)

Where, $(V_c, V_o) > 0$, and $K$ is some constant such that $0 < K \leq 1$.

The capacity of the ASP to offer service is constrained by its ability to absorb the cost of providing that service. For example, to offer sales-force automation services on the Internet, the ASP needs to allocate network bandwidth, application run times (that determine the number of concurrent users), file storage, data security, customer support, and so on. These services, which are an integral part of the SRA, carry a price tag. The ASP must factor in the cost of these services to arrive at the SRA value. If there is no option embedded in the contract, the SRA value $V_o$ is simply $S$ (i.e., the current market value of the applications software plus the present value of the software maintenance fee charged over the planning horizon $T$)$^{15}$. The software maintenance fee is used as a proxy for the cost that the ASP expects to incur to manage and support the desired applications solution. If the renter is entitled to the

$^{14}$ For simplifying the results of the compound option model (discussed later in this study) the rent changes from the first year to the second, and then is fixed at the second-year level for the remainder of the SRA’s life.

$^{15}$ Throughout the study, $S$ is denoted as the (underlying) software price for the sake of brevity.
embedded flexibility, the corresponding option price is imputed in the SRA value \( V_c \). The option value \( c(T) \) delineates the unexpected cost component (cost to the ASP if the in-project opportunity is invoked) resident in application hosting over \( T \) years. In view of the upcoming annual renewal, the ASP is exposed to the risk of a low-cost competitor reproducing the hosting firm’s level of service in a more cost-effective manner and luring the end user away. Further, a more flexible hosting service level offered by a rival firm could invariably increase the current ASP’s exposure to the risk of losing the end user in the future. Intuitively, a flexible SRA should command a higher value. The posited direct relationship between flexible service levels and SRA values is likely to become more pronounced over time as software applications and/or hardware components become obsolete.

Since the embedded flexibility contained in the option-laden SRA has potential value, the ASP will expect a premium, as reflected in the RHS of (25), for offering such flexibility. The option premium serves to compensate the ASP for the lost revenue potential resulting from an ‘abrupt’ termination of the option-embedded SRA. Such an unforeseen termination would invariably precede the expiration of the continuing contract. Simultaneously, the opportunity to terminate (such as, by purchasing the underlying software and implementing a viable internal IT solution) provides a valuable decision alternative to the renter. Once the renter selects the option-laden SRA in lieu of the continuing agreement, it essentially agrees to pay for the right to exit the contract at certain dates. In compensating the ASP, the recipient firm will pay no more than the value (risk premium) that the option-laden SRA adds to the continuing contract over the planning horizon \( T \). Therefore,
\[ V_c = V_0 + C(T) \]  \hspace{1cm} (26)

Where, \( C(T) \) denotes the aggregate price of the embedded sequential call option. By evaluating the aggregate call option \( C(T) \), one is able to ascertain the flexible SRA value since the continuing contract price is known at present. Below, a mathematical equivalent for the notation \( C(T) \) in (26) is derived, and equation (25) is validated.

With the option-laden SRA, the renter has the right to invoke the exit decision at an anniversary date on or before the contract’s maturity. Let \( p_i \) represent the exit probability for the year-\( t \) anniversary date. Therefore, \( 1 - p_i \) denotes the probability of failure to exercise the exit option at the end of year \( t \). If \( c(t) \) is the \( t \)-year call option value, and there exist \( T \) possible anniversary dates (i.e., the original rental contract is set to mature in \( T \) years), then the following options series can be extracted from the \( T \)-year SRA.

\[ c(t) = p_t c(t) + (1 - p_t) c(t) \]

\[ c(t+1) = p_{t+1} c(t+1) + (1 - p_{t+1}) c(t+1) \]

\[ : \]

\[ c(T-1) = p_{T-1} c(T-1) + (1 - p_{T-1}) c(T-1) \]

\[ c(T) = p_T c(T) + (1 - p_T) c(T) \]  \hspace{1cm} (27)

Where, \( c(t) \), \( c(t+1) \), \ldots, \( c(T-1) \), \( c(T) \) are call option values embedded in the SRA with \( t \), \( t+1 \), \ldots, \( T-1 \), \( T \) number of exit points, and \( p_{t+1} \), \ldots, \( p_{T-1} \), \( p_T \) are the associated exit probabilities that are contingent upon the final-period exit decision. Let \( D_{T-1} \) be the decision made at the end of year \( T-1 \), which can result in one of two outcomes (exit and renewal) that are respectively represented by 1 and 0. Then, the likelihood that the
renter would exercise the year $T-1$ exit option given that it must exit at the close of year $T$ is the $T-1$ exit probability expressed as

$$p_{T-1} = \Pr\{D_{T-1} = 1 \mid D_T = 1\} \quad (28)$$

If for any $T$, $p_T = 1$, $c(T)$ describes a "simple" call option that matures at the close of year $T$. Calls, denoted by values $c(t)$, $c(t+1)$, ... , $c(T-1)$, can be characterized as "compound" call options (Carr, 1988) since each is derived from another potential opportunity. Whereas the simple call option can be valued using (3), (6), or (10), the compound exit opportunity is evaluated as a nested exchange using (14). Further, if the SRA is terminated at the end of year $t$ ($p_t = 1$), all subsequent decision probabilities become meaningless, or $p_{t+1} = 0$, ... $p_{T-1} = 0$, $p_T = 0$. In this case, the aggregate call option $C(T)$ is $c(t)$. Moreover, if there exists a year $T$ exit opportunity, the original SRA remains in effect at the end of year $T-1$, and $c(T)$ is the relevant terminal simple option value for year $T$. On the other hand, if exit is imminent at the end of year $T-1$, $c(T-1)$ becomes the only relevant terminal option value since $p_T c(T)$ is worthless.

Table 4 presents a matrix of decision probabilities and corresponding $C(\cdot)$ formulae for $T = 4$.

<table>
<thead>
<tr>
<th>Probabilities $(T = 4)$</th>
<th>$P_1$</th>
<th>$P_2$</th>
<th>$P_3$</th>
<th>$P_4$</th>
<th>$C(\cdot)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_1$</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>$c(1)$</td>
</tr>
<tr>
<td>$P_2$</td>
<td>(0,1)</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>$c(1) + c(2)$</td>
</tr>
<tr>
<td>$P_3$</td>
<td>(0,1)</td>
<td>(0,1)</td>
<td>1</td>
<td>0</td>
<td>$c(1) + c(2) + c(3)$</td>
</tr>
<tr>
<td>$P_4$</td>
<td>(0,1)</td>
<td>(0,1)</td>
<td>(0,1)</td>
<td>1</td>
<td>$c(1) + c(2) + c(3) + c(4)$</td>
</tr>
</tbody>
</table>
Notes:

1) $p_t$ and $c(t)$ respectively denote the exit probability and price of the year $t$ option ($t = 1, 2, 3, 4$).

2) For $T = 4$, the SRA is set to mature in four years. The expiration of the year-four exit option coincides with contract maturity. Since exit is imminent at contract maturity, $c(4)$ is always treated as a simple call option.

3) The probability $p_t = 1$ implies that exit is certain. The probability $p_t = (0, 1)$ describes the likelihood that exit/renewal cannot be gauged with certainty.

Equation (27) represents a nested series of reciprocal exit opportunities such that one is written on another. For nested options, the value of a compound option is proportional to the terminal simple option price (Carr, 1988). Therefore,

$$c(t) \propto c(t+1) \ldots \propto c(T-1) \propto c(T)$$  \hspace{1cm} (29)

Where, $c(t)$ through $c(T)$ signify the values of the SRA-embedded exit opportunities available over $T$, and $\propto$ is the constant of proportionality. Converting proportionality in (29) into parity, one obtains:

$$c(t) = k(t) \cdot c(t+1) = \ldots = k(T-2) \cdot c(T-1) = k(T-1) \cdot c(T)$$ \hspace{1cm} (30)

Where, $k(T-1)$ is some ‘discount’ factor applied to $c(T)$ to obtain the $T-1$ year option price such that $0 \leq k(\cdot) \leq 1$. Equation (30) implies that:

$$c(t) = k(t) \cdot c(t+1) = k(t) \cdot k(t+1) \ldots k(T-1) \cdot c(T)$$ \hspace{1cm} (31)

$$c(t+1) = k(t+1) \cdot c(t+2) = k(t+1) \cdot k(t+2) \ldots k(T-1) \cdot c(T)$$ \hspace{1cm} (32)

$$c(T-1) = k(T-1) \cdot c(T)$$ \hspace{1cm} (33)

Collecting all the relevant exit option prices from (27), one can write the expected value of the aggregate exit flexibility over $T$ years as:

$$C(T) = p_t \cdot c(t) + p_{t+1} \cdot c(t+1) + \ldots + p_{T-1} \cdot c(T-1) + p_T \cdot c(T)$$ \hspace{1cm} (34)
Substituting from (31), (32), and (33) in (34), one obtains:

\[ C(T) = p_t k(t) \cdots k(T-1) c(T) + p_{r,1} k(t+1) \cdots \]
\[ \cdots k(T-1) c(T) + \cdots + p_{r,l} k(T-1) c(T) + p_T c(T) \]  

(35)

Taking \( k(0) = 1 \), one can express equation (35) as:

\[ C(T) = c(T) \sum_{t=1}^{T} p_t \frac{\prod_{i=0}^{T-1} k(i)}{\prod_{j=0}^{T-1} k(j)} \]  

(36)

or,

\[ C(T) = c(T) [p \ k + 1] \]  

(37)

Where, \( p \) is the \( 1 \times (T-1) \) row matrix of conditional probabilities, and \( k \) is the \( (T-1) \times 1 \) column matrix of discount factors. Equation (37) clearly suggests that a nested series of call options may be priced by simply evaluating the terminal opportunity in the series, and weighing the resultant value with the summation of the product of conditional probabilities and discount factors calculated through the penultimate period in the planning horizon. Equations (26) and (37) give:

\[ c(T) = \frac{V_c - V_o}{1 + p \ k} \]  

(38)

If \( K = (1 + p \ k)^{-1} \), then (38) yields (25). Since the divisor \((1 + p \ k) \geq 1\), \( K \in (0, 1] \).

Given that \( K \) is non-negative, and \( c(T) \) is positive, the option-laden SRA commands a higher value than its 'closed-ended' counterpart. Further, (37) implies that \( K \) may be expressed as a ratio of the simple option value to the aggregate call price. This inference allows us to 'plug' different \( K \)-ratios in (37) and substitute the resultant aggregate call prices \( C(T) \) in (26) to obtain a range of option-laden SRA values.

Clearly, as previously noted in (30), a change in the terminal call premium \( c(T) \)
produces a corresponding movement in the aggregate call price $C(T)$ with the magnitude of the discount proportion $K$ being determined by the level of the terminal option $c(T)$. Among the factors affecting a simple exit option $c(T)$, the underlying volatility tends to exert a dominant influence.

**Volatility**

The variability underlying an exit option originates in fluctuating applications software price and opportunity cost. As firms enter the ASP industry and the technological environment continues to change, competing services are increasingly differentiated to gain loyal clientele in the market (Schmerken, 2000). This differentiation may be in the form of instituting a unique way of hosting on-line applications, providing cutting-edge network performance and connectivity, offering ‘extra’ services for less, or simply introducing software enhancements to prevent obsolescence. Given an efficient ASP market, only the anticipated changes in technology (incorporated in the differentiation strategy) are reflected in the current applications software price (equivalent to the present value of monthly rental revenues $v_0$ derived from the continuing SRA). Further, the cost incurred by the ASP when the renter exercises the exit option is the lost revenue potential$^{16}$. The closer the renter exits to the SRA’s maturity date, the lower is the lost revenue potential. As a result of early exit, the ASP faces the risk of incurring a loss from renting. The greater the number of exit opportunities over a designated contract duration, the higher is the risk borne by the ASP in comparison with a comparable continuing contract. With premature exit, the outright sale of applications software retrospectively is the value-
maximizing decision if the SRA-inception software price exceeds the present value of rental revenues actually collected. Clearly, the opportunity cost (or, exercise price) volatility arises as a consequence of the uncertainty surrounding the timing of the SRA exit decision that ultimately depends on the state of technological development in the market for comparable ASP offerings. Therefore, a lengthy SRA with embedded flexibility tends to be riskier than a similar contract with no exit opportunities. Unexpected innovations that eventually serve to differentiate the SRA cause the underlying prices to fluctuate just as the advent of “new” information in capital markets results in stock price volatility (Fama, 1970).

The SRA value reflects what a monopolistically competitive ASP will expect to bear for hosting software applications during a given time period. The value of the SRA-embedded exit option varies according to both the underlying applications software price volatility $\sigma_x \sqrt{T}$ and the variability in opportunity cost $\sigma_a \sqrt{T}$. As the state of technological development in the rental applications software arena becomes uncertain over time, the underlying software price volatility increases with the frequency of unanticipated innovations. Consequently, the ASP’s opportunity cost (contingent upon the renter’s terminal decision) variability increases with the length of the planning horizon\(^{17}\). This implies that the small increment to the Wiener process of $X$ moves in direct correspondence with that of $S$ over time, or correlation coefficient

\(^{16}\) Incidentally, this is also the cost to the end user for purchasing the underlying applications software upon exiting the SRA, or, conversely, for keeping the rental contract alive by subsequent renewal. \(^{17}\) If the renter exits early, the ASP loses the opportunity of imposing higher, innovation-adjusted exercise prices on subsequent anniversary dates.
\( \rho_{\pi\tau} \) in (8) is positive\(^{18} \). The risk (option) premium (value) is reflected in the current-period subscription fees \( v_c \) following the presumption that the renter would allow the embedded option to lapse at the end of the first year. Moreover, whether the exit option materializes or otherwise lapses without exercise at expiration, the option writer (i.e., the ASP) keeps the risk premium. A higher expected compensation for risk translates into a higher embedded-option value \( c() \) and associated SRA price \( V_c \).

Clearly,

\[
V_c = f(S, c) \tag{39}
\]

Following Black and Scholes (1973), the approach proposed herein is to express the value of an SRA-embedded exit flexibility (call option) as a function of five parameters – the underlying software price: \( S \), the cost of purchasing the underlying applications software (option’s exercise price): \( X \), the instantaneous variance of the proportional change in the ratio of \( S \) to \( X \): \( \sigma^2 \), the option’s time to maturity: \( \tau \), and the risk-free interest rate: \( r \). Therefore, the year-\( T \) exit option can be stated using the functional notation below.

\[
c(T) = g(S, X, \sigma^2, \tau, r) \tag{40}
\]

Where, \( \sigma^2 = h(\sigma^2, \sigma_{\tau}, \rho_{\pi\tau}) \) if both \( S \) and \( X \) are stochastic processes with instantaneous correlation coefficient \( \rho_{\pi\tau} \). From (39) and (40):

\[
V_c \propto \sigma^2 \tag{41}
\]

Where, \( \sigma^2 = \sigma^2_\tau \), given that \( X \) is invariant. However, for a nested series of exit opportunities comprising the option-laden SRA, the present value \( X \) of a random

\(^{18} \) However, since opportunity cost manifest in forgone revenues tends to decrease over time (eventually becoming constant at the end of the planning horizon), the magnitude of risk borne by the ASP as a consequence of the exit uncertainty diminishes as the SRA approaches maturity.
stream of forgone software revenues is essentially stochastic through the last rental period\(^{19}\). As the returns on \(S\) as well as the proportional changes in \(X\) become more volatile (i.e., both \(\sigma_x \sqrt{T}\) and \(\sigma_x \sqrt{T}\) increase given \(\rho_x < 0\)), the likelihood that \(S\) would exceed \(X\) improves. Since call options signify claims solely on the upper tail of the probability distribution of terminal-date software prices, the higher the underlying volatility the more valuable the exit option. Further, all determinants with the exception of volatility, in (40) are readily observable. The volatility underlying a known stochastic process is simulated in the following section using an approach described in Hull (1997).

**Volatility Simulation Procedure**

Let the applications package price \(S\) follow a Markov process. Then, in discrete time, a small price change \(\Delta s\) over a small interval of time \(\Delta t\) may be represented by the equation:

\[
\frac{\Delta s}{s} = \mu \Delta t + \sigma \varepsilon \sqrt{\Delta t}
\]

Where, \(\varepsilon\) is a random drawing from a standardized normal distribution, \(\Delta s/s\) is the proportional return from the software package in a short interval of time, \(\Delta t\). The parameters, \(\mu \Delta t\) and \(\sigma \sqrt{\Delta t}\) are, respectively, the expected value of the proportional return and volatility of the software price \(S\) such that,

\[
\mu = \frac{1}{n} \sum \ln(s_t / s_{t-1})
\]

\[
\sigma = \sqrt{(n-1)^{-1} \sum (s - \mu)^2}
\]

\(^{19}\) If the rent is charged at the beginning of a month, then the last rental period would commence with the first day of the concluding month of the final anniversary year.
Where, $s$ is the difference between the natural logarithm of $s_t$ and that of $s_{t-1}$, and $n$ is the number of observations. From (42), then, $\Delta s/s$ is normally distributed with $\mu \Delta t$ as instantaneous mean and $\sigma \sqrt{\Delta t}$ as instantaneous standard deviation, or

$$\Delta s/s \sim \mathcal{N}(\mu \Delta t, \sigma \sqrt{\Delta t})$$ (45)

Subsequent to calculating mean and standard deviation of the available yearly software prices, a Markov process over one-month time increments is simulated by repeatedly sampling values $v_1$ from a standardized normal distribution, i.e., $v_1 \sim \mathcal{N}(0, 1)$, and then converting these numbers to sample values $v_2$ via equation:

$$v_2 = \mu \Delta t + \sigma \sqrt{\Delta t} v_1$$ (46)

Each outcome $v_2$ can be regarded as a random sample from the distribution of month-end software prices that are generated by a single trial (or, a set of sampling values $v_1$). The exact number of random values $v_1$ required for the process corresponds to the length of the planning horizon $T$ over which the volatility measure is to be estimated. For example, to determine the volatility parameter for an SRA that is set to mature in $T$ years, the number of random values required for the process is $12T$. The simulation is conducted for a total of thirty trials to produce a complete probability distribution of monthly returns (or, proportional changes in the application package price). The standard deviation of the annualized return (or, natural logarithm of consecutive monthly price movements) is calculated for each trial and then averaged to proxy for volatility in the Black and Scholes (1973) OPM given by (3).

An option that has a stochastic exercise price requires an additional volatility parameter estimate. The exercise price volatility is delineated by the average annual
standard deviation of the proportional change in forgone revenues over the SRA's life. The estimating procedure begins by expressing the stochastic exercise price $X$ as a discretized Markov process:

$$\Delta x/x = \alpha \Delta t + \phi \varepsilon \sqrt{\Delta t}$$ \hspace{1cm} (47)

Where, $\varepsilon \sim \mathcal{N}(0, 1)$, $\Delta x/x$ is the proportional change in forgone revenues over a short interval of time, $\Delta t$. The parameters, $\alpha \Delta t$ and $\phi \sqrt{\Delta t}$ are, respectively, the expected value of the proportional change in and volatility of the exercise price $X$. Next, the present value of the expected stream of forgone revenues for exit options that are set to expire in $t, t+1, \ldots, T-1, T$ years is obtained. The mean and standard deviation of the exercise prices calculated for given option maturities are then substituted in (46) to initiate the simulation process.

**Outsourcing Scenario II Illustration**

The data employed for the purpose of this illustration comprise year-end software prices by concurrent user count for three consecutive years of 1998, 1999, and 2000 sampled at an existing software development firm. The single-user application package prices in effect on the last day of the final month of each given year are deemed adequate for numerically demonstrating the simulation and the valuation processes. Table 5 below shows the mean and standard deviation parameter values give the three single-user price points.

---

$^{20}$ A one month-increment seems appropriate since rent under an SRA is charged on a monthly basis.
Table 5. Parameter Values for Single User Software Prices

<table>
<thead>
<tr>
<th>Date</th>
<th>$S_i$</th>
<th>(\ln(S_i/S_{i-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>12/30/98</td>
<td>$21,535</td>
<td></td>
</tr>
<tr>
<td>12/30/99</td>
<td>$21,105</td>
<td>-0.02017</td>
</tr>
<tr>
<td>12/30/00</td>
<td>$15,930</td>
<td>-0.28131</td>
</tr>
</tbody>
</table>

\[
\begin{array}{|c|c|}
\hline
\mu & -0.15074 \\
\sigma & 0.18465 \\
\Delta t & 0.08333 \\
\sqrt{\Delta t} & 0.28868 \\
\hline
\end{array}
\]

Denote SRA-I as the four-year continuing rental contract with monthly subscription fees of $484.76 as calculated in Table 6 below.

Table 6. Continuing SRA Data and Rent Calculation \((T = 4)\)

\[
v_o(r) = V_o \left(\frac{r/12}{r/12 + 1}\right)^{48} \frac{(r/12 + 1)^{48} - 1}{(r/12 + 1)^{48}}
\]

| Software price                  | \(S\)       | $15,930 |
| SRA duration (months)           | \(T\)       | 48      |
| ASP discount rate               | \(r\)       | 20%     |
| Monthly rental values \(v_o(r)\)| $484.76     |
| Overall rental revenues         |             | $23,268 |
| Overall ASP return              |             | 46%     |

Another rental contract SRA-II offers three premature termination opportunities on the first three anniversary dates with a ‘mandatory’ exit at the end of four years following the contract’s inception. SRA-II also stipulates that the renter has the right, but not an obligation, to purchase the underlying applications software (currently priced at
$15930) for $1000 on the fourth and final anniversary date. Moreover, the renter enjoys the flexibility of invoking the exit option by purchasing the applications package on any of the three intermediate anniversary dates at the ASP’s ‘relevant’ opportunity cost. Table 7 below presents a schedule of the opportunity cost based on the current software price and a four-year SRA.

Table 7. Forgone Revenues/Exercise Price for t-year Options

<table>
<thead>
<tr>
<th>Number of Exit Points</th>
<th>Exercise Price (PV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>$10,697</td>
</tr>
<tr>
<td>3</td>
<td>$6,406</td>
</tr>
<tr>
<td>2</td>
<td>$2,886</td>
</tr>
<tr>
<td>1</td>
<td>$1,000</td>
</tr>
</tbody>
</table>

Notes:

1) A set of four exit points implies a year $t=4$ option, a set of three exit points implies a year $t=3$ option, and so on. The four-year option’s exercise price is the sum total of the rental revenues that are forgone over three years (three exit points remain after the first year of service). In essence, the exercise price is the cost to the renter if it decides to purchase the underlying software at the time of exit.

2) An SRA embedding four exit points may be thought of as being analogous to a contract combining a one-year, a two-year, a three-year, and a four-year option.

3) The exercise price is expressed in present-value terms using the ASP discount rate of 20%. When the SRA offers a single exit opportunity, the ASP establishes the present value of the exercise price as $1000.

For example, given that four exit opportunities are available to the end user, the present value of forgone rental revenues over thirty-six (twenty-four, twelve) months is the relevant year-four (year-three, year-two) opportunity cost or exercise price. The year-one exercise price is fixed by the ASP at the SRA’s inception. If the renter
decides to continue outsourcing applications to the ASP beyond four years, a new SRA would be drafted upon the expiration of SRA-II. However, the subsequent subscription fees may or may not equal the rent charged under SRA-II. In terms of options valuation, the ASP faces an exercise price that is fixed for the fourth anniversary date, but stochastic for intermediate (first through third) anniversary dates.

**Exit Times**

Outsourcing, as opposed to purchasing, applications software is an irrefutably valuable proposition if it is offered as a ‘perpetual’ service agreement embedding an end-user exercisable right to sever the contractual relationship at any point in time. The ‘exit-at-will’ clause built into an outsourcing contract provides the renter with an excellent avenue for evaluating service especially in view of numerous competing alternatives, and the flexibility of deferring exit until a value-maximizing investment decision is obtained. An outsourcing contract may originate under any one of two possible scenarios.

Under the first scenario, the software development firm originates the outsourcing contract, and directly renders service. The subcontractor incurs an opportunity loss by claiming ownership of the applications package, and not making the product available for sale in the market. Therefore, the opportunity cost of employing an applications software license within the outsourcing framework is tantamount to making an initial outlay of \( S_t \). In addition, the subcontractor writes a call option on the underlying software. The call option gives the renter a right to postpone contract termination at the decision implementation point. If \( V_t \) is the market value of the outsourcing contract, and \( S_t \) is the price of the underlying applications software at
time \( t \), the renter (call holder) will exercise the deferral option (continuing with the original SRA) if \( V_t > S_t \). The subcontractor’s profit position is enhanced by the premium received today for offering exit flexibility to the renter as a hedge against obsolescence risk in the future. The payoff to the software development firm is shown in Table 8 below.

**Table 8. Software Development Firm or Option Writer Payoff**

<table>
<thead>
<tr>
<th>Investment Position</th>
<th>( S_t \geq V_t )</th>
<th>( S_t &lt; V_t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Long Software</td>
<td>( S_t )</td>
<td>( S_t )</td>
</tr>
<tr>
<td>Short Call Option</td>
<td>-(( V_t - S_t ))</td>
<td></td>
</tr>
<tr>
<td>Net Payoff</td>
<td>( S_t )</td>
<td>( 2S_t - V_t )</td>
</tr>
</tbody>
</table>

Under the second scenario, a third-party subcontractor (or, ASP) that purchases the software license from the original applications developer originates the outsourcing contract. As such, the subcontractor establishes a short call and a long put on the underlying applications software. The call option, in effect, provides the renter a right to defer exit (i.e., reallocate investable funds from the current outsourcing contract to the next best alternative – backsourcing). The renter is likely to exercise its deferral option, if the outsourcing contract value \( V_t \) exceeds the market price of software \( S_t \), where \( t \) is the relevant decision implementation point. On the other hand, as a holder of the put option, the subcontractor retains the right to sell the underlying applications software in the market for a predetermined exercise price, \( X \)\textsuperscript{21}. The payoff to the subcontractor is shown in Table 9 below.

Tables 8 and 9 illustrate that the servicization of software via an outsourcing mechanism offering embedded options tends to result in multiple payoffs. As the

\textsuperscript{21} The subcontractor sells the original license at \( X \), and purchases a new one from the developer for \( S_t \).
software price declines, the downside potential of the service provider may be limited to or governed by some known value.

Table 9. Subcontractor or Call Option Writer and Put Option Holder Payoff

<table>
<thead>
<tr>
<th>Investment Position</th>
<th>$X &gt; S_t \geq V_t$</th>
<th>$S_t &lt; V_t$ &amp; $S_t &lt; X$</th>
<th>$S_t \geq V_t$ &amp; $S_t \geq X$</th>
<th>$X &lt; S_t &lt; V_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Long Software</td>
<td>$S_t$</td>
<td>$S_t$</td>
<td>$S_t$</td>
<td>$S_t$</td>
</tr>
<tr>
<td>Long Put Option</td>
<td>$X-S_t$</td>
<td>$X-S_t$</td>
<td>$X-S_t$</td>
<td>$X-S_t$</td>
</tr>
<tr>
<td>Short Call Option</td>
<td>0</td>
<td>$-(V_t-S_t)$</td>
<td>0</td>
<td>$-(V_t-S_t)$</td>
</tr>
<tr>
<td>Net Payoff</td>
<td>$X$</td>
<td>$X-(V_t-S_t)$</td>
<td>$S_t$</td>
<td>$2S_t-V_t$</td>
</tr>
</tbody>
</table>

For example, columns two and three of Table 9 depict a relatively low software price with the corresponding net payoff amounts that are largely determined by the designated put-option exercise price. The incidence of multiple payoffs is in sharp contrast to the simple case of end-user software ownership where the outlay occurs as one complete lump sum payment $S$ that solely determines the future investment payoff. Additionally, the profit on the writer’s option-embedded position (such as, that in Table 8) may be significantly improved as a result of collecting the option premium to complement (offset) the gains (losses) from software price appreciation (erosion).

For the renter, the American-style option attached to an outsourcing mechanism supplies an insurance policy against the risk of obsolescence. If the subcontractor fails to render service commensurate with end-user expectations, the renter may fail to invoke the right to defer exit, thereby terminating further investment in the potential benefits of the current outsourcing agreement. An optimal approach to selecting early exit dates ensures that the value of the payoff function is maximized\(^\text{22}\). In essence, the decision implementation points are outcomes of a random time variable.

\(^{22}\) The finance literature commonly refers to these dates as optimal stopping times.
that assumes non-negative, finite values. In practice, a ‘perpetual’ outsourcing contract remains in effect so long as the subcontractor renders service. If the outsourcing firm is expected to operate its business for an additional $t$ periods, and each period is of length $n$, then $nt = T$. A finite operating horizon imposes an upper time bound on the deferral option embedded in the outsourcing mechanism. Therefore, the renter’s terminal option premium $c$ is given by:

$$c(V_T, S_T, T) = \max (V_T - S_T, 0)$$

(48)

Moreover, there exists a possibility that the renter may decide to backsource and terminate the outsourcing contract prior to $T$. Such a case may occur at some critical time(s) delineated by $t \in [0, T]$, such that:

$$V_t < S_t$$

(49)

and

$$V_t - S_t < e^{-r(T-t)} \max (V_T - S_T, 0)$$

(50)

Where, $r$ is the risk-free rate. To determine an optimal decision implementation point, the renter must similarly evaluate the likelihood of premature exit in successive future time periods until $T$. 

- Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.
CHAPTER 6

RESULTS

Renter Perspective

Given the firm's constrained resources and the management's apprehensions, the rental software alternative seems viable since it requires a low initial capital outlay as well as provides the flexibility of migrating to an alternative applications environment if necessary. A cursory analysis suggests that SRA-II caters to the specific business needs of the firm at a lower monthly rental cost and a higher benefit-cost ratio (see Tables 10 and 11 below). Further, since the NPV of both SRAs is negative, the traditional capital budgeting rule dictates that neither SRA be accepted and that other decision alternatives be explored. However, the analysis is not complete unless the optionality embedded in rental software investments is also evaluated.

Table 10. Preliminary SRA Comparison

<table>
<thead>
<tr>
<th>Outsourcing Contract Type</th>
<th>Present Value</th>
<th>NPV</th>
<th>PV(B) ÷ PV(C)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Monthly Costs PV(C)</td>
<td>Monthly Benefits PV(B)</td>
<td></td>
</tr>
<tr>
<td>SRA-I</td>
<td>$19,267</td>
<td>$16,162</td>
<td>-$3,105</td>
</tr>
<tr>
<td>SRA-II</td>
<td>$16,348</td>
<td>$13,873</td>
<td>-$2,475</td>
</tr>
</tbody>
</table>
Notes:

1) Discount rate $r$ used for present value calculations equals 6%.

2) PV for monthly costs is calculated as if rent is charged at the beginning of the month, and PV for benefits is computed by assuming that benefits accrue as lump sum at the end of the month.

<table>
<thead>
<tr>
<th>Month</th>
<th>Costs</th>
<th>Benefits</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SRA-I</td>
<td>SRA-II</td>
</tr>
<tr>
<td>1</td>
<td>1,650</td>
<td>1,400</td>
</tr>
<tr>
<td>2</td>
<td>1,650</td>
<td>1,400</td>
</tr>
<tr>
<td>3</td>
<td>1,650</td>
<td>1,400</td>
</tr>
<tr>
<td>4</td>
<td>1,650</td>
<td>1,400</td>
</tr>
<tr>
<td>5</td>
<td>1,650</td>
<td>1,400</td>
</tr>
<tr>
<td>6</td>
<td>1,650</td>
<td>1,400</td>
</tr>
<tr>
<td>7</td>
<td>1,650</td>
<td>1,400</td>
</tr>
<tr>
<td>8</td>
<td>1,650</td>
<td>1,400</td>
</tr>
<tr>
<td>9</td>
<td>1,650</td>
<td>1,400</td>
</tr>
<tr>
<td>10</td>
<td>1,650</td>
<td>1,400</td>
</tr>
<tr>
<td>11</td>
<td>1,650</td>
<td>1,400</td>
</tr>
<tr>
<td>12</td>
<td>1,650</td>
<td>1,400</td>
</tr>
<tr>
<td>Present Values</td>
<td>19,267</td>
<td>16,348</td>
</tr>
<tr>
<td>Net Benefits</td>
<td>-3,105</td>
<td>-2,475</td>
</tr>
</tbody>
</table>
(B) Valuation of cash flows expected over $T+1 - T$ (Year 2)

<table>
<thead>
<tr>
<th>Month</th>
<th>Costs</th>
<th>Benefits</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SRA-I</td>
<td>SRA-II</td>
</tr>
<tr>
<td>13</td>
<td>1,650</td>
<td>1,400</td>
</tr>
<tr>
<td>14</td>
<td>1,650</td>
<td>1,400</td>
</tr>
<tr>
<td>15</td>
<td>1,650</td>
<td>1,400</td>
</tr>
<tr>
<td>16</td>
<td>1,650</td>
<td>1,400</td>
</tr>
<tr>
<td>17</td>
<td>1,650</td>
<td>1,400</td>
</tr>
<tr>
<td>18</td>
<td>1,650</td>
<td>1,400</td>
</tr>
<tr>
<td>19</td>
<td>1,650</td>
<td>1,400</td>
</tr>
<tr>
<td>20</td>
<td>1,650</td>
<td>1,400</td>
</tr>
<tr>
<td>21</td>
<td>1,650</td>
<td>1,400</td>
</tr>
<tr>
<td>22</td>
<td>1,650</td>
<td>1,400</td>
</tr>
<tr>
<td>23</td>
<td>1,650</td>
<td>1,400</td>
</tr>
<tr>
<td>24</td>
<td>1,650</td>
<td>1,400</td>
</tr>
<tr>
<td></td>
<td>Present Values</td>
<td></td>
</tr>
<tr>
<td></td>
<td>18,148</td>
<td>15,398</td>
</tr>
<tr>
<td></td>
<td>Net Benefits</td>
<td></td>
</tr>
<tr>
<td></td>
<td>-2,925</td>
<td>2,112</td>
</tr>
</tbody>
</table>

Notes:

1) Productivity gains or cost savings due to software enhancements accrue once learning is complete beyond the sixth month of deployment. While additional productivity gains for the currently selected outsourcing contract (SRA-II) accrue in month seven, those for the competing contract (SRA-I) are likely to occur in month nineteen if the switch or exchange actually takes place.

2) Since the renter has the option of inheriting SRA-I in lieu of SRA-II at $T$, the appropriate individual net-benefit amounts are those expected in the $T+1 - T$ year discounted to the present.

Equation (10) is used to assess the current value of the embedded option that allows exiting SRA-II and adopting SRA-I with a view to take advantage of higher productivity gains from deploying a technologically superior rental applications.
package at the end of one year. Once option prices are obtained, (24) can be employed
to calculate SRA investment values. Tables 12 through 14 present \( c \) and \( V \) values for
different estimates of \( \sigma_1, \sigma_2, \) and \( \rho_{12}. \)

Table 12. Exchange Option Price and SRA Opportunity Value Calculations (\( \sigma_1 = \sigma_2 \))

<table>
<thead>
<tr>
<th>( X_1 )</th>
<th>( X_2 )</th>
<th>( \sigma_1 )</th>
<th>( \sigma_2 )</th>
<th>( B_1 )</th>
<th>( B_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>$23,000</td>
<td>$18,000</td>
<td>0.2</td>
<td>0.2</td>
<td>-$2,925</td>
<td>$2,112</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( \rho_{12} )</th>
<th>-0.9</th>
<th>-0.5</th>
<th>0</th>
<th>0.5</th>
<th>0.9</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N(d_1) )</td>
<td>0.7949</td>
<td>0.8108</td>
<td>0.8433</td>
<td>0.9075</td>
<td>0.9973</td>
</tr>
<tr>
<td>( N(d_2) )</td>
<td>0.6678</td>
<td>0.7035</td>
<td>0.7658</td>
<td>0.8698</td>
<td>0.9965</td>
</tr>
<tr>
<td>( c )</td>
<td>$6,263</td>
<td>$5,986</td>
<td>$5,610</td>
<td>$5,216</td>
<td>$5,002</td>
</tr>
<tr>
<td>( V )</td>
<td>-$774</td>
<td>-$1,051</td>
<td>-$1,427</td>
<td>-$1,821</td>
<td>-$2,035</td>
</tr>
</tbody>
</table>

Table 13: Exchange Option Price and SRA Opportunity Value Calculations (\( \sigma_1 > \sigma_2 \))

<table>
<thead>
<tr>
<th>( X_1 )</th>
<th>( X_2 )</th>
<th>( \sigma_1 )</th>
<th>( \sigma_2 )</th>
<th>( B_1 )</th>
<th>( B_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>$23,000</td>
<td>$18,000</td>
<td>0.5</td>
<td>0.3</td>
<td>-$2,925</td>
<td>$2,112</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( \rho_{12} )</th>
<th>-0.9</th>
<th>-0.5</th>
<th>0</th>
<th>0.5</th>
<th>0.9</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N(d_1) )</td>
<td>0.7594</td>
<td>0.7581</td>
<td>0.7617</td>
<td>0.7824</td>
<td>0.8551</td>
</tr>
<tr>
<td>( N(d_2) )</td>
<td>0.4694</td>
<td>0.5001</td>
<td>0.5513</td>
<td>0.6347</td>
<td>0.7865</td>
</tr>
<tr>
<td>( c )</td>
<td>$9,016</td>
<td>$8,435</td>
<td>$7,598</td>
<td>$6,570</td>
<td>$5,512</td>
</tr>
<tr>
<td>( V )</td>
<td>$1,979</td>
<td>$1,398</td>
<td>$561</td>
<td>-$467</td>
<td>-$1,525</td>
</tr>
</tbody>
</table>

Table 14. Exchange Option Price and SRA Opportunity Value Calculations (\( \sigma_1 >> \sigma_2 \))

<table>
<thead>
<tr>
<th>( X_1 )</th>
<th>( X_2 )</th>
<th>( \sigma_1 )</th>
<th>( \sigma_2 )</th>
<th>( B_1 )</th>
<th>( B_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>$23,000</td>
<td>$18,000</td>
<td>0.8</td>
<td>0.4</td>
<td>-$2,925</td>
<td>$2,112</td>
</tr>
</tbody>
</table>
Table 14 continued:

<table>
<thead>
<tr>
<th>( \rho_{12} )</th>
<th>-0.9</th>
<th>-0.5</th>
<th>0</th>
<th>0.5</th>
<th>0.9</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N(d_1) )</td>
<td>0.7868</td>
<td>0.7766</td>
<td>0.7646</td>
<td>0.7581</td>
<td>0.7747</td>
</tr>
<tr>
<td>( N(d_2) )</td>
<td>0.3529</td>
<td>0.3830</td>
<td>0.4313</td>
<td>0.5029</td>
<td>0.6107</td>
</tr>
<tr>
<td>( c )</td>
<td>$11,746</td>
<td>$10,967</td>
<td>$9,824</td>
<td>$8,383</td>
<td>$6,826</td>
</tr>
<tr>
<td>( V )</td>
<td>$4,709</td>
<td>$3,930</td>
<td>$2,787</td>
<td>$1,346</td>
<td>-$211</td>
</tr>
</tbody>
</table>

The numerical simulation indicates that the outsourcing decision commands a higher value as the price of embedded flexibility escalates with rising software price volatility underlying each applications solution. At low expected rates of software price fluctuations (i.e., \( \sigma_I = 0.2 \) and \( \sigma_2 = 0.2 \)), the extended analysis (Table 12) yields the same "no-go" investment decision as the standard NPV calculation (Table 10) since the outsourcing value is negative regardless of the correlation between individual software price volatilities. At moderate-to-high expected rates of software price fluctuations (i.e., \( \sigma_I = 0.5 \) and \( \sigma_2 = 0.3 \)), (24) returns positive SRA values for non-positive correlation coefficient (Table 13). The NPV technique, unaffected by the joint distributional characteristics of the underlying software prices, produces the same investment evaluation. For the volatility pair, (\( \sigma_I = 0.8 \), \( \sigma_2 = 0.4 \); Table 14), the rental software investment values are the highest at any given level of correlation (Figure 2). Thus, as the advent of new technological information becomes more random in the forecastable future, sequential exchanges between IT investments (as well as overall outsourcing value) tend(s) to grow in value. The exchange option price for any volatility pair is maximized if the impact of technological innovations on competing IT solutions is contrary.
Figure 2. Relationship between $\nu$ and $\rho_{12}$ for Different $(\sigma_1, \sigma_2)$ Combinations

**Subcontractor Perspective**

Table 15 below presents a simulated path of the software package price over a one-year period by evaluating equation (32) at twelve sample values that are randomly selected from a standardized normal distribution\(^2\). For example, the first sample value from $\nu_1$, 0.9863, generates a proportional return of 0.04001 given $\nu_2 \sim \mathcal{N}(-0.01256, 0.053305)$. The product of the beginning-period (month 1) software price, i.e., $\$15930$, and one plus the rate of change in price, i.e., 1.04001, gives the ending-period (month 2) software price that in turn becomes the beginning-period value at the next step in the simulation process. The procedure is replicated for a two-year (three-year, four-year) SRA by evaluating (46) at twenty-four (thirty-six, forty-eight) random values.
Table 15. Simulation of Software Price over Twelve Monthly Intervals

<table>
<thead>
<tr>
<th>Mo.</th>
<th>Random Sample</th>
<th>Proportional Return</th>
<th>Software Price (Beg)</th>
<th>Change ΔS</th>
<th>Software Price (End)</th>
<th>Annual Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.98630</td>
<td>0.04001</td>
<td>15930.00</td>
<td>637.40</td>
<td>16567.40</td>
<td>0.47079</td>
</tr>
<tr>
<td>2</td>
<td>-1.29980</td>
<td>-0.08185</td>
<td>16567.40</td>
<td>-1355.98</td>
<td>15211.42</td>
<td>-1.02468</td>
</tr>
<tr>
<td>3</td>
<td>-0.49860</td>
<td>-0.03914</td>
<td>15211.42</td>
<td>-595.36</td>
<td>14616.06</td>
<td>-0.47911</td>
</tr>
<tr>
<td>4</td>
<td>0.76507</td>
<td>0.02822</td>
<td>14616.06</td>
<td>412.47</td>
<td>15028.53</td>
<td>0.33395</td>
</tr>
<tr>
<td>5</td>
<td>-0.41303</td>
<td>-0.03458</td>
<td>15028.53</td>
<td>-519.65</td>
<td>14508.88</td>
<td>-0.42228</td>
</tr>
<tr>
<td>6</td>
<td>-0.26678</td>
<td>-0.02678</td>
<td>14508.88</td>
<td>-388.58</td>
<td>14120.30</td>
<td>-0.32577</td>
</tr>
<tr>
<td>7</td>
<td>0.17504</td>
<td>-0.00323</td>
<td>14120.30</td>
<td>-45.62</td>
<td>14074.68</td>
<td>-0.03884</td>
</tr>
<tr>
<td>8</td>
<td>0.99580</td>
<td>0.04052</td>
<td>14074.68</td>
<td>570.29</td>
<td>14644.97</td>
<td>0.47664</td>
</tr>
<tr>
<td>9</td>
<td>0.52889</td>
<td>0.01563</td>
<td>14644.97</td>
<td>228.91</td>
<td>14873.88</td>
<td>0.18612</td>
</tr>
<tr>
<td>10</td>
<td>-0.19636</td>
<td>-0.02303</td>
<td>14873.88</td>
<td>-342.52</td>
<td>14531.36</td>
<td>-0.27957</td>
</tr>
<tr>
<td>11</td>
<td>1.12779</td>
<td>0.04755</td>
<td>14531.36</td>
<td>691.03</td>
<td>15222.39</td>
<td>0.55750</td>
</tr>
<tr>
<td>12</td>
<td>0.32433</td>
<td>0.00473</td>
<td>15222.39</td>
<td>71.95</td>
<td>15294.34</td>
<td>0.05659</td>
</tr>
</tbody>
</table>

Stdev\(^{24}\) 47.98%

The respective software-price volatility estimates (averaged over thirty trials) for one-, two-, three-, and four-year SRAs are 59.22%, 63.78%, 62.48% , and 63.26%. The corresponding exercise price volatility figures are 67.42%, 65.06%, 63.74%, and 64.52%. Since the illustration subsumes a four-year SRA, the relevant software price and opportunity cost volatility estimates are 63.26% and 64.52% respectively. Table 16 below combines the variance estimates for the two stochastic processes given in (2)

\(^{23}\) The number of sample values corresponds to that of months per year. However, the simulation may be replicated repeatedly for any time interval.

\(^{24}\) The price volatility measure actually used in options calculations is averaged over thirty trials, with each trial comprising twelve random sample values.
and (9) into a single average volatility parameter. The combined volatility measure is used as an input for computing the aggregate exit option value manifest in (37).

Table 16. Average Volatility \( (T = 4) \)

<table>
<thead>
<tr>
<th>( \sigma_s = 0.6326 )</th>
<th>( \sigma_x = 0.6452 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho_{sx} )</td>
<td>( \sigma )</td>
</tr>
<tr>
<td>-0.5</td>
<td>1.1066</td>
</tr>
<tr>
<td>0</td>
<td>0.9036</td>
</tr>
<tr>
<td>0.5</td>
<td>0.6390</td>
</tr>
</tbody>
</table>

Notes:

1) Equation (7) is used to calculate \( \sigma \).

2) The range for the correlation coefficient \( \rho_{sx} \) is chosen to show the volatility spread. A negative (positive) correlation coefficient implies that the increments to the Wiener processes for \( S \) and \( X \) are inversely (directly) related. In the case of zero correlation, software price movements have no impact on the exercise price. However, \( \rho_{sx} = 0.5 \) seems to be the most realistic estimate for the ASP scenario.

Table 17 below presents the opportunity cost fractions necessary to "exchange" the year-four exit opportunity for a premature termination.

Table 17. Exchange Ratios as Fractions of Year Four Option Exercise Price

<table>
<thead>
<tr>
<th>Exercise Price (PV)</th>
<th>Exchange Ratio (q)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10,697</td>
<td>1.0000</td>
</tr>
<tr>
<td>$6,406</td>
<td>0.5988</td>
</tr>
<tr>
<td>$2,886</td>
<td>0.2698</td>
</tr>
<tr>
<td>$1,000</td>
<td>0.0935</td>
</tr>
</tbody>
</table>

The individual \( q \)-ratios are used as inputs in (15) to generate corresponding price ratios \( P^* \) via an iterative process. The univariate and bivariate probabilities in (14) are corrected for the rate of forgone revenues or return shortfall (estimated as the ASP's
cost of capital) before the equation is applied to compute three sets of the pair-wise combination options series. Table 18 below shows the compound and the simple exchange option values.

Table 18. Exit Option Values \((T = 4)\) given a Stochastic Exercise Price

<table>
<thead>
<tr>
<th>Exit Points</th>
<th>(S) or (c(\cdot))</th>
<th>(X)</th>
<th>(q)</th>
<th>(P^*)</th>
<th>(c(\cdot))</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>$15,930</td>
<td>$10,697</td>
<td>1.0000</td>
<td>1.0000</td>
<td>$6,441</td>
</tr>
<tr>
<td>3</td>
<td>$6,441</td>
<td>$6,406</td>
<td>0.5988</td>
<td>1.3418</td>
<td>$5,528</td>
</tr>
<tr>
<td>2</td>
<td>$5,528</td>
<td>$2,886</td>
<td>0.2698</td>
<td>0.8785</td>
<td>$2,342</td>
</tr>
<tr>
<td>1</td>
<td>$2,342</td>
<td>$1,000</td>
<td>0.0935</td>
<td>0.5428</td>
<td>$1,260</td>
</tr>
</tbody>
</table>

Notes:

1) Equation (6) is used to calculate the year-four exit option, where \(c(\cdot) = c\{S, X, \tau\}\) and \(\tau = \tau_1 - \tau_2 = 1\).

2) \(c(3) = c\{c(4), q_3 X, \tau\}\) where, \(q_3\) is the exchange ratio for the year-three option.

3) A slightly modified equation (14) is used to calculate the truncated nested options series. For example,

\[c(3) = c(4) N_2(a_1, b_1, \rho) - X_3 N_2(a_2, b_2, \rho) - q_3 X_3 N_1(a_2)\]

Where, \(a_1 = \ln(P/P^*) + (\eta + 2\sigma^2)\tau_1\), \(b_1 = \ln(S/X) + (\eta + 2\sigma^2)\tau_2\), \(\eta\) is the rate of foregone revenues or return shortfall estimated as the ASP discount rate, and \(P^*\) is the associated critical ratio given by equation (15).

As expected, the year-four option possesses the highest value with the year-one-through-three compound options exhibiting a decreasing price trend. Such multi-stage options characteristic is also suggested in Trigeorgis (1993a), and Panayi and Trigeorgis (1998). The year-one, -two, -three, and -four options are then weighted by their respective conditional probabilities, following equation (34), to produce an
expectations measure of the cumulative exit option as shown in Table 19 below.

Substituting this value in (26), one obtains the option-laden SRA value.

Table 19. Aggregate Exit Option Price and Associated SRA Value \((T = 4)\)

| Exit Points | \(c(\cdot)\) | \(k(\cdot)\) | \(\Pr\{D_{r_4}=1|D_{r_5}=1\}\) \((a = c, s)\) | \(\Pr \ast c(\cdot)\) |
|-------------|-------------|-------------|---------------------------------|-----------------|
| 4           | $6,441      | 1.0000      | 1.00                            | $6,441          |
| 3           | $5,528      | 0.8582      | 0.74                            | $4,096          |
| 2           | $2,342      | 0.4236      | 0.28                            | $646            |
| 1           | $1,260      | 0.5381      | 0.20                            | $257            |

\[ C(4) = \sum \Pr \ast c(\cdot) \]
\[ V_c = V_0 + C(4) \]

Notes:

1) Equation (30) is used to calculate the discount factors \(k(\cdot)\).

2) Exit probabilities are based on the rationale underlying equation (28). For example, the exit probability for the year-four option implies the likelihood of exit on the fourth anniversary date given that termination of hosting services under the original SRA is inevitable at the end of year four. Since exit is certain for \(T=4\), given a four-year SRA, this probability is 1. Similarly, the exit probability for the year-three option denotes the likelihood of exit on the third anniversary date given that exit must occur at the end of year four. Such probability is calculated by subtracting \(N_{a_2, b_2, \rho}\) from 1, where \(N_{a_2, b_2, \rho}\) is the bivariate renewal probability for the year-three option. The modified equation (14) gives \(N_{a_2, b_2, \rho}\). The calculation is repeated for year-two and -one options.

3) Equation (34) yields the expected aggregate exit option price, and (26) then gives the option-laden SRA value.
Exit Times

As a rational, value-maximizing investor, the renter will exit at the decision implementation point $t$, where two qualifying conditions are jointly fulfilled. First, the respective DCF profiles of ownership and outsourcing intersect at $t$, and, second, the cumulative benefits of ownership are greater than that of outsourcing beyond $t$. Figure 3 below illustrates an optimal decision implementation point. The value-maximizing end user will backsource IT (i.e., exit the outsourcing mechanism by purchasing the applications software) at $t$ since the cumulative present value of the expected cash flows from software ownership exceeds that from outsourcing subsequent to $t$.

Figure 3. Cumulative DCF Profiles – Backsourcing vs. Outsourcing

Figure 4 introduces a switching-scenario DCF profile to the original situation depicted in Figure 3. One can easily infer from Figure 4 that the end user will exit at $t$ by switching to an alternative outsourcing mechanism.
Figure 4. Cumulative DCF Profiles – Backsourcing vs. Switching
CHAPTER 7

CONCLUSIONS

Renter Perspective

The Margrabe exchange-offer valuation model (1978) is applied for evaluating the flexibility captured in applications software outsourcing investments. A rapidly growing ASP market is expected to mature in the next four years. The IT landscape founded on the potential advantages of outsourcing business applications is evolving from a hybrid environment of failing and thriving niche firms to a tightly consolidated marketplace of few entrenched software solution vendors.

The applications software alternatives available today, which must also continuously evolve to outpace technological obsolescence, tend to follow the same volatile pattern that characterizes the ASP industry on the whole. This implies that while competing software applications offer marginally differentiated business functionality, the asking price for acquiring such functionality widely fluctuates across different hosting firms. One possible explanation is the speed at which the various solution vendors expect to integrate technological innovations into base software configurations.
Since prices impute future expectations (given a rental software market that is efficient in the Fama sense), uncertainty about sequential development in technology underlying software applications is manifest in price fluctuations. The same source of uncertainty that affects applications software price also impacts the value of flexibility embedded in rental contracts written on the applications software. While the traditional capital budgeting technique cannot appropriately price the risk that delineates uncertainty in rental software contracts, the options pricing analysis is able to quantify such risk. This assessment of risk affords the decision maker the ability to evaluate rental software contracts that offer sequential exchange opportunities at the end of a finite planning horizon. The exchange option is valuable if the decision implementation points coincide with technological changes that produce more efficient business solutions. Given the pace of technological obsolescence, the scenario assumed the decision implementation point to be one year. Further, the value of such embedded flexibility (given applications software alternatives) is found to increase at higher volatility levels, with the highest option prices (and investment values) tending to occur where the technological divergence between underlying applications environments is the greatest (correlation coefficient is the most negative).

Intuitively, if the advent of new technological information produces the same conversion effect on the respective base configurations for two software-package alternatives, the expected post-conversion software prices would quickly tend towards equilibrium levels as a result of competition in an efficient market. Thus, in the case of evaluating an investment that allows migration between competing outsourcing solutions with similar base configurations, the volatility impact of an innovation on
investment value is likely to be dampened. This means that at the decision implementation point, the decision maker would be less inclined to migrate to an alternative that offers similar sequential net benefits from the innovation. Conversely, with disparate base configurations, new technological information will give rise to innovations that are peculiar to each outsourcing solution. As the potential to integrate the individual innovations with the respective base configurations is impounded in underlying software prices, the efficient-market competition would tend to pit one outsourcing solution against the other causing software prices to fluctuate widely thereby reinforcing the volatility impact on investment value. In such a case, the flexibility to migrate to a competing rental software solution with higher sequential net benefits of conversion bears more value at the decision implementation point.

This analysis required comparison of two investment alternatives with the same finite planning horizon of one year. In more complex scenarios, where the decision maker must evaluate multiple investment alternatives, several unique two-project combinations can be constructed by using the expression, \( \frac{N!}{2! (N-2)!} \), where \( N \) is the total number of projects and \( ! \) is the factorial operator. The two projects comprising the highest-value combination should be evaluated for possible sequential exchange opportunity.

In summary, this analysis offers three major implications for IT managers evaluating rental software investments. One, the traditional NPV technique will consistently undervalue option-laden projects that involve outsourcing IT applications. By evaluating rental software investments using only the NPV approach, IT managers may reject potentially valuable projects. Two, a dynamic technological
environment requires flexible decision implementation points over the length of the planning horizon. Outsourcing contracts with a short expiration generate potential migration opportunities for IT managers that seek value-maximizing projects in a market characterized by rapid technological obsolescence. Three, while evaluating rental software alternatives, IT managers should also consider the underlying technology in terms of the direction of impact of new information. The outsourcing investment value tends to increase (as the embedded option value rises) given that new information causes the competing applications software prices to fluctuate in opposite directions.

**Subcontractor Perspective**

The analysis presents an original attempt to introduce and value exit flexibility embedded in software rental agreements using a sequential options valuation approach. The prime objective of any subcontractor or ASP, needless to say, is to offer an outsourcing mechanism that provides the 'best' value to the end user. As a result, the ASP industry has seen a rapid evolution of software rental strategies in the past few years. However, when growth occurs at such a phenomenal pace, it usually comes at a price. The current stagnation in the ASP market partially caused from a spillover effect of the recent debacle within the United States technology sector, may point to a weakness in the current practice of establishing appropriate software rental prices and promotions given the risk of technological obsolescence and client migration. The passive capital budgeting analysis incorporating, at the most, multiple discount-rate net present value calculations is ill equipped to handle risk resulting from a dynamic technological environment. A decision paradigm that incorporates such risk is
constructed by valuing a flexible SRA within an options framework. The option-embedding contract provides a means for a technologically sensitive end user to migrate to more efficient software applications in the course of business not only with a view to minimize losses from technological obsolescence, but also as a means of capitalizing on productivity gains.

The SRA embedding the option to terminate use of rental software creates an opportunity for the end user in the future to exploit technological innovations in IT, and thus maximize the net benefits from outsourcing. As the writer of this exit option, the ASP bears the risk of not recovering, at the minimum, the market price of the comparable shrink-wrapped software. The longer the end user outsources business applications to the ASP, the greater is the revenue generating potential of the rental contract. The present value of revenues obtained by the ASP as monthly subscription fees (or rent) over $T$ equals the current software price under two provisos. First, the underlying applications software is not subjected to any unanticipated technological developments during the SRA's life. If an innovation is inadvertently introduced in the future to the base applications package, the software price will increase or decrease depending on the usefulness and patentability of the innovation. The uncertainty regarding technological advancement in the ASP industry tends to cause the software prices to fluctuate. We further assume that the software-price volatility is invariant with respect to the SRA’s duration. Second, the renter is locked in for $T$ years without the possibility of exit. As with capital markets in Fama (1970), given an informationally efficient ASP market, any benefits of an early termination (if allowed) would be reflected in the present value of rental payments. Therefore, the SRA value
tends to diverge from the current software (continuing contract) price in the face of early exit opportunities.

A typical rental agreement usually stipulates the level of service that the ASP must provide the recipient over the contract's duration. If the ASP cannot meet the stipulated performance criteria, it is contractually bound to compensate the renter for any lost business. Faced with unreliable service, the renter may compel the ASP to surrender the contract releasing the renter from additional obligations. Since the purpose of this study is to extend the scope of real options framework to include rental software valuation as well as price the flexibility resident therein, the imposition of such arbitrary service-level penalties is a minor consideration and can be safely ignored. Moreover, the applications software solution is rented with complete functionality (module for module) as is precisely offered with the commercially available product.

The analysis presents a modification to the Carr’s compound exchange option formula to factor in the ASP risk premium (return shortfall) prior to calculating a truncated series of nested options. The calculations demonstrate that the option-laden SRA adds value to the outsourcing mechanism as predicted. Moreover, such value is a linear combination of the prices of individual embedded exit opportunities. Since the intermediate exit options can be expressed in terms of the terminal exit opportunity, the analyst need only calculate the simple option value for the final exit point. The most important benefit of such calculation is that it avoids using time consuming and obfuscating analysis. Moreover, the value of building flexibility into an SRA can be
easily simulated for a perceived set of discount factors and exit probabilities. Tables 20 through 23 below describe one such example.

Assume that an analyst (after studying the market behavior of ASP clients) determines that certain end users, exploring the rental alternative, will terminate the contract eighty percent of the time given one exit point. Further, the same recipient firms display a reduced propensity to terminate as the number of exit options available with the SRA increases until all must exit at the end of the contract’s life. Table 20 shows the relevant exit probabilities. In addition, the analyst gathers information about the ASP’s required marginal or incremental returns given the number of exit points embedded in the SRA. These returns are represented by the discount factors in Table 20. A simple option price of $5000 (given the discount factors, exit probabilities, and exit points) yields an aggregate expected flexibility value of $7960 as shown in Table 23. In essence, this is the value of the ‘cushion’ or the risk premium required to offer exit flexibility with the SRA.

Table 20. Four-Year SRA Decision Example

<table>
<thead>
<tr>
<th>Exit Points</th>
<th>Discount Factors k(·)</th>
<th>Exit Probabilities Pr{·}</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simple Terminal Exit Option Value</td>
<td>c(4) = $5,000</td>
<td>1.0</td>
</tr>
<tr>
<td>3</td>
<td>0.8</td>
<td>0.26</td>
</tr>
<tr>
<td>2</td>
<td>0.5</td>
<td>0.72</td>
</tr>
<tr>
<td>1</td>
<td>0.3</td>
<td>0.80</td>
</tr>
</tbody>
</table>
Table 21. 3 x 1 Discount-Factor Matrix

<table>
<thead>
<tr>
<th>Matrix Elements</th>
<th>k</th>
</tr>
</thead>
<tbody>
<tr>
<td>k(3)</td>
<td>0.80</td>
</tr>
<tr>
<td>k(2) * k(3)</td>
<td>0.40</td>
</tr>
<tr>
<td>k(1) * k(2) * k(3)</td>
<td>0.12</td>
</tr>
</tbody>
</table>

Table 22. 1 x 1 Probability-Discount-Factor Matrix

\[
p_k = p_3 k(3) + p_2 k(2) k(3) + p_1 k(1) k(2) k(3) = 0.592
\]

Table 23. Aggregate Option Value

\[
C(4) = c(4) [p_k + 1] = 7,960
\]

Notes:

1) The value of the year-four exit option can be derived from the underlying applications software price using equation (3) or (6) regardless of the number of available intermediate exit opportunities.

2) The aggregate option value calculated for the given decision scenario is based on equation (37).

The option-laden SRA adds value to the rental mechanism since the end user enjoys the privilege of surveying the IT landscape in an attempt to capitalize on favorable applications software innovations at one of several exit points. The ability to create a range of risk premiums for offering rental software with similar sequential option(s) presents an important practical implication to managers seeking firm-value maximization. In general, omitting flexibility from IT mechanisms offered in today’s technologically volatile markets could unfavorably affect provider revenues.
Additionally, the failure to assess an appropriate value for in-project flexibility can have a deleterious impact on the provider's long-term financial viability.

**Exit Times**

A perpetual software outsourcing software contract places an extraordinary burden of risk on both the subcontractor and the renter. The former party is faced with the possibility of future capital as well as operating losses in the presence of technological obsolescence. Similarly, the renter is exposed to the likelihood of business attrition if outsourcing fails to deliver potential economies. On the other hand, the two parties have the opportunity to gain from a flexible outsourcing arrangement. The subcontractor reaps a premium for assuming the additional risk of opportunity loss, and the renter enjoys the option of migrating to alternative outsourcing mechanisms with the objective of capitalizing on beneficial software innovations.

However, the full valuation impact of offering options with IT services, namely, software renting, may not be discerned without further study. One possible future research direction envisioned for the type of analysis described in this study incorporates the valuation of software applications that are rented by module with the number of modules varying stochastically.
BIBLIOGRAPHY


Fox, P., "IT Services for Rent." Computer World (November 20, 2000).


Keegan, P., "The Death of Software?" *Upside* (October 1999).


Laberis, B., "Some of the Myths and Realities Behind ASPs." *Computer World* (May 1, 2000).


_________. "ASPs Gain Ground." *Information Week* (October 9, 2000b).


Schmerken, I., "Everybody Wants to be an ASP." *Information Week* (April 24, 2000).


Turek, N., "Competition Means More Choices in Web Hosting: Providers offer more extensive services, scores of hosting centers, custom services." *Information Week* (June 12, 2000).


Whiting, R., "Software Morph’s into a Service!" *Information Week* (October 11, 1999).