Spring 2003

Design and fabrication of a microscale Joule-Thomson refrigerator

Lionel Gamini Hewavitharana

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We hereby recommend that the thesis prepared under our supervision by Lionel Gamini Hewavitharana titled Design and Fabrication of a Micorscale Joule-Thomson Refrigerator be accepted in partial fulfillment of the requirements for the Degree of Doctor of Philosophy.

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ABSTRACT

A simple thermodynamic, heat transfer, and fluid flow model was developed for microscale Joule-Thomson refrigerators (JT devices). For a given geometry, the model predicted that the cooling capacity of the refrigerator increased with the inlet refrigerant pressure. The effectiveness of the JT device also increased with the inlet pressure, and the heat exchanger channel length. At a constant inlet pressure, the effectiveness, and the refrigeration capacity of a given JT device increased as the aspect ratio of heat exchanger channels was increased. For nitrogen refrigerant, the model predicted that it was possible to obtain approximately 250 mW of refrigeration capacity at 82 K with 10 MPa (100 atm) of inlet pressure and a flow rate of 15.17 ml/s at standard pressure and temperature (STP). This prediction was justified by experimental values of Little (1984) who obtained 250 mW of refrigeration capacity at 83 K with 10 MPa (100 atm) of inlet pressure and a flow rate of 18 ml/s at STP. The simulation model was also used to design a novel JT device based on a layered arrangement of the evaporator, capillary, and the heat exchanger. The proposed JT device would have produced approximately 250 mW of refrigeration capacity at 100 K, for an inlet pressure of 6 MPa (60 atm). This proposed JT device was fabricated on silicon wafers using photolithography. The heat exchanger channels had a cross section of 50 X 20 μm and a length of 6 cm. The capillary channel cross section was 20 X 20 μm and its length was 6 cm. Both the length and the width of
the evaporator was 30 mm, and its depth was 20 μm. Pyrex 7740 glass wafers (3 mm thick) were used to separate the evaporator from the capillary and the capillary from the heat exchanger. The heat exchanger was bonded with a top glass cover plate. Most layers were successfully bonded using the anodic bonding procedure. After bonding the evaporator to a glass wafer, subsequent anodic bonding was carried out by applying voltage from sides of each glass and silicon wafer. This bonding attempt demonstrated that the anodic bonding procedure could be used in packaging several silicon and glass wafers. The packaged device held together briefly but later separated due to poor bonding quality of the capillary and the heat exchanger. This poor bonding quality may have resulted from inadequate surface quality of silicon wafers. However, the knowledge and the experience gained in this work will be very useful in future development of JT devices.
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CHAPTER 1

INTRODUCTION

Our ever-increasing desire to pack more and more transistors into a single chip to enhance the processing power of computers led to significant developments and improvements in the process of photolithography. In 1975, Moor presented his famous curve that predicted the packaging density of computer chips. Over the years, this curve has served the electronic industry well by projecting the photolithography requirements. The technology used in electronic industry opened up new possibilities in other areas of engineering, especially in mechanical and biomedical engineering fields. The emergence of microsystem engineering is a direct result of the influence of photolithography technology on other branches of engineering. The technology is successfully exploited to create three-dimensional Microelectromechanical Systems (MEMS). While the electronics are fabricated using integrated process sequence, the micromechanical components are fabricated using compatible micromachining processes that selectively etch away parts of silicon wafer or deposit three-dimensional layers to construct mechanical devices. Because silicon is the base material for both electronic chip fabrication and micromechanical components, it is possible to integrate electronics and micromechanical components on the same silicon wafer to build a complete system, which is called a system on a chip.
The market potential of MEMS devices was realized by universities and government agencies, compelling them to undertake technology demonstration projects. Figures 1.1 through 1.3 present samples of some MEMS devices developed in university laboratories.

The MEMS devices have now ventured out of research laboratories and are being successfully exploited in the market atmosphere. Commercial applications are mainly found in the automotive, information, and biotechnology industries. For example, MEMS accelerometers are already incorporated into crash-air-bag technology of modern automobiles. These accelerometers are more reliable, more functional, lighter, and cheaper than the conventional counterpart.

The MEMS technology has continued to expand and influenced the miniaturization of fluidic devices. However, miniaturization of fluidic devices is more than reducing the geometry. Some aspects of fluid flow behavior in microgeometries may
be different from macroflow situation. In a microflow situation, the surface-to-volume ratio is very high, and the effect of surface tension might be a significant problem that requires attention. Therefore, the development of microfluidic devices demanded related theoretical developments in fluid flow and heat transfer. During the last decade, theoretical understanding of flow and heat transfer in microgeometries has improved considerably. The commercial development of microfluidic devices is growing. Some of these components are micropumps, microvalves, microdiffusers, micromixers, and microcooling devices. The most visible microfluidic application is found in the ink-jet printer. Figures 1.3 and 1.4 represent some of the microfluidic components for which the fabrication technology has already been established.
As the miniaturization continued, researchers started developing microcooling devices using MEMS technology to meet cooling requirements of small electronic components and computer chips. Microheat exchangers, microheat pipes, and microrefrigerators based on Joule-Thomson cooling have drawn attention for further development.


Figure 1.3 Microfluidic Valve - University of California, Berkley

Joule-Thomson coolers have been used in many applications because of their portability, fast cool-down time, constant operating temperature, and simplicity of operation. They are also finding many applications with the military industry. Small, compact Joule-Thomson coolers were used to cool infrared devices such as airborne surveillance systems. Those coolers were rugged, reliable, quiet, inexpensive, and could be operated over a wide range of ambient temperatures. Very often, Joule-Thomson coolers operated on the open cycle. In the 1960's, closed-cycle Joule-Thomson coolers were developed and found to be as reliable and good as Stirling coolers. However, over time, these coolers lost out to Stirling coolers mainly due to their lower thermodynamic efficiency.
The unprecedented developments in semiconductor technology opened up new opportunities for Joule-Thomson refrigerators. As the component density of computer chip increases due to smaller line width, so does the power dissipation. Therefore, new cooling devices, methods, and techniques are required for satisfactory removal of heat from computer chips and other small power-dissipating components. Using the same fabrication technology as in the semiconductor industry, microscale cooling devices can be fabricated to meet most cooling requirements. Joule-Thomson refrigerators are prime candidates for such miniaturization and therefore have drawn renewed interest from many researchers.

This dissertation deals with the design and fabrication of a microscale Joule-Thomson refrigerator (JT device). First, a thermodynamic, heat transfer, and fluid flow
simulation model for JT device is presented. This model is then used to optimize the geometry of a JT device, based on a layered arrangement, for the maximum possible cooling capacity. Based upon these simulation results, a prototype device was constructed using a layered arrangement of silicon and glass layers.

Chapter 2 of this dissertation presents a survey of previous work related to Joule-Thomson refrigerators. A brief description of the theoretical background of Joule-Thomson refrigeration is presented in Chapter 3. Chapter 4 presents a brief overview of photolithography and wafer bonding process. Chapter 5 is devoted to the development of a simulation model. Chapter 6 describes the use of the model to obtain the geometry of a JT device based on a layered arrangement. Chapter 7 presents the fabrication process while Chapter 8 describes the packaging of the device. Finally, Chapter 9 presents the discussions and conclusions.
CHAPTER 2

RELATED RESEARCH

Previous research related to Joule-Thomson cooling has been carried out in mainly in three areas: (1) thermodynamic studies (2) Joule-Thomson cryocoolers, and (3) microminiature Joule-Thomson refrigerators. Some of thermodynamic studies included an investigation of the Joule-Thomson inversion curve, transient characteristics of Joule-Thomson cooling, and the development of the concept of an integral inversion curve. In this chapter, the emphasis is placed upon the work related to Joule-Thomson cryocoolers and microminiature Joule-Thomson refrigerators.

2.1 Thermodynamic Studies

Thermodynamic studies on the Joule-Thomson effect have concentrated on studying the inversion curve, transient characteristics of Joule-Thomson cooling (JT cooling), prediction of the temperature after expansion, performance of ideal flow regulated coolers, and the development of the concept of an integral inversion curve. The inversion curve of a cryogenic refrigerant provides the acceptable pressure-temperature range for JT cooling. Therefore, establishment of the inversion curve was considered a preliminary step in the design of a liquefaction system for a particular refrigerant. Relevant literature in this regard can be found in the work of Gunn et al. (1966).

Joule-Thomson refrigerators very often operate as an open-cycle system to avoid the possible complexities of using compressors. Therefore the refrigerant is usually drawn from a high-pressure cylinder. Under such condition, available operating time should be determined. Studies related to cool down similarity, operating time, and cool-down time can be found in the work of Bodio and Wilczek (1981), Bodio and Wilczek (1984), Maytal (1992), Maytal (1994), and Chou et al. (1995).

The temperature of the refrigerant after expansion is an important parameter in simulation studies of JT coolers. This temperature can be determined by equating the enthalpies of refrigerant before and after expansion. Chou et al. (1993) presented two methods of determining the temperature of the refrigerant after expansion. These two methods were unreliable for gas temperatures below 200 K before the expansion.

2.2 Joule-Thomson Cryocoolers

Joule-Thomson cryocoolers were mainly used to cool infrared devices such as airborne surveillance systems. These coolers operated as open cycle systems and had larger dimensions.

Geist and Lashmet (1966) investigated a cryostat with two stages. The first stage of the cryostat used nitrogen to pre-cool hydrogen. Pre-cooled hydrogen then underwent Joule-Thomson expansion in the second stage. The device achieved a maximum cooling capacity of 4 Watts at 20 K. They compared throttling and non-throttling devices and found that the throttling type performed better due to its ability to respond to varying cooling loads.
Geist and Lashmet (1966) discussed a small JT cryocooler, basically a test apparatus, to evaluate the performance of different types of heat exchangers. They studied a pin fin heat exchanger, a coiled fin heat exchanger, a spiral coiled tube heat exchanger, and a helically finned and coiled heat exchanger. Heat exchangers were evaluated on the basis of time taken to attain the boiling temperature of hydrogen. They concluded that helically finned and coiled heat exchangers were the best for JT cryocoolers.

Currie (1967) describes a three-stage JT cryocooler designed to achieve a cooling temperature of 4 K. The first stage consisted of a nitrogen bath to cool hydrogen. In the second stage, hydrogen underwent Joule-Thomson expansion, and the cold hydrogen gas was used to pre-cool helium in a counter flow heat exchanger. Final cooling was achieved by Joule-Thomson expansion of helium at a temperature of 4 K. The notable feature of this JT cryocooler is the use of nitrogen bath in the first stage for a Joule-Thomson cooling stage with nitrogen to cool hydrogen. The author's main argument for the use of nitrogen bath was that it would reduce the complexities of the overall system.

Buller (1971) investigated a small Joule-Thomson refrigerator. The heat exchanger of the cryostat was 0.208 inch in diameter and 2 inches long. A small tube with an outside diameter of 0.208 inch and an inside diameter of 0.012 inch was used to send high-pressure gas. The tube was wrapped around the body of the heat exchanger and consisted of an extended heat-transfer surface soldered to the outside surface of tubing. The gas expanded through an orifice at the end of tubing and was controlled by a needle. The needle was fitted onto a bellow super critically charged with the refrigerant. This bellow responded to temperature changes and controlled the movement of the needle.
Buller (1971) made a theoretical prediction of 50% improvement in operating time for throttling-type cryostat than a fixed-orifice type. However, he experienced an improvement of more than 500%, for which he could not give a satisfactory explanation.

Buller (1971) also undertook a study to predict the operating time of JT cryocoolers. Operating times for both throttling and non-throttling were predicted. He predicted the operating time of non-throttling type cryocoolers directly from mathematical relations. However, a numerical procedure had been used to predict the operating time of throttling type cryocoolers because the refrigerant flow rate varied according to the cooling load.

The design of a multi-stage JT cryocooler capable of producing 4 K has been discussed by Chan et al. (1981). The design consisted of three stages: in the first stage, pre-cooled hydrogen using JT cooling of nitrogen was used; in the second stage, hydrogen underwent JT cooling and pre-cooled helium; in the third stage, JT expansion of helium produced a final cooling temperature of 4 K. The heat load designed to be removed at 4 K was 1 mW. The authors proposed charcoal adsorption desorption compressors for all three stages.

An experimental and numerical study on the steady-state characteristics of a Joule-Thomson cooler was done by Ng et al. (2002). They developed the governing equations for heat transfer in high-pressure gas flow, low-pressure gas flow, finned mandrel, and shield tube. The modified Benedict-Webb-Rubin (MBWR) equation of state was used to determine the thermo-physical properties of argon, the refrigerant in their model. Temperature dependency of specific heat capacity, viscosity, and thermal conductivity was also included in the numerical model. They solved the governing
differential equations using a finite volume approach. During the iteration, the thermo­
physical properties of argon were updated according to the local pressure and
temperature. Simulated results for cold fluid outlet temperature were in good agreement
with the experimentally obtained values for cold fluid outlet temperature.

2.3 Microscale Joule-Thomson Refrigerators

The fast growth of computer technology increased the speed of processing power
mainly by the increased electronic component density in the microchip. The inevitable
outcome of this process was increased power dissipation. Therefore, in addition to
conventional convection type cooling, novel techniques of cooling have been pursued.
Little (1978a) at Stanford University first proposed the idea of developing microscale
Joule-Thomson refrigerators (JT devices) to cool an electronic chip. He derived a set of
scaling laws and showed that JT cryocoolers can further be miniaturized so that they can
be used in miniature electronic component cooling. For the same effectiveness and
pressure drop of two heat exchangers, the hydraulic diameter and length of tubing would
scale according to following relations.

\[ d \approx m^{0.5} \]  \hspace{1cm} \text{first scaling law} \hspace{1cm} (2.1)

\[ l \approx m^{0.6} \]  \hspace{1cm} \text{second scaling law} \hspace{1cm} (2.2)

cool down time \approx m^{0.6} \hspace{1cm} \text{third scaling law} \hspace{1cm} (2.3)

operating time \approx m^{0.6} \hspace{1cm} \text{fourth scaling law} \hspace{1cm} (2.4)

where \( d \) is the diameter of the channel, \( l \) is the length of the channel, and \( m \) is the
mass flow rate of the refrigerant.
Little (1978b) also reported the first attempt to manufacture a microscale JT device. He used photolithography to fabricate the device. Operation with CO$_2$ gas was reported to be successful. However, experiments with N$_2$ gas did not produce successful results. He attributed the poor result mainly to wall conduction.

Little (1980) discussed the effect of Reynolds number in designing the heat exchanger of the JT device. He extended his scale analysis to show that laminar flow conditions alone can be considered for design of the heat exchanger of the JT device.

Hollman and Little (1980) reported the first successful construction of a JT device suitable for nitrogen refrigerant. This device was fabricated on amorphous low conductivity glass material. The authors mentioned that they had to develop a new technique to etch channels on glass but did not divulge any information of the process. The refrigeration capacity was 100 mW at 122 K with inlet nitrogen gas pressure being 11 MPa. The authors did not provide any information on the geometry of the device. They merely mentioned that the dimensions of the channels were on the order of 100 micrometers.

The idea of scaling down the geometry for same operating conditions and effectiveness of heat exchanger was further discussed by Little in another paper published in 1982. He showed that, in addition to his previously published scaling relations, the Reynolds number scaled with the mass flow rate according to Equation 2.5.

\[
Re \approx \frac{m^{0.5}}{\eta}
\]  

(2.5)

In this paper he advocated the design of the counter flow heat exchanger to meet laminar flow conditions and proceeded to fabricate it on a glass wafer. The heat exchanger was also roughened by using 27 μm Al$_2$O$_3$ to increase the roughness. The JT device so
fabricated had a capacity of 250 mW and operated at 83 K. Nitrogen gas at 100 atm pressures was used as the refrigerant. The device had a gas flow rate of 18 ml/s at standard temperature and pressure (STP).

Garvey et al. (1983) described a JT device having a refrigeration load of 25 mW. The fabrication of the device was influenced by the work of W. A. Little at Stanford University. The high-pressure 4.8 cm long inlet channel had a width of 50 μm and a depth of 20 μm. The capillary was 1.7 cm long and had a channel width of 50 μm and a depth of 7μm. Low-pressure return was 1.3 cm long and had a 1500 μm channel width. The depth of the return channel varied from 10 μm at the reservoir to 20 μm at the outlet.

Wu and Little (1983) investigated the friction factor in microchannel fluid flow. They undertook this study primarily to examine whether the Moody chart can be used to design gas channels of counter flow heat exchanger of JT devices. Three test channels were etched using the photolithography procedure and on three Si wafers with different dimensions. The relevant hydraulic diameters were 55.81 μm, 55.92 μm, and 72.38 μm. The surface of the third test channel (hydraulic diameter 72.38 μm) was smooth. The surface of the first channel (hydraulic diameter 55.81 μm) was also smooth except for a few discrete protuberances on the bottom surface. The second test channel (hydraulic diameter 55.92 μm) had a rough section distributed over a part of the channel surface. They used 0.27 μm Al₂O₃ powder and abrassistively etched five test channels on five different glass wafers. The reported friction factor data in laminar flow region were apparently greater than theoretical values for rough channels. Their data for smooth channels seemed in agreement with the Moody chart but again contradicted the findings of Choi et al. (1991) and Pfahler et al. (1991). The greatest deviation was reported for
glass channels. This study also showed that, in rough glass channels, transition from laminar to turbulent occurred at a Reynolds number close to 350. However, for relatively smoother channels on glass, the transition occurred at a Reynolds number of 900. Correlations proposed by Wu and Little (1983) for friction factor in each flow regime for glass channels are presented in following equations.

\[
f = \frac{(110 \pm 8)}{Re} \quad \text{for } Re \leq 900 \quad (2.6)
\]

\[
f = 0.165(3.48 - \log Re)^{3.4} + (0.081 \pm 0.007)
\quad \text{for } 900 < Re < 3000 \quad (2.7)
\]

\[
f = \frac{(0.195 \pm 0.017)}{Re^{0.11}} \quad \text{for } 3000 < Re < 15000 \quad (2.8)
\]

In 1984 W. A. Little produced an excellent paper that summarized the knowledge developed throughout a period of ten years. A variety of topics relevant to the design of JT devices were discussed. Such discussion included scale consideration, flow regime consideration, roughness effects, influence of temperature and pressure-dependent fluid properties, fabrication techniques, performance characteristics, closed-cycle operation, and future applications.

Having studied the friction factor in microchannel fluid flow, Wu and Little (1984) examined the heat transfer in microchannel counter flow heat exchangers because such a heat exchanger is an integral part of a JT device. The objective was to obtain a correlation for the Nusselt number (\(Nu\)) under similar conditions experienced by a JT heat exchanger. Microchannels of three different cross-sectional shapes were used in their tests. The channel sizes were larger when compared with channel sizes used by other investigators. For instance, channel width ranged from 312 \(\mu\)m to 548 \(\mu\)m and channel...
depths ranged from 89 to 97 μm. They drew nitrogen gas from a high-pressure cylinder at ambient temperature, filtered and sent through a channel in the test piece. Nitrogen gas at the outlet was directed through a small tube wrapped around a nitrogen liquid cylinder to the other channel in the test piece. Two test pieces had the conventional heat exchanger lay out where heat transfer occurred across the separating wall. In one test section heat transfer occurred across a separating plate because two channels were on the base and the cover plate respectively. In the other test section, there was a channel having a width of 526 μm. Two narrow channels were on either side of the central channel and each narrow channel had a width of 32 μm. They established a relation for the Nusselt number which is given by Equation (2.9).

\[ Nu = 0.00222 \Pr^{0.4} \text{Re}^{1.09} \]  

(2.9)

In this investigation, they also noted that the Reynolds’s analogy did not hold for roughened channels. In the turbulent region, for the same Reynolds number, the Nusselt number for the channel heated from two sidewalls was greater than the one for channels heated from a single side wall. This difference was more pronounced at higher Reynolds numbers. The use of Equation 2.9 is erroneous in the turbulent region for channels heated from two walls.

Alfeev et al. (1973) from the Soviet Union first found out that nitrogen and hydrocarbon gas mixture produced higher cooling efficiencies. This fact was subsequently verified in the U.S.A. (Little, 1990). Use of gas mixtures in JT devices produced a substantial increase of cooling capacity. This result should be mainly due to the different thermodynamic properties of the mixture. At certain pressures, there were oscillations of cold stage temperature. This finding was attributed to the coexistence of
liquid phases of two different compositions and different boiling points in the evaporator and the heat exchanger.

Mikulin et al. (1992) reported on the construction of metallic planar JT devices. They reported about constructing rectangular type and circular disk type refrigerators. They constructed two rectangular type JT devices having two steel plates (thickness 0.3 mm) and a dividing copper plate (thickness 0.2 mm) between two steel plates. Each steel plate had a high-pressure 0.2 m long gas channel with a rectangular cross-section (0.65 x 0.13 mm) and two low-pressure gas channels with a cross-section of 2.25 x 0.12 mm. The channels were etched in zigzag arrangements. The plates also had 2 mm long restrictors with triangular cross sections. However, they did not describe the packaging clearly. It was not clear whether they were serially connecting channels or the device is a layered arrangement. The other design was a conventional type JT device. All channels and the reservoir were etched on one steel plate and covered by a steel cover plate. In the circular type device, gas channels spiraled towards the center of the plate. A circular plate, that has a central orifice, separated the other plane of construction. Because the geometry of the device was vaguely described, the flow of refrigerant could not be fully understood. Their experimental results showed that the circular type had the lowest cool down time. The packaging of this device was also not clearly presented.

Despite apparent variation of the friction factor from theoretical values, they noted that the relation between the friction factor ($f$) and the Reynolds number ($Re$) remained linear and the slope of the straight line was the same as in the Moody chart.

The counter flow heat exchanger is a very important component in the JT device. In case of nitrogen refrigerant, it should be able to cool down the hot fluid up to about
150 K before the Joule-Thomson expansion. Two-phase conditions will not exist in the evaporator of JT cryocoolers for heat exchanger effectiveness less than 85 percent (Barron, 1985). It is believed that the effectiveness should be even higher in JT devices primarily due to wall heat conduction.

Yuen (1996) at the University of California at Santa Barbara investigated the two-phase flow in counter flow heat exchangers used in JT devices. He fabricated three test heat exchangers. The first heat exchanger had an aspect ratio (between the depth and the width) of 1.6:1, but the second and third were fabricated with an aspect ratio of 3.4:1 primarily to increase the heat transfer area. The depth of microchannels was 813 μm and width was either 508 or 239 μm. The wall thickness, in all cases, was 102 μm with 36 parallel channels in the first heat exchanger. The other two heat exchangers that had an aspect ratio of 3.4:1 had 65 parallel channels. These depths and widths are quite large compared with actual channel depths and widths used in JT devices. The channel length was about 3 cm.

Hewavitharana (1997) examined a LIGA-fabricated JT device and developed a simulation model using TK SOLVER software. However, this model was specific to the particular JT device and could be used as the basis of a new general-purpose simulation model.

Sreedhar and Sreedhar (1998) examined the possibility of using mixture of nitrogen and neon in JT devices to achieve cooling temperatures below 77 K. They derived an equation for the maximum inversion temperature of a binary mixture of gases as a function of the composition of the mixture. The lower cooling temperature that can
be obtained with such a mixture depends on the molar ratio of nitrogen. They estimated that a reduction of 11 K would be attainable.

Frank T. Hartley and Jack A. Jones (1998) at California Institute of Technology undertook to find a solution to the cooling requirement problems of smaller spacecraft and portable terrestrial equipment. They proposed a JT device that uses a mixture of gas on a closed cycle with a miniature peristaltic pump to circulate the refrigerant. Even though they seem to have obtained the patent rights for the pump and gas mixture, there were no reports of fabrication of an actual device.

Narayanan and Venkatarathnam (1999) examined the performance of the counterflow heat exchanger with heat loss through the wall at the cold end. They considered the energy balance in the heat exchanger and developed the governing equations. They assumed negligible thermal resistance between the cold wall end and the surroundings in the evaporator. It was essential for them to assume this condition to obtain closed form solutions for the governing equations. They obtained theoretical expression for hot fluid outlet temperature, cold fluid outlet temperature, and effectiveness of the heat exchanger for balanced operation. They in fact defined two nondimensional effectiveness expressions—one for hot fluid and one for cold fluid and related each other through the nondimensional heat transfer at the cold end. For unbalanced operation they implemented a numerical procedure to obtain solutions. According to the findings of Narayanan and Venkatarathnam (1999a), when the hot fluid is the minimum fluid or the flow is balanced, the heat leak at the cold end decreased with increased Number of Transfer Units (NTU). However, when the cold fluid is the minimum fluid, as exists in JT devices, the cold end heat loss increased with NTU. They suggested that performance of JT
devices would be better with mixed refrigerants because they tend to have a higher heat
capacity rate in the cold stream.

Narayanan and Venkatarathnam (1999b) undertook another study to understand
the effects of winding flow arrangements in the counter flow heat exchanger on its
performance. The considered a channel layout that had a central hot fluid channel with a
cold fluid channel on either side. Governing equations were derived using the energy
balance for the heat exchanger. The equations showed that they had considered two
dimensional heat transfers in the solid section of the heat exchanger as well as in fluid
that flowed through hot and cold fluid channels. They used the finite difference method
to solve the governing equations along with the relevant boundary conditions. A
minimum number of 845 grids each for the solid section and the fluid channels were
used. They found that the performance of the heat exchanger was strongly dependent
upon the number of U-turns in the heat exchanger because the wall length is greater and
therefore so is the thermal resistance of the wall. They believed that the counter flow
heat exchanger with many U-turns in the flow arrangement would perform similar to a
pure counter flow heat exchanger without any heat leak from the cold end. However, an
increased number of U-turns would also increase the pressure drop and therefore needs to
be kept at a permissible level.

Ameel and Hewavitharana (1999) studied the counter current heat exchanger of
a JT device under external heat leaks to both fluid streams. Their work suggested that the
performance of the heat exchanger was severely affected by the external heat transfer in
some situations, and hot fluid and cold fluid temperature profiles crossed each other
resulting hot fluid heating instead of cooling.
Hewavitharana and Hegab (2000) presented a preliminary simulation work of a JT device based on layered arrangements. Both the counter flow heat exchanger and the capillary were assumed to have multiple channels. The simulation assumed that there was no longitudinal wall conductance because the heat exchanger channels were not connected to the evaporator. They also assumed that there were no external heat leaks to both fluids in the heat exchanger because the heat exchanger would be sandwiched between a top glass cover plate and another separating glass plate placed between the capillary and the heat exchanger. This simulation study showed that it would be possible to achieve about 250 mW of cooling at an initial nitrogen gas pressure of 60 atm. According to previously reported data, such a cooling was possible around 100 atm pressure.

This literature survey finds that W. A. Little at Stanford University did the pioneering contribution to the development of JT devices. He scaled down known JT cryocoolers and fabricated prototype models. He also undertook friction and heat transfer studies on counter flow heat exchangers fabricated using photolithography technology. One suspects that scaled down counter flow heat exchanger did not produce satisfactory performance and Little had to carry out friction and heat transfer studies in micro channels to understand the actual physics of the fluid flow system in microgeometries. However, his contribution is indisputable.

Subsequent studies on JT device confined mainly to the counter flow heat exchanger because of its importance in the microscale JT system. Effect of external heat transfer to both fluids, longitudinal wall conduction, and winding flow arrangements in the counter flow heat exchanger have been studied. Limited information related to the
fabrication and the operation characteristics is also available in the literature. However, there have been no attempts to develop a general-purpose simulation model for the complete JT device and use it as a design tool. This dissertation addresses this issue and presents a general-purpose simulation model for JT devices as well as the construction of a prototype device from that model.
CHAPTER 3

JOULE-THOMSON REFRIGERATION

The isenthalpic expansion of a real gas experiences a temperature change due to the reduction in pressure. If the expansion produces a temperature drop, then the process is known as Joule-Thomson cooling (JT cooling). Joule-Thomson cooling is one of the oldest cooling methods known to researchers. Figure 3.1 shows an isenthalpic expansion of a real gas. It can be seen that at some enthalpies, temperature increases as the pressure drops, reaching a maximum. However, further decrease of pressure produces a decrease of temperature. The cooling region enclosed by the inversion curve and the temperature axis is identified as the cooling region. Figure 3.2 shows the Joule-Thomson curve for nitrogen. The JT coefficient is positive in the cooling region and negative outside the inversion curve. If a real gas has a positive JT coefficient, it will produce cooling after expansion. Some gases, such as hydrogen and helium, have a maximum inversion temperature (point T₁ in Figure 3.1) lower than room temperature, requiring that they be precooled to produce cooling after expansion.

JT cooling is found mainly in liquefaction systems. In a simple liquefaction system, as shown in Figure 3.2, the cryogenic refrigerant is isothermally compressed to a very high pressure and then sent through a counter-flow heat exchanger. The compression process is shown in the temperature-entropy (T-S) diagram (Figure 3.4) of.
the system as 1-2. In liquefaction systems, the heat exchanger is designed to have a minimal pressure drop and a very high effectiveness. It is interesting to note that no liquid could be produced in the evaporator for effectiveness less than 85 percent (Barron. 1985).
Figure 3.3 shows the Joule-Thomson curve for nitrogen. It is observed from Figure 3.2 that the optimum inlet nitrogen gas pressure that will produce cooling after expansion is about 40 MPa (400 atm). The maximum inversion temperature is around 600 K, and the minimum inversion temperature is 100 K. Nitrogen has a fairly wide range of inversion temperatures so that room-temperature nitrogen can produce cooling after expansion. In JT devices, the incoming nitrogen gas is cooled in a heat exchanger to about 150 K to obtain a two-phase condition in the evaporator. Figure 3.3 also shows that nitrogen should not be cooled below 100 K in the heat exchanger because the minimum inversion temperature of nitrogen is 100 K, and at any temperature below 100 K, gas the heating occurs after expansion. The compressed refrigerant stream is usually known as hot fluid stream. A significant temperature reduction of the hot fluid stream is achieved in the heat exchanger (process 2-3 in Figure 3.4). Subsequently, the hot fluid stream undergoes an isenthalpic expansion (process 3-4 in Figure 3.4) producing a mixture of the cold gas and liquid refrigerant. The liquid is collected in the liquid reservoir while the
cold gas is sent through the heat exchanger and sent to the compressor.

Small refrigerators based on JT cooling have been used to cool infrared devices, cryopumps, electron microscopes, and ion emission microscopes. These refrigerators are known as cryostats, cryocoolers, or micro JT refrigerators. Such a refrigerator is shown in Figure 3.5. They are highly reliable, have low noise, and have a fast cool-down time.

When the Joule-Thomson coefficient of a refrigerant has a negative value at room temperature, it should be pre-cooled by a refrigerant, which has a positive JT coefficient at room temperature. Therefore, when very low temperatures are required, hydrogen, helium, or neon should be used as the refrigerant. Under such conditions, JT refrigeration
consists of multiple stages. The first stage is JT pre-cooling using nitrogen to cool hydrogen, helium, or neon.

Cooling capacity of JT cryocoolers exceed the heat removal requirement of most electronic devices, causing a mismatch between the cooling requirement and the actual cooling. Therefore, the need for further miniaturization of JT refrigerators has arisen to overcome this cooling mismatch. This need was also highlighted by the developments in electronics. In most cases, cooling of selected components, rather than all components may be sufficient. This type of cooling is known as spot cooling. The available methods for spot cooling include jet impingement and spray cooling, thermo-electric, Joule-
Thomson coolers, and pulse-tube refrigerators. Other cooling devices such as the Split-Stirling or Gifford-McMahon are mechanical devices with moving parts, making them poor candidates for miniaturization. Thermo-electric coolers become very inefficient when it is necessary to obtain cryogenic temperature ranges. Even though the pulse-tube refrigerator has no moving parts, vibration and acoustic noise would preclude its use in most low noise applications. Thus, one of the best choices for cryogenic spot cooling is Joule-Thomson refrigeration.

The JT refrigerator is usually based on the simple Linde-Hampson liquefaction system. This simple system is shown in Figure 3.6 and its thermodynamic cycle (T-S diagram) in Figure 3.7. In a Joule-Thomson cooler, similar to a liquefaction system,
refrigeration is produced by passing a high-pressure gas down a counter-current heat exchanger and then expanding it to a lower pressure. However, the gas is drawn from a high-pressure gas cylinder and therefore compression stage is not shown in Figure 3.7. The cycle begins as the high-pressure gas (state point 2) is sent through the counter flow heat exchanger. However, in microminiature JT devices, there may be an appreciable pressure drop due to the channel roughness and small cross-section resulting in a pressure drop of the gas in the heat exchanger. Therefore, the thermodynamic state of the refrigerant coming out of the heat exchanger can be represented by the point 3' in Figure 3.7. The isenthalpic expansion which follows is shown by the process 3'-4'. The same process with negligible pressure drop is also shown as 3-4. Process 4'-5'-1' shows the subsequent passage of the low-pressure refrigerant. This low-pressure, low-temperature gas will pre-cool the incoming high-pressure gas in the heat exchanger. This pre-cooling will cause the expanding gas to become colder over time until the liquefaction of the gas occurs. Finally, a stable minimum temperature is reached at the boiling point of the gas corresponding to the pressure existing in the boiler or low-temperature reservoir. The pressure drop of the cold stream is small, as its channel width is usually larger than that of the hot stream channel. Figure 3.7 also shows that the enthalpy of the out-going refrigerant is greater than the enthalpy of the inlet stream but its temperature is lower than that of the inlet stream.

The application of the conservation of energy (First Law of Thermodynamics) to the system shown in Figure 3.6 yields an equation for the possible cooling capacity of the JT device. This equation assumes that there are no external heat leaks from the heat exchanger and capillary section of the refrigerator and is presented in Equation (3.2).
\[ Q = m(h_s - h_i) \]  

(3.2)

A typical JT device consists of four main components: (1) inlet and outlet ports, (2) heat exchanger, (3) capillary expansion channel, and (4) liquid collector (evaporator). One of the early designs of the JT device is shown in Figure 3.8. In this design, the

(Fig: Narayanan and Venkatarathnam, 1999)

**Figure 3.8 A Microscale Joule-Thomson Refrigerator with One Hot Fluid Channel and One Cold Fluid Channel in the Heat Exchanger**

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counterflow heat exchanger consisted of a hot fluid channel and a cold fluid channel. Later designs incorporated two cold fluid channels that are separated by channels walls and the hot fluid channel. This new modification increased the heat transfer area between the hot fluid and the cold fluid in the counter-flow heat exchanger. Figure 3.9 is an illustration of this later design. IMM Germany fabricated a JT device using the LIGA fabrication method. The device was built using the design shown in Figure 3.9. The LIGA process involves both the lithography and mould preparation, and it produces very high quality, high aspect ratio geometries. Figure 3.10 shows the negatives of the heat
Figure 3.10a Heat Exchanger of LIGA Fabricated JT Device by iMM Germany

Figure 3.10b Capillary and the Evaporator of LIGA Fabricated JT Device by iMM Germany
exchanger and the capillary of the JT device manufactured by IMM Germany. In chapter 2, background research revealed that the fabrication of microscale JT devices pioneered by Dr. W. A. Little at Stanford University. He used lithography to fabricate heat exchanger and capillary channels. Figure 3.11 shows the section of an early design by Dr. Little. In Figure 3.11a, the high-pressure channel is 300 μm wide and the separating wall has a thickness of 150 μm. The capillary channels, shown in Figure 3.11b have a width of 100 μm, and the depth is 50 μm.

(Ref: Holman and Little. 1980)

**Figure 3.11a Section of the Heat Exchanger**
This chapter presented a brief description of Joule-Thomson cooling, Joule-Thomson cryocoolers, and microscale Joule-Thomson refrigerators. The next chapter provides an overview of the photolithography process and the wafer bonding used in fabricating a micro JT refrigerator.
CHAPTER 4

PHOTOLITHOGRAPHY AND SILICON

WAFER BONDING

Photolithography is defined as the transfer of a pattern on to a substrate through the exposure of a photo mask to ultraviolet light. The process involves several steps-wafer cleaning, formation of a masking layer, application of photoresist, exposure of mask through UV light, development, and patterning. Depending on geometrical requirements, a further etching (e.g., KOH etching) step is carried out to obtain desired dimensions. Because of the importance of photolithography and wafer bonding in this research work, this chapter presents a brief survey of the subject.

4.1 Silicon

The base material used in photolithography is silicon (Si). In 1824, silicon was discovered in Sweden, and the credit of the discovery is given to Jöns Jacob Berzelius. Silicon has a diamond structure and is known to have "two interpenetrating face-centered cubic" primitive lattices. Figure 4.1 shows the arrangements of atoms in a primitive cell. The distance between two base atoms in the unit cell is 0.543 nm. The numbers in Figure 4.1 (b) indicate height of the atom above the base of the cube as a fraction of the cell dimension.
Silicon is the indispensable material in the electronic industry. When doped with boron, gallium, phosphorus, or arsenic, etc. silicon can be used to produce transistors, solar cells, rectifiers, and other electronic solid-state devices. Silicon is brittle at room temperature but is a very strong material. Figure 4.2 shows the most commonly known crystal planes of the silicon structure. Mechanical properties are usually dependant upon the direction of the crystal plane. For example, Young's modulus in the direction <100> is about 131 GPa where as it is about 169 GPa in the <110> direction. Wet chemical etching rates also depend on the crystal direction. The etch rate with potassium hydroxide (KOH) at 85 °C in <100> is 1.4 μm/min. The etch rate in <111> direction is about 1/400th of the etch rate in <100> direction (Madou, 2002). The linear

Crystal plane (100)

Crystal plane (110)

Crystal plane (111)

Figure 4.2 Crystal Planes in Silicon Crystal Structure
coefficient of thermal expansion increases with the temperature. Thermal conductivity of Si also depends on the temperature. Below room temperature, the thermal conductivity increases and reaches the maximum in the vicinity of 20 to 30 K. Further decrease of temperature accompanies the decrease of thermal conductivity.

4.2 Photoresists

Photoresists are polymers that contain several components. They are tuned to respond to specific wavelengths of light and different light sources. There are about four main components in a photoresist: polymer, solvent, sensitizer, and additives. When exposed to UV light, polymer structure changes and may become polymerized from soluble (or vice versa). The solvent makes it possible to apply the polymer to the wafer surface. The sensitizer usually controls the chemical reactions in the resist due to exposure. Additives in the photoresist may absorb light rays in the resist film or control the dissolution of the non-exposed portion of the resist during development. There are two main categories of photoresists, the positive and negative. In positive photoresists, exposed areas become soluble, whereas in the negative photoresists, exposed areas become insoluble because of polymerization.

The well-known positive resist is polymethylmethacrylate (PMMA). It has a maximum sensitivity of about 220 nm. This photoresist is used in deep ultraviolet lithography and also can be used in electron beam, ion beam, and X-ray lithography. Table 4.1 provides some positive and negative photoresist examples.

Selection of a photoresist in a process depends on many factors. The most important consideration is sensitivity, the ability of the photoresist to produce the
minimum feature size. The photoresist should also have very good adhesive properties, be pin hole resistant, and have a high thermal stability. Table 4.1 lists some commonly used photoresists and their sensitivity.

**Table 4.1 Negative and Positive Photoresists**

<table>
<thead>
<tr>
<th>Class of Resist</th>
<th>Resist Name</th>
<th>Tone</th>
<th>Lithographic Sensitivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optical</td>
<td>Kodak 747</td>
<td>Negative</td>
<td>9 mJ/cm²</td>
</tr>
<tr>
<td></td>
<td>Az-1350J</td>
<td>Positive</td>
<td>90 mJ/cm²</td>
</tr>
<tr>
<td></td>
<td>Kodak KTFR</td>
<td>Negative</td>
<td>9 mJ/cm²</td>
</tr>
<tr>
<td></td>
<td>PR 102</td>
<td>Positive</td>
<td>140 mJ/cm²</td>
</tr>
<tr>
<td>e-beam</td>
<td>Copolymer acrylate</td>
<td>Negative</td>
<td>0.3 μC/cm²</td>
</tr>
<tr>
<td></td>
<td>Germanium selenide</td>
<td>Negative</td>
<td>80 μC/cm²</td>
</tr>
<tr>
<td></td>
<td>PMMA</td>
<td>Positive</td>
<td>50 μC/cm²</td>
</tr>
<tr>
<td>X-ray</td>
<td>PMMA</td>
<td>Positive</td>
<td>1000 mJ/cm²</td>
</tr>
</tbody>
</table>

(Source: Fundamentals of Microfabrication, Marc Madou, 2002)

4.3 Process of Photolithography

Photolithography processes involve several steps: wafer cleaning, adhesion of photoresist, soft baking, exposure, development, hard baking, and etching. In the first step, the wafers are chemically cleaned to remove particulate matter on the surface as well as any traces of organic, ionic, and metallic impurities. After cleaning, silicon dioxide, (or other masking layer) which serves as a barrier, is deposited or thermally grown on the surface of the wafer. After the formation of the SiO₂ layer, photoresist is applied to the surface of the wafer. High-speed centrifugal whirling of the silicon wafer is the standard method for applying photoresist coatings. This technique, known as spin...
coating, produces a thin uniform layer of photoresist on the wafer surface. The spin coating is followed by the soft baking. During soft baking, almost all of the solvents are removed from the photoresist coating. Soft baking plays a very critical role in photo imaging. The photoresist coatings become photosensitive, or imageable, only after soft baking. Over-soft baking will degrade the photosensitivity of resists by either reducing the developer solubility or actually destroying a portion of the sensitizer, and under-soft baking will prevent light from reaching the sensitizer. Positive resists are incompletely exposed if considerable solvent remains in the coating. This under-soft baked positive resist is then readily attacked by the developer in both exposed and unexposed areas, causing less etching resistance. Therefore, strict adherence to the established soft baking recipe is essential. After proper soft baking, the photoresist is exposed to UV light through the photo mask or a film mask. Photo mask is a glass plate with a patterned emulsion of metal film on one side. Film mask resembles a photographic film with the pattern printed on it. If several photo masks or film masks are required, proper alignment of them with the wafer is extremely important. The next step is the exposure of photoresists through the mask to UV light. The exposure time can be estimated from the following relation.

\[ t_{\text{exposure}} = \frac{J}{I} \]  \hspace{1cm} (4.1)

In Equation (4.1), \( J \) is the sensitivity of photoresist per 1 \( \mu \text{m} \) thickness and \( I \) is the lamp power intensity. However, final exposure time should be established after a series of test runs that involve exposure and development. The exposure is then followed by the development. In the development, the substrate is washed with a solvent that
preferentially removes the areas of higher solubility. Depending on the type of photoresist (positive or negative), the washed away areas may be either illuminated or shadowed areas of the photoresist coating. After the development, the pattern should be examined under yellow light through a microscope. This microscopic examination will help determine whether the pattern is satisfactorily transferred to the photoresist. Proper development ensures no patches of photoresist left on soluble areas and can be clearly recognized in microscopic examinations. After the development, the wafer should be rinsed with deionized water and dried in a nitrogen gas stream. This step is then followed by hard baking. The purpose of hard baking is to improve the adhesion of the photoresist to the substrate. Typical hard baking temperatures are 150 °C for negative resist and 130 °C for positive resists. The next step is to etch the masking layer in exposed areas so that the pattern can be established on the silicon wafer. If the masking layer is SiO₂, then the etching is carried out with a Buffered Oxide Etchant (BOE). The commonly used BOE is a mixture of hydrogen fluoride (HF) and ammonium fluoride (NH₄F). Subsequently, the photoresist is dissolved in a solvent such as acetone. Depending on the size requirements, one more etching step with KOH or other dry etching techniques is undertaken to obtain the desired pattern. Figure 4.3 is an illustration of the steps involved in photolithography.

The minimum feature size that can be patterned on the photoresist depends upon the resolution of the photolithographic system. The printing of the pattern on the photoresist can be done in three different ways: contact printing, proximity printing, and projection printing as shown in Figure 4.4. Theoretical minimum feature size patterned by a conventional photoresist is given by Equation (4.2).
\[ 2b_{mn} = 3\sqrt{\frac{\lambda(s + Z)}{2}} \]  

(4.2)

1. Photore sist spun on Si wafer

2. Exposure to UV light

3. Development

4. Acid etching (Pattern transfer)

(Original diagram by Georgia Tech; Substantial improvements by the author)

**Figure 4.3 Illustration of Steps Involved in Photolithography**

where \( b_{mn} \) stands for half the minimum feature size, \( s \) for the gap between the photoresist and the photo mask, \( \lambda \) for the wave length of the exposing light, and \( Z \) for the photoresist thickness. Equation (4.2) shows that the wavelength \( \lambda \) should be very small to produce higher resolution. The equation (4.2) also indicates that a smaller thickness of
photoresists would enhance the resolution. In contact printing, the gap $s$ is zero. Therefore, for contact printing, Equation (4.2) reduces to Equation (4.3).

$$2b_{\text{min}} = 3\sqrt{\left(\frac{\lambda}{2}\right) s}$$  \hspace{1cm} (4.3)

Even though the contact printing produces the highest resolution, it causes damages to both the substrate and the photo mask and also introduces contaminants. Therefore, contact printing is unsuitable for most modern microcircuit fabrication.

The damages to the substrate and the photo masks can be minimized in proximity printing. However, diffraction of the transmitted light reduces the resolution and distorts the pattern. The amount of degradation depends on the wafer to substrate distance (thickness of resist coating) which may vary across the wafer. If the gap $s$ is considerably larger than the resist thickness ($s \gg Z$), then Equation (4.2) can be rearranged for proximity printing as

$$2b_{\text{min}} = 3\sqrt{\lambda s}$$  \hspace{1cm} (4.4)

In most practical UV proximity exposure printings, approximately, 2 to 3 $\mu$m of resolution is attainable.

Projection printing is used for resolutions smaller than 2 $\mu$m. In projection printing, the image of the mask is projected onto the wafer through a high-resolution lens. The lens provides diffraction-limited performance and is similar to that of a microscope. However, it is much more complicated than the microscope objective. The projection lens should have larger field size, lower distortion, and smaller field curvature. The resolution $R$ in projection printing is given by Equation (4.5).
where $k_l$ is an experimentally determined parameter, $\lambda$ is the wave length of the exposing light, and $NA$ is the numerical aperture of the projection system. The parameter $k_l$ depends on process conditions and resist parameters, and can vary between 0.5 and 1.0. For single-layer photoresists a value greater than 0.7 is preferred for $k_l$. Numerical aperture, $NA$, can be re-written in terms of refractive index $n$ and the maximum acceptance angle $\theta$ that can be focused by the optical system. Therefore, equation (4.5) is re-written in the form given by Equation (4.6).
The numerical aperture can be written in another form using the depth of the field ($f$) of the optical system and the diameter ($D$) of the projecting lens. The resulting equation is

$$R = \frac{k \lambda}{n \sin \theta} \left( \frac{D}{2f} \right)$$  \hfill (4.7)$$

The diffraction of light at the edge of the pattern on the mask has a negative effect on the resolution. Figure 4.5 presents the variation of intensity of light after passing through the mask and the corresponding resist response. The figure implies that the resist adjacent to the edge does not receive the correct amount of dose in the exposure. This insufficient exposure causes the image of a perfect edge to become blurred or diffused.

![Figure 4.5 Light Distribution Profile on a Photoresist](image)
The incident light that passes through the resist is reflected by the substrate. This reflected wave and the incident light wave produce a standing wave through the coherent interference. This coherent interference causes a periodic intensity distribution in the direction perpendicular to the surface. Figure 4.6 shows the effect of standing waves on the photoresist. The vertical surface of the resist has peaks and valleys like pattern due to the effect of standing waves.

(Source: University of Kiel, Denmark)

Figure 4.6 Effect of Standing Waves on Photoresist Profile

4.4 Etching

After completion of the development and hard baking, the next important step in photolithography is etching. Etching can be mainly categorized into two, wet etching and dry etching.
4.4.1 *Wet Etching*

Wet etching itself falls into two broad categories: isotropic etching and anisotropic etching. In isotropic etching, etching takes place in all direction at the same rate. In anisotropic etching, etching takes place at different rates in different direction, and so there is more control of the shape produced. Figure 4.7a and Figure 4.7b illustrates the isotropic and anisotropic etching.

Because the material removal takes place in all direction at the same rate, isotropic etching produces undercutting, the process of material removal horizontally underneath the photoresist. In advanced circuit designs, undercutting should be taken into account to prevent shortening of adjacent patterns. Severe undercutting can occur due to etching temperature, strong etch mixture, and excessive etching time. Poor adhesion of the photoresist to the wafer can also be responsible for undercutting. The remedial measures include applying an adhesion promoter (priming) before resist coating, and proper soft baking and hard baking. The most common etched layer is thermally grown silicon dioxide (SiO₂).

4.4.2 *Dry Etching*

In dry etching, etching agents come from gases used in the process, whereas in wet etching, etching ions come from the liquid chemicals. In dry etching, the surface to be etched is bombarded by ions, causing physical removal of the base material. Chemical reaction with a reactive species occurs at the surface aiding the etching. Sometimes the etching can occur due to the combined effect of ion bombardment and chemical reaction at the surface. Dry etching is usually employed for feature sizes less than 3 μm. There are
Figure 4.7a Isotropic Etching

Figure 4.7b Anisotropic Etching

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three dry etching techniques: plasma, ion beam etching (ion milling), and reactive ion etching (RIE).

4.4.2.1 **Plasma Etching**

This is a chemical process but uses gases and plasma energy to cause the chemical reaction. A plasma etching system consists of a chamber, vacuum system, gas supply, and a power supply unit. The wafers are loaded into the chamber and the pressure inside is reduced by the vacuum system. After the vacuum has been established, the reactive gas is sent into the chamber. In SiO₂ masking layer etching, fluorocarbon (CF₄) mixed with Oxygen (O₂) is used as the reactive gas. The power supply creates a radio frequency (RF) field through electrodes in the chamber and energizes the gas mixture to a plasma state. In the plasma state fluorine attacks SiO₂ and converts it into a volatile component. This volatile component is removed from the chamber by a vacuum system. The etch rate depends on a number of factors such as system design, chemistry, ion density, and system pressure.

4.4.2.2 **Ion Beam Etching**

Ion beam etching is a physical process as opposed to the chemical plasma system. The silicon wafers are placed on a holder in a vacuum chamber and a stream of argon is introduced into the chamber. This argon stream is then collided with a stream of high energy electrons causing the ionization of argon atoms and bringing them to a high energy state with a positive charge. The wafers held on a negatively grounded holder attract argon ions. As the argon ions travel to the wafer surface they accelerate picking up
energy. These highly accelerated argon ions crash into the exposed wafer surface blasting away a small amount of material from the wafer surface. This material removal occurs due to the transfer of momentum by argon ions. The etching is highly anisotropic and can be used to obtain highly defined small openings.

4.4.2.3 Reactive Ion Etching

Reactive ion etching (RIE) is a combination of plasma etching and ion beam etching principles. This combination brings the benefits of chemical plasma etching along with the benefits of ion beam etching. Selectivity for Si/ SiO₂ (etch ratio) depends upon the gas that is mixed with CF₄ in the plasma. If hydrogen (H₂) is mixed with CF₄, then the SiO₂ etches more than the base material Si. If the gas mixed with CF₄ is Oxygen (O₂) then Si etches faster than the SiO₂ masking layer.

4.5 Wafer Bonding

Wafer bonding is very often required in MEMS especially in packaging sensors and microfluidic devices. Three methods are available for wafer bonding: fusion bonding, anodic bonding, and adhesive bonding.

4.5.1 Fusion Bonding

Fusion bonding, sometimes known as direct wafer bonding, permanently bonds two silicon wafers. The bonding process consists of four steps: the cleaning and surface activation steps, washing and drying, forming primary bonds, and establishing the final bond. In the cleaning and surface activation step, two mirror-polished surfaces are made hydrophilic and then sprayed with de-ionized water while wafers are being spun at rates
up to about 50 rpm. The spinning rate is increased up to about 3000 rpm after water stream has been shut off. Then the wafers are heated by an infrared lamp to remove any remaining water from the surfaces. Dried surfaces are then mated in a clean environment forming the primary bond. Usually wafers are separated by mechanical separators and the first contact between surfaces is made at the center. As the centers of wafers are pressed against each other, mechanical separators are withdrawn to allow the bond wave to propagate towards the periphery forming a uniform bond. Wafers are subsequently examined under infrared microscope to detect any voids present. If the primary bonding is satisfactory then the wafers are heated in a furnace at about 300° C to initiate the chemical bonding. Subsequently, temperature is increased above 800° C and maintain at a constant level for more than five hours. During this annealing time, interfacial silicon oxide layer, typically 10-15 Å, is formed causing the permanent bonding. However, one has to establish the actual process recipe depending on the circumstances. The bond thus made is very strong and irreversible, and the bonded wafers retain bulk properties. However, the process requires stringent surface-quality requirements.

4.5.2 Anodic Bonding

The anodic bonding procedure was first introduced by G. Wallis and D.I. Powerantz in 1969. Anodic bonding relies on the migration of charges to produce bonded surfaces. The bonding occurs between a highly polished silicon wafer surface and a smooth surface of a pyrex borosilicate glass. The glass should have alkali metals to facilitate the charge migration. The typical glass material used in bonding is borosilicate (Pyrex 7740), and it has a sodium oxide (Na₂O) content of 3.5%. Moreover, linear
coefficient of thermal expansion of Pyrex 7740 closely matches that of silicon so that
the bonding will not induce unacceptable thermal stresses. High negative potential,
approximately 1000V, is applied to the glass at a temperature about 500 °C to attract
sodium ions (\(Na^+\)) to the cathode. The glass is softened at temperatures above 500 °C
facilitating the mobility of \(Na^+\). At the cathode \(Na^+\) ions are neutralized. Such a removal
creates a space charge at the glass silicon interface creating a strong electrostatic
attraction between the glass and silicon wafers. This attraction holds both pieces firmly in
place. Furthermore, driven by the existing electric field, oxygen present in glass is
transported to the silicon glass interface. At the interface, oxygen combines with silicon
to form a SiO\(_2\) layer to create the permanent bond. Figure 4.8 illustrates the anodic
bonding process. The bonding of silicon to glass is hermetic, and the strength is reported
to be higher than the substrate (Johanson et al. 1988). This method is very popular in
packaging sensors.

![Figure 4.8 Illustration of the Anodic Bonding Process](image-url)
4.5.3 **Adhesive Bonding**

Adhesive bonding uses an intermediate material to bond silicon wafers. These intermediate materials include polymides, epoxies, thermoplast adhesives, and photoresists. Some of the advantages of adhesive bonding are: (1) many different substrate materials can be joined; (2) bonding temperature can be lower than 100° C, and therefore the process is CMOS compatible; (3) high bond strength; (4) elastic properties of some polymers can reduce the stress in the bonded material; (5) low cost; and (6) compatibility with many clean room processing. Some of the disadvantages are: (1) the bonding may not lead to a hermetic seal; and (2) limited long term stability of many polymer materials. In addition, adhesive material will soften during curing and flow over the pattern. In microfluidic devices, channels may be completely blocked due to the overflow.

The Frank Nicklaus group at the Royal Institute of Technology in Sweden investigated several materials for adhesive bonding. They examined Photoresist 1818 manufactured by Shipley, and benzocyclobutane (BCB) in wafer bonding. Their findings suggested that BCB and Photoresist 1818 could be successfully used to bond silicon wafers with 0% voids. The BCB was found to be the best with strong bond whereas Photoresist 1818 reported low bond strength.

This chapter presented a brief overview of photolithography and wafer bonding processes. The next chapter is devoted to the development of a simulation model for microscale Joule-Thomson refrigerators.
CHAPTER 5

A SIMPLE THERMODYNAMIC, HEAT TRANSFER, AND FLUID FLOW SIMULATION MODEL FOR MICROSCALE JOULE-THOMSON REFRIGERATORS

The review of literature presented in Chapter 3 shows that microscale JT refrigerators are currently manufactured on a commercial basis. These refrigerators fabricated according to designs developed by W. A. Little at Stanford University. He initially simply scaled down heat exchangers of known performance to microscale level and proceeded to develop microscale JT devices. However, the fact that he performed some theoretical and experimental investigations on the heat exchanger suggested that a simple scaling-down procedure alone was not good enough in the design process. Most subsequent research work has been directed to study the heat exchanger of the device because it is very important to have a high-effectiveness heat exchanger to cool down incoming hot fluids to sufficiently low temperatures to produce two-phase conditions in the evaporator. It was evident from the literature review that there had been no attempts to consider the whole device as one unit and optimize the geometry to obtain the maximum flow rate while maintaining a high effectiveness of the heat exchanger. Such
an attempt requires the development of a simulation model for the JT device. Also, the simulation model would be an ideal tool in the design of JT devices.

This study was therefore undertaken to develop a simple thermodynamic, heat transfer, and fluid flow model for microscale JT devices and use it to design and fabricate a novel microscale JT device. As mentioned previously, a simulation model can be used to optimize the geometry with respect to the cooling capacity of the device. The cryogenic refrigerant considered in this study is nitrogen because it has a positive Joule-Thomson coefficient at room temperature. Therefore, it is not necessary to have pre-cooling in the system and nitrogen will undergo cooling after the expansion in the capillary.

5.1 Development of the Simulation Model

The proposed simulation model was based on the simple Linde-Hampson system shown in Figure 5.1. This system has been reproduced here. The development of the simulation model included the formation of thermodynamic, heat transfer, and fluid flow equations related to the heat exchanger, the capillary, and the evaporator. This formation of related equations is presented in the following sections.

5.1.1 Heat Exchanger

Figure 5.1 represents the Linde-Hampson system on an open cycle. At the inlet state “1”, the nitrogen refrigerant enters the heat exchanger with a known pressure ($P_1$) and a temperature ($T_1$). Mass flow rate at the inlet can be written according to Equation (5.1).
where $V$ is the inlet volumetric flow rate and $\rho$ is the density of inlet refrigerant.

The head loss in duct flows, for any cross-section and, for laminar and turbulent flow, is given by the *Darcy-Weisbach* equation (White 1999) presented in Equation (5.2).

$$h_f = f \frac{L V^2}{d 2g}$$  \hspace{1cm} (5.2)

Equation (5.2) can be written in terms of the pressure drop in the duct because the head loss $h_f$ is given by

$$h_f = \frac{\Delta p}{\rho \cdot g}$$  \hspace{1cm} (5.3)

and therefore the pressure drop can be written by combining Equations (5.2) and (5.3). This combining will yield Equation (5.4).
Equation (5.4) is used to write the pressure drop in the hot fluid channels and the cold fluid channels of the heat exchanger. However, fluid properties in the equation are determined at appropriate average temperatures. The Darcy friction factor \( f \) is determined according to the fluid flow regime.

\[
\Delta p = \rho f \frac{L V^2}{d} \quad (5.4)
\]

Equation (5.4) is for smooth circular tubes. In the present study, flow channels have rectangular cross sections and therefore the hydraulic diameter is used in Reynolds number calculations. According to Incropera and Dewitt (2000), the Blasius approximation given in Equation (5.6) reasonably represents the friction factor \( f \) in turbulent flow for smooth surfaces.

Engineering Equation Solver (EES) software, chosen to build the simulation model, has a built in property data base for nitrogen, which is the refrigerant selected for this present study. At state “1”, the density \( (\rho_1) \), enthalpy \( (h_1) \), thermal conductivity \( (k_1) \), specific heat capacity \( (c_{p1}) \), and viscosity \( (\mu_1) \) of nitrogen can be determined from known inlet pressure and temperature \( (P_1 \text{ and } T_1) \) using following expressions.

\[
f = \frac{64}{Re} \quad \text{for } 0 < Re < 2300 \quad (5.5)
\]

\[
f = 0.316 Re^{-0.25} \quad \text{for } 4000 < Re < 10^5 \quad (5.6)
\]

\[
\rho_1 = \text{density(Nitrogen, } P=P_1, T=T_1) \quad (5.7)
\]

\[
h_1 = \text{enthalpy(Nitrogen, } P=P_1, T=T_1) \quad (5.8)
\]

\[
k_1 = \text{conductivity(Nitrogen, } P=P_1, T=T_1) \quad (5.9)
\]
\[ c_{pl} = \text{specheat(Nitrogen, } P=P_i, T=T_i) \]  
\[ \mu_i = \text{viscosity(Nitrogen, } P=P_i, T=T_i) \]  

Similar expressions can be written for other thermodynamic states 2, 4, and 5 that are related to the heat exchanger.

The Nusselt's number \( Nu \) is written in the following forms:

\[ Nu = \frac{hD_h}{k} \]  

In equation (5.12), \( h \) represents the average convective heat transfer coefficient, \( D_h \) represents the hydraulic diameter, and \( k \) represents the thermal conductivity. In the laminar flow region, Nusselt number depends on the cross section of the channel. Applicable Nusselt number values for constant heat flux and constant surface temperature conditions are dependant upon the channel cross section and are provided by Kays and Crawford (1993) and by Incropera and Dewitt (2000). In this model, in laminar flow region, a value for Nusselt number will be generated based on the hydraulic diameter. In the turbulent flow region, the Nusselt number given by the Colburn equation (5.13) is used. The Dittus-Boelter equation is not used because it tends to over predict the Nusselt number as much as 20 percent (Kays and Crawford, 1993).

\[ Nu = 0.023 \, Re^{4/5} \, Pr^{1/3} \]  

In JT devices, counter flow heat exchangers are used to obtain high heat transfer rates for given conditions. The effectiveness of such a heat exchanger can be written in two different forms. According to the definition given by Barron (1985), the effectiveness
is the ratio between the actual heat transfer and the maximum possible heat transfer (ideal heat transfer) in the heat exchanger. Accordingly, the effectiveness (E) is given by:

\[
E = \frac{h_s - h_i}{h_j - h_i}
\]  (5.14)

where \(h_s\) represents the enthalpy of the refrigerant at the exit under ideal conditions. Enthalpies \(h_i\) and \(h_s\) represent the actual enthalpy at the inlet and the exit of cold refrigerant. According to Kays and Crawford (1993), the effectiveness (E) for counter flow heat exchanger is also given by:

\[
E = \frac{1 - e^{-NTU_{\text{max}}} (1 - C_{\text{max}} / C_{\text{min}})}{1 - (C_{\text{min}} / C_{\text{max}}) e^{-NTU_{\text{max}}} (1 - C_{\text{min}} / C_{\text{max}})}
\]  (5.15)

When two fluids of a heat exchanger are separated by a wall, the overall heat transfer coefficient (U) is determined from the two local convective heat transfer coefficients on either side of the wall, plus the thermal resistance of the wall material, and any other thermal resistance that may be present. The counter flow heat exchanger of the proposed JT device consists of two silicon wafers that have channels etched on their top surfaces. Backsides of these two wafers are bonded to form the heat exchanger. Therefore, the resulting configuration is similar to the one shown in Figure 5.3. According to Kays and Crawford (1993), neglecting other thermal resistance, the overall heat transfer coefficient \(U\) is given by Equation (5.16). In Equation (5.16), \(\eta_1\) and \(\eta_2\) are total overall surface effectiveness of the side 1 and side 2 and the total base thickness (sum of two wafer thicknesses) is given by \(t\). The two surface effectivenesses, \(\eta_1\) and \(\eta_2\), can also be expressed by Equations (5.17) and (5.18).
A1 - Total heat transfer area on side 1 (wafer 1)  
A2 - Total heat transfer area on side 2 (wafer 2)  
Aw - Base area of each side  

Figure 5.3 Cross Section of a Micro Heat Exchanger

\[
\frac{l}{U} = \frac{l}{\eta_1 h_1} + \frac{A_1}{\kappa A_w} + \frac{l}{\eta_2 h_2 A_2}
\]

(5.16)

\[
\eta_1 = 1 - \frac{A_{f1}}{A_1} (1 - \eta_{f1})
\]

(5.17)

\[
\eta_2 = 1 - \frac{A_{f2}}{A_2} (1 - \eta_{f2})
\]

(5.18)

The surface areas of fins on sides 1 and 2 are given by \(A_{f1}\) and \(A_{f2}\) and respective fin heat transfer effectivenesses are presented in \(\eta_{f1}\) and \(\eta_{f2}\). The total heat transfer area on a side \((A)\) contains both the total channel base area and the fin area. Therefore, the ratio of total fin area to total heat transfer area is less than one. Even if fin effectiveness is about 50%, the product of terms that would be subtracted from ‘1’ in both Equations (5.17) and (5.18) would be small. Therefore, it is reasonable to assume that \(\eta_1\) and \(\eta_2\) are close to one. The ratio \(\frac{A_1}{A_w}\) needs to be computed according to channel geometry of the heat
exchanger. Here $A_w$ represents the base area that separates channels on side 1 (wafer 1) and side 2 (wafer 2). In the present study, the channel width and the wall thickness are same (50 μm) so the total fin base area would be half of the total base area. The height of channels is 20 μm and this is close to half the size of channels width. Therefore, $\frac{A_L}{A_w}$ roughly should be between 0.75 and 1. If we assume that $\frac{A_L}{A_w}$ is close to unity, then the equation (4.16) can be simplified to Equation (5.19).

$$\frac{l}{U} = \frac{l}{h_1} + \frac{t}{\kappa} + \frac{l}{h_2} \frac{A_L}{A_w}$$ (5.19)

Here, the overall heat transfer coefficient $U$ is based on the surface area of side 1. If different numbers of channels are present on sides 1 and 2, the heat transfer areas $A_L$ and $A_2$ can be modified accordingly. After such modification, the overall heat transfer coefficient used in the simulation model can be expressed as:

$$\frac{l}{U} = \frac{l}{h_1} + \frac{t}{\kappa} + \frac{l}{h_2} \frac{N_{ch} w_1 L_h}{N_{ch} w_2 L_c}$$ (5.20)

In Equation (5.20), $N$ represents the number of channels, $w$ represents the width of channels, and $L$ represents the length of channels. Equation (5.20) does not contain the depths of channels because the equation has been written for the case where the heat transfer occurs across the base of the heat exchanger. However, if the simulation model is used to analyze existing JT devices where heat transfer occurs across the separating walls, the depth of channels should be used in place of the widths of channels.
The capacity rate ratio of the hot and cold fluid streams is given by Equation (5.21) as presented below.

\[ C_r = \frac{C_{\text{min}}}{C_{\text{max}}} \]  

(5.21)

where \( C = mc \) is the capacity of the fluid.

The number of transfer units (NTU) is written in the usual form:

\[ NTU = \frac{UA}{C_{\text{min}}} \]  

(5.22)

In equation (5.22), the area \( A \) represents the total heat transfer area of the hot fluid side which may be written as \( A = N_c h w h L_h \).

It is assumed that the heat exchanger is not exposed to any external heat leaks from the surroundings and the wall conduction is negligible. The energy balance in the heat exchanger is then given by Equation (5.23).

\[ h_1 - h_2 = h_5 - h_4 \]  

(5.23)

The left hand side of Equation (5.23) is the amount of energy released by the incoming hot fluid to the outgoing cold fluid stream in the counter flow heat exchanger.

Equations similar to the set of equations given by (5.1) to (5.13) were written for the cold fluid stream to complete the mathematical representation of the counter flow heat exchanger.

5.1.2 Capillary

The capillary section is an extremely important component of the microscale JT device. Most of the refrigerant high-pressure drop occurs across the capillary, and the refrigerant starts undergoing phase transformation in this section. The capillary end is

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exposed to the evaporator pressure. In this simulation model it is assumed that the refrigerant leaves the evaporator at dry saturated conditions. A set of equations similar to Equations (5.7) to (5.11) is written for the density, enthalpy, heat conductivity, specific heat capacity, and the viscosity of the refrigerant in the capillary. These equations are as follows:

\[ \rho_{\text{cap}} = \text{density(Nitrogen, } P = P_{\text{cap}}, T = T_{\text{cap}}) \]  
\[ h_{\text{cap}} = \text{enthalpy(Nitrogen, } P = P_{\text{cap}}, T = T_{\text{cap}}) \]  
\[ k_{\text{cap}} = \text{conductivity(Nitrogen, } P = P_{\text{cap}}, T = T_{\text{cap}}) \]  
\[ c_{p,\text{cap}} = \text{specheat(Nitrogen, } P = P_{\text{cap}}, T = T_{\text{cap}}) \]  
\[ \mu_{\text{cap}} = \text{viscosity(Nitrogen, } P = P_{\text{cap}}, T = T_{\text{cap}}) \]

In addition, the pressure drop across the capillary is given by Equation (5.29).

\[ \Delta p = (p_2 - p_4) = \rho_{\text{cap}} f_{\text{cap}} \frac{L_{\text{cap}} V_{\text{cap}}^2}{d^2} \]  

Here, fluid properties are determined at the average temperature. Therefore, in the capillary, average temperature and average pressure are given by

\[ T_{\text{cap}} = \frac{T_2 + T_3}{2} \]  
\[ P_{\text{cap}} = \frac{P_2 + P_3}{2} \]  

The capillary expansion is shown in Figure 5.4. Both the state points "3" and "4" have the same saturation temperature and pressure. The capillary expansion from the state "2" to state "3" is an isenthalpic process. Therefore, the enthalpy of the refrigerant at state "3" is the same as that of at state "2".

\[ h_2 = h_3 \]
In the two-phase region, pressure and temperature are not independent. Therefore, if either the saturation pressure \((P_4)\) or the temperature \((T_4)\) is known, it can be used with Equation (5.32) to determine the state point “3”, which specifies the quality of the refrigerant. In the simulation model, this pressure or temperature is independently determined and used with the equation (5.32) to obtain the state point “3”. The refrigerant at the state point “3” is a mixture of gas and liquid. Therefore, the quality of the refrigerant at the state point “3” is represented by Equation (5.33).

\[
x = \frac{h_j - h_f}{h_g - h_f}
\]

(5.33)

where, \(h_f\) is the liquid state enthalpy of the refrigerant and the \(h_g\) is the saturated vapor state enthalpy. In Figure 5.4, the saturated vapor state condition is represented by the state point “4”. Therefore, \(h_g\) can be replaced by \(h_d\). The EES software also has a built-in function to determine the quality of refrigerant.

5.1.3 **Evaporator**

In the evaporator, two-phase fluid flow condition exists. Usually, Lockhart-Martinelli semi-empirical equation is used to calculate the two-phase flow pressure drop. However, the evaporator is exposed to heat flux from the heat source, and therefore an adiabatic condition does not prevail. In such a situation, a modification introduced to Lockhart-Martinelli equation by Martinelli and Nelson (1948) should be used. Then, according to Barron (1985), the frictional pressure drop is given by Equation (5.34).

\[
\Delta p = \frac{L(dP/dl)}{(x_i - x_f)} \int_{x_i}^{x_f} (l - x)^{n-2} \phi L \, dx
\]

(5.34)
Here, \((dp/dl)\) is the pressure drop per unit length that would exist if the liquid were flowing at the total mass flow rate. The length between the inlet and the outlet is given by \(L\), and \((x_2-x_1)\) is the change of the quality over the length. The \(\Phi_L\) is expressed as a combination of Lockhart-Martinelli parameters.

\[
\Phi_L = \frac{(X^2 + CX + 1)^{1/2}}{X}
\]

where \(X\) is given by:

\[
X^2 = \frac{C_L (Re_G)^m \rho_G}{C_G (Re_L)^m \rho_L} \left( \frac{1-x}{x} \right)^2
\]

The constants \(C_L, C_G, m\), and should be read from a table for a given flow regime.

In addition, if significant, pressure drop due to momentum change should be added.

It is clear that the pressure drop calculation in the evaporator involves the introduction of number of complex equations. Consequently, those equations increase the level of difficulty in obtaining convergence of the simulation model. In the ideal cycle, there is no pressure drop in the evaporator. Therefore, the model is solved for the ideal condition to obtain the mass flow rate. Then the mass flow rate is used along with assumed dimensions for the evaporator and Equations 5.34 - 5.36 to find the pressure drop in the evaporator. The geometry of the evaporator is changed until the pressure drop is negligible so that the ideal condition is approached. This process should result in larger dimensions for the evaporator so that the fluid flow velocity would be very small and the pressure drop in the evaporator is negligible.
5.2 Solution Method

Engineering Equation Solver (EES 4.95) was selected to solve the model presented earlier. The EES software is a rule-based equation solver, and it contains a database of real fluid properties including nitrogen. Nitrogen is the chosen refrigerant in this simulation work because it is the most commonly used refrigerant in JT devices. Thermal/fluid properties of nitrogen can be readily incorporated into the simulation model using the appropriate function with required inputs.

The simulation model consisted of about 94 equations. The readily known input variables are (1) inlet pressure ($P_i$); (2) inlet temperature ($T_i$); and (3) exit pressure ($P_j$). Properties at other thermodynamic states are not known. Therefore, the model can be solved only for a given geometry. This geometry itself is a set of assumed values for length, width, and depth of fluid transporting channels. All fluid properties required in the
model are estimated at average temperatures. The EES software allows the user to
define the range of the guess value for each variable in a variable sheet. This feature
allows the user to define the range of values that the variable should assume in the
iteration process.

In the solution procedure, first, the known input parameters, \( P_i, T_i, \) and \( P_3 \) are
provided. The minimum possible line width that can be successfully achieved with a film
mask is about 20 \( \mu \)m and this value is given to both the \textit{width} (\( w_{\text{cap}} \)) and the \textit{depth} (\( d_{\text{cap}} \))
of capillary channels because, usually, the capillary has the smallest dimension in a JT
device.

The pressure drop in the heat exchanger is usually small. Therefore, a slightly
lower pressure than the inlet pressure can be assigned as a guess value for pressure \( p_2 \) in
the variable sheet. The examination of the T-s diagram of nitrogen shows that the
incoming hot refrigerant should be cooled down to around 150 K to produce cooling after
expansion. Therefore, \( T_2 \) is assigned a guess value in the vicinity of 150 K. The guess
values for the evaporator temperature \( T_4 \) is set in the range of 75-100 K. Exit temperature
\((T_9)\) should be provided a guess value that is closed to the inlet temperature. This is
because the heat exchanger normally has a very high effectiveness. The associated
thermal and fluid properties will be determined by EES according to the guess values of
pressure and temperature provided by the user. The system of equations is iteratively
solved to obtain the convergence of variables. If the system does not converge then the
guess values of variables are adjusted and the solution is re attempted. If the solution
cannot be obtained and the range of values given to variables is exhausted, then a new set
of values is provided for the geometry and the iteration process is continued. The solution procedure is schematically illustrated in Figure 5.5.

In the simulation model, thermal conductivity of silicon should be provided. Thermal conductivity of silicon depends on temperature. The temperature within the heat exchanger of the JT device varies between 300 K to about 80 K. Therefore, the thermal conductivity is selected at an average temperature. In this present simulation work, thermal conductivity of silicon is taken as 264 W/m K. This is the thermal conductivity of silicon around 200 °C. The two refrigerant temperature extremes in the device would be the inlet refrigerant temperature which is about 300 K. The lowest temperature is in the evaporator and would be between 80-100 K. Therefore, it was advisable to use an average value for the thermal conductivity. In this case, the value was determined from Handbook of Semiconductor Silicon Technology, edited by O'Mara, Herring, and Hunt and published by Noyes Publications in New Jersey in 1990. The thickness of the wall between the hot fluid stream and cold fluid stream is taken as 150 μm, the thickness commonly used in existing JT devices.

The simulation model was used to predict the cooling capacity of a JT device that has single hot fluid channels and two cold fluid channels. The main difficulty encountered in this attempt was the incomplete information available in the literature with regard to channel dimensions. Very often complete geometric information of a JT device is not readily available in the literature.

Little (1984) mentioned a JT device capable of producing 250 mW of cooling at 83 K with an inlet pressure of 100 atm. The flow rate of the device at Standard Temperature and Pressure (STP) was 18 ml/sec. He did not provide the geometric details
of channels. The present simulation model was used to check whether such a claim can be justified. The results obtained in the simulation process showed that it was possible to obtain 250 mW of cooling capacity at 82.45 K with an inlet pressure of 100 atm and a heat exchanger effectiveness of 96.7 percent. However, the flow rate predicted by the model at STP was 15.17 ml/s, less than the flow rate mentioned by Little (1984). For a given inlet pressure, the geometry of the JT device determines the flow rate, and close matching of the simulated flow rate to the experimentally determined flow rate would give reasonable credibility to the simulation model. The flow rate obtained by Little (1984) is higher than the predicted value and the deviation is about 20 percent from the simulated results. However, if one considers the experimental uncertainty, the predicted value could be considered reasonable. The geometric dimensions for which above mentioned results were obtained are presented in Table 5.1. The relevant temperature–entropy (T-s) diagram produced by EES is presented in Figure 5.6.

Usually the heat exchanger channels have a longer length, and they are accommodated in a winding flow arrangement on the silicon wafer.

The present model was also used to predict the performance of a single-layer JT device. The main parameters examined were refrigeration capacity, effectiveness, evaporator temperature, and quality of the refrigerant after expansion. In predicting the performance, the following additional quantities were defined.

$$ p_m = \frac{p_1}{(p_{cr} - p_f)} $$  \hspace{1cm} (5.37)

Where $p_m$ represents the non-dimensional inlet pressure, $p_{cr}$ is the critical pressure, and $p_f$ is the evaporator pressure.
The non-dimensional evaporator temperature $TT_e$ is defined according to Equation (5.38).

$$TT_e = \frac{T_e}{(T_{cr} - T_e)}$$  \hspace{1cm} (5.38)

Here, $T_e$ represents the evaporator temperature, and $T_{cr}$ is the critical temperature of nitrogen refrigerant.

The non-dimensional refrigeration capacity $QQ_{Re,fr}$ is defined by Equation (5.39).

$$QQ_{Re,fr} = \frac{Q}{(m(h_{cr} - h_e))}$$  \hspace{1cm} (5.39)

where $m$ is the mass flow rate and $Q$ is the refrigeration capacity. The enthalpy at the critical point is represented by $h_{cr}$ and the enthalpy of dry saturated refrigerant that leaves the evaporator is given by $h_e$.

The model was used to examine the effect of inlet pressure on the cooling capacity ($Q$), effectiveness ($e$), mass flow rate ($m$), evaporator temperature ($T_e$), and the exit temperature ($T_j$) of a JT device with a fixed geometry. This fixed geometry was responsible for producing 250 mW of cooling capacity mentioned previously.

Figure 5.7 shows the variation of refrigeration capacity with the inlet pressure. The refrigeration capacity increases with the increase of inlet pressure. The JT curve for nitrogen presented earlier (Chapter 3) shows that the optimum inlet pressure before expansion is about 40 MPa (400 atm.) Therefore, it can be expected that the refrigeration capacity will continue to increase up to inlet pressures about 40 MPa. The variation of refrigeration capacity with the inlet pressure is almost linear within the pressure range considered in this study.
Figure 5.8 shows that the mass flow rate increases with the increase of inlet pressure. The increase can be considered linear within the pressure range considered in this study. However, the mass flow rate can reach a maximum if choking occurs in the capillary at inlet pressures beyond the range considered here. The refrigeration capacity is influenced by the mass flow rate, and the two graphs should therefore have a similar variation with inlet pressure.

The enthalpy of refrigerant \( h_3 \) at the outlet also increases with the inlet pressure but the refrigeration capacity is mostly influenced by the mass flow rate in the JT device.

The flow of refrigerant does not seem to be choking in the capillary or in other fluid channels. The Mach numbers in cold fluid channels, hot fluid channels, and the capillary are 0.1569, 0.03437, and 0.1254 respectively. Therefore, it can be assumed that the flow is subsonic even at a pressure of 100 atm and the assumption of incompressible flow is appropriate.

The effect of inlet pressure on the effectiveness \( \varepsilon \) of the heat exchanger was also examined. Figure 5.9 represent the variation of effectiveness \( \varepsilon \) with the inlet pressure for a given heat transfer area of the heat exchanger. The effectiveness \( \varepsilon \) increases with the increase of inlet pressure \( p_{in} \). The counter flow heat exchanger in the JT device is somewhat unusual. The same refrigerant is flowing through the heat exchanger in opposite direction. The inlet temperature at the cold end is not independent of the temperature and the pressure at the hot end. The temperature and the pressure at the hot end determine the pressure and temperature in the evaporator for a given geometry. Because the cold fluid inlet temperature is the saturation temperature at the conditions prevailing in the evaporator, it is in turn dependent upon the pressure and the
Solve
Variable range exhausted? Yes Yes Physically Meaningful?
No
Converged? Yes
No
Solution
Open the variable setting window of EES
Provide guess values for \( p_2 \) and \( p_4 \)
Provide a guess value in the range 150-170 K for \( T_2 \)
Provide the guess value in the range 75-100 K for \( T_4 \)
Define the iterative range for other variables
Select the initial pressure and temperature and assign them to \( p_1 \) and \( T_1 \)
Assign atmospheric pressure to \( p_3 \)
Assign the minimum dimension for width \( (w_{cap}) \) and the depth \( (d_{cap}) \) of the capillary
Assume geometry of hot fluid and cold fluid channels
Assume negligible pressure drop in the evaporator

Figure 5.5 Schematic Representation of the Solution Procedure

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Table 5.1 Simulated Geometric Parameters of a JT Device with a Capacity of 250 mW at a Refrigeration Temperature of 83 K (Evaporator not Included).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Dimension</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hot fluid channel length</td>
<td>8 cm</td>
</tr>
<tr>
<td>Cold fluid channel length</td>
<td>8 cm</td>
</tr>
<tr>
<td>Hot fluid channel width</td>
<td>50 μm</td>
</tr>
<tr>
<td>Cold fluid channel width</td>
<td>300 μm</td>
</tr>
<tr>
<td>Hot fluid channel depth</td>
<td>150 μm</td>
</tr>
<tr>
<td>Cold fluid channel depth</td>
<td>150 μm</td>
</tr>
<tr>
<td>Capillary channel length</td>
<td>4.4 cm</td>
</tr>
<tr>
<td>Capillary channel width</td>
<td>25 μm</td>
</tr>
<tr>
<td>Capillary channel depth</td>
<td>50 μm</td>
</tr>
</tbody>
</table>

temperature at the hot end. Therefore, any changes to the geometry or to the hot end inlet conditions change all thermodynamic and heat transfer parameters of the device.

Figure 5.10 represents the variation of evaporator temperature with the inlet pressure. For a given JT device, backpressure in the evaporator should increase as the inlet pressure increases. Because two-phase conditions exist in the evaporator the pressure and the temperature are not independent. If the pressure in the evaporator increases so does the temperature.
After the expansion, the refrigerant becomes wet as the inlet pressure (for a given inlet temperature) increases. The results obtained for the refrigerant quality after expansion show that thermodynamic state point (after the expansion) is pushed more towards the saturated liquid line indicating the production of more liquid refrigerant in the evaporator. This liquid removes more latent heat, improving the cooling capacity of the JT device.

The variation of the quality of refrigerant with the inlet pressure is almost linear within the pressure range considered in this study and presented in Figure 5.11.
The effect of the channel aspect ratio on the cooling capacity, mass flow rate, and effectiveness was also examined. Here, the inlet pressure was assumed to be 120 atm. Because the hot fluid and cold fluid channel widths are different, aspect ratios of both fluid channels were different but had the same depth in order to have the same heat transfer area between hot fluid and cold fluid channels on either side. As the aspect ratio increases, the mass flow rate increases but reaches a maximum as shown in Figure 5.12. As the mass flow rate increases with the aspect ratio, we can expect refrigeration capacity to increase with the aspect ratio. Figure 5.12 confirms that refrigeration capacity is actually increasing with the aspect ratio within the region of aspect ratios considered in this study.
Figure 5.8 Variation of Mass Flow Rate with Inlet Pressure ($p_1/(p_{cr}-p_d)$)

The effectiveness also increases with the aspect ratio as shown in Figure 5.14. As the aspect ratio increases, the depth of hot and cold fluid channels increases so that a greater heat transfer area is available in the heat exchanger to transfer more heat to the cold fluid. The improved heat transfer in the heat exchanger results in a higher effectiveness ($\varepsilon$) for the heat exchanger.

This chapter described the development of a simple thermodynamic, heat transfer and fluid flow model for microscale JT devices. In the absence of reliable and accurate microflow relations, commonly used macroflow equations and conditions were used to build the model. The simulation model described in this chapter is presented in Appendix A.
Figure 5.9 Variation of Heat Exchanger Effectiveness with Inlet Pressure \( \frac{p_i}{p_{cr} - p} \)

Figure 5.10 Variation of Evaporator Temperature \( \frac{T_j}{T_{cr} - T_j} \) with the Inlet Pressure \( \frac{p_i}{p_{cr} - p} \)
Figure 5.11 Variation of Quality of Refrigerant with Inlet Pressure \( \left( \frac{p_i}{p_{cr} - p_d} \right) \)

Figure 5.12 Variation of Mass Flow Rate with Aspect Ratio

Cold fluid channel:
- width = 300 \( \mu \)m
- depth = same as hot hot fluid channel

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Figure 5.13 Variation of Refrigeration Capacity with Aspect Ratio

Figure 5.14 Variation of Heat Exchanger Effectiveness with Aspect Ratio
As mentioned previously, the simulation model closely predicted the performance of an existing JT device. In the absence of complete geometric information of JT devices in the literature, comprehensive analysis was not possible, but the predicted performance is reasonable and agreeable with generally known practical performance of such devices.

This simulation model can also be used to design new JT devices. In the next chapter, a description of an attempt to design a novel JT device based on a layered arrangement is presented.
CHAPTER 6

DESIGN OF A MICROSCALE JOULE-THOMSON REFRIGERATOR BASED ON A LAYERED PACKAGING

Microscale Joule-Thomson devices usually consist of a single substrate and a cover plate. In Chapter 5, a simple thermodynamic, fluid flow, and heat transfer model was presented. Such a model will be very useful in predicting the performance of the device and can also be used in the design of such devices. However, one of the main problems of JT devices is the possible external heat transfer to fluid streams including the heat exchanger. This problem is significant when a cryogenic fluid such as nitrogen is used as a refrigerant. Under external heat transfer conditions, heat exchanger performance deteriorates, and consequently the incoming hot refrigerant may not be sufficiently cooled to produce low temperature refrigerant after expansion. Moreover, the capillary end of current JT devices is exposed to the evaporator. Because of this exposure, one end of the separating wall is at a colder temperature than the other end. Unless the JT device is adequately insulated, there will be wall conduction heat transfer within the device. Both of these problems cited above should be minimized in JT devices to obtain better performance. One of the solutions to minimize external heat leaks and wall conduction heat transfer is to have a layered arrangement of the heat exchanger.
capillary, and the evaporator. This chapter presents an attempt to design such a JT device using the model presented in chapter 5.

6.1 Proposed Design of a Novel Joule-Thomson Device

A simple thermodynamic, fluid flow, and heat transfer model presented in chapter 5 can be used to design layered JT devices. In a layered design, heat transfer between the cold fluid and the hot fluid in the counter flow heat exchanger occurs across the base of the silicon wafer used to etch fluid channels. In addition, the heat exchanger and capillary section will have multiple channels. In the heat exchanger multiple channels will provide a greater heat transfer area so that channel length can be reduced. In a conventional single layer arrangement, heat exchanger channels are long, and the heat transfer occurs across the separating wall. To accommodate the long length required, usually a winding flow (zigzag) channel arrangement is adopted.

Some of the problems that limit the performance of existing JT devices are (1) external heat leaks and (2) conduction heat transfer in the channel separating wall. Because JT devices use a cryogenic refrigerant such as nitrogen, external heat transfer to fluid flow channels is a significant problem. It has been found that the external heat transfer to the heat exchanger of a JT device severely reduces its performance (Hewavitharana, 1997). In existing JT devices the capillary channel end is exposed to the evaporator so that the channel's end is at the cold temperature. If the JT device is not properly insulated, heat leaks from environment can conduct through the separating wall
reducing the device performance. Therefore, it is necessary to minimize those problems to achieve better performance in JT devices.

This study proposes a JT device based on a layered arrangement. In a layered arrangement, the heat exchanger can be sandwiched between two glass plates, one being the top cover plate, and other being the separating plate between capillary and the heat exchanger. Such an arrangement will minimize the external heat leaks to the heat exchanger. In addition, channel separating walls in the capillary are not directly exposed to the evaporator. The schematic diagram of the proposed JT device is presented in Figure 6.1.

The incoming refrigerant (hot fluid), normally at very high-pressure and room temperature, will be drawn from a gas tank and directed towards the inlet gas manifold of the heat exchanger. The refrigerant then flows down in hot fluid channels (red arrow), and then flows into the inlet manifold of the capillary channels without interfering with the flow in cold fluid channels. In the process, hot fluid is cooled by the cold fluid flowing in the opposite direction in cold fluid channels. In Figure 6.1, cold fluid channels are shown on the top surface of the wafer. Actually, those channels are etched on the backside of the wafer and projected on to the top surface for viewing only. The heat transfer to cold fluid occurs through the thickness of two wafers used in the heat exchanger. In the capillary, refrigerant undergoes expansion and liquid is collected in the evaporator. In the evaporator, liquid refrigerant absorbs heat and leaves the evaporator as a vapor. In this present study, it is assumed that refrigerant leaves the evaporator at a dry
Cold fluid channels have been projected onto the top surface. They are actually etched on the back side.

Figure 6.1 Schematic Arrangement of the Proposed JT Device
saturated state. The refrigerant leaving the evaporator is directed to the cold fluid inlet manifold. In Figure 6.1, cold fluid channels are projected on to the top surface to show the flow diagram but channels are actually etched on the backside. In cold fluid channels, the refrigerant flows in the opposite direction to incoming refrigerant, and absorbs heat from hot fluid across the thickness of two wafers. In the proposed device, fluid flow channels would be etched on silicon wafers, and Pyrex 7740 glass plates would be used as separating plates. These plates improve the structural integrity of the device and can be used in wafer bonding processes.

6.2 Use of Simulation Model in the Design

The first task in the design process is to determine the geometry of the JT device and it would be accomplished using the simulation model presented in Chapter 5. In the proposed device, heat transfer between the cold fluid and hot fluid occurs across the thickness of the silicon wafers. In addition, the counter flow heat exchange and the capillary will have multiple channels. In existing JT devices, the counter flow heat exchanger consists of a single hot fluid channel and two cold fluid channels that are running on either side of the hot fluid channel. Also, the counter flow heat exchanger has winding flow arrangements to accommodate long fluid channels. In such cases the heat transfer occurs across the channel separating wall. Winding flow arrangements also provide the large heat transfer area needed to achieve high heat exchanger effectiveness. In the proposed design, large heat transfer areas can be achieved by having multiple channels in the counter flow heat exchanger, and consequently channel length can be shortened so that straight channels can be used instead of winding flow channels.
The simulation model presented in Chapter 5 was modified according to requirements discussed in the previous section. The convergence of the model was attempted following a similar procedure outlined in Figure 5.5 of Chapter 5. After the convergence has been obtained for a given inlet pressure and an assumed geometry, refrigeration capacity was predicted. First, the refrigeration capacity of the proposed device was predicted for different inlet pressures and different number of fluid flow channels keeping the number of flow channels in the capillary constant. This process helped identify the optimum inlet pressure that should be used in the device. Having identified the inlet pressure, then the number of capillary channels was changed and the convergence was obtained. The process continued until the best number of capillary channels was identified.

As mentioned in Chapter 5, capillary channel width and depth were fixed at 20 \textmu m because that was the minimum line width that can be successfully etched on silicon wafers with a film mask.

The variation of refrigeration capacity for different inlet pressures beginning 3500 kPa to 6000 kPa was simulated. Figure 6.2 graphically present the results. The refrigeration capacity increases as the inlet pressure increases. The Figure shows that about 100 cold fluid and hot fluid channels would be sufficient to obtain maximum refrigeration capacity at 6000 kPa.

Variation of refrigeration capacity with number of capillary channels was examined. Here, the inlet pressure was fixed at 6000 kPa, and the number of hot and cold fluid channels was fixed at 100. It was found that the refrigeration capacity linearly
increased as the number of capillary channels is increased. Figure 6.3 presents the results obtained in the study. This increase of refrigeration capacity should be mainly due to the increase of mass flow rate. However, the model did not converge for capillary channels above 30 because, after the expansion, the refrigerant did not fall into two-phase region. Because of the convergence limitations, the range of number of capillary channels used in this study is not sufficient to observe such a behavior. To examine the effect of heat
exchanger channel length, the number of cold and hot fluid channels was fixed at 100. The inlet pressure was fixed at 6000 kPa. In the previous section, it was found that refrigeration capacity was higher for 30 capillary channels. Therefore, in examining the effect of heat exchanger channel length on the refrigeration capacity, the number of capillary channels was set at 30. The simulation was done for heat exchanger channel lengths beginning 4.5 cm to 6 cm. Figure 6.4 presents the results obtained in the simulation. As the length increased, refrigeration capacity increased. Figure 6.5 presents the heat exchanger effectiveness with channel length. As the channel length increases,
heat transfer area between cold fluid and hot fluid should increase enabling more heat transfer between the cold fluid and the hot fluid. Therefore, the increase of effectiveness with the length can be anticipated. Results presented in Figure 6.5 confirm this anticipation.

The variation of refrigeration capacity with change of width was also studied keeping the aspect ratio at 0.4. This is the original aspect ratio at which the convergence of the model was obtained. However, when the width is changed, depth also changes keeping the aspect ratio unchanged. In the simulation, the model showed a decrease in refrigeration capacity. Therefore, width of heat exchanger channels was kept at 50 µm.

Figure 6.4 Variation of Refrigeration Capacity with Heat Exchanger Length (Heat Exchanger Channels = 100, Capillary Channels = 30, Inlet Pressure = 6000 kPa)
and the aspect ratio was kept at 0.4 because these dimensions produced the greater refrigeration capacity.

The capillary length affects the evaporator temperature as well as the refrigeration capacity. As the capillary length increases, evaporator temperature drops and the refrigeration capacity is reduced. The refrigeration capacity is reduced mainly because the mass flow rate decreases as the capillary length increases. Shorter capillary lengths produce a higher refrigeration capacity due to the increase of mass flow rate. The drawback of shorter capillary lengths is the increase of evaporator temperature. In this study, evaporator temperature is intended to be maintained below 100 K. and therefore a length...
of 6 cm is selected for capillary length. Because heat exchanger channels are also of 6 cm length, a similar length for capillary will help reduce problems associated with overall physical laying out of channels on wafers. Table 6.1 presents the effect of capillary length on the refrigeration capacity, evaporator temperature, and the mass flow rate.

**Table 6.1: Effect of Capillary Length on Refrigeration Conditions**

<table>
<thead>
<tr>
<th>Capillary length/ (cm)</th>
<th>Evaporator temp./ (K)</th>
<th>Refrigeration capacity/ (mW)</th>
<th>Mass flow rate/ (kg/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.5</td>
<td>102.3</td>
<td>340.2</td>
<td>0.00005494</td>
</tr>
<tr>
<td>5.0</td>
<td>100.9</td>
<td>306.0</td>
<td>0.00004535</td>
</tr>
<tr>
<td>6.0</td>
<td>98.8</td>
<td>254.3</td>
<td>0.00003371</td>
</tr>
</tbody>
</table>

The evaporator size was determined by using results obtained by solving the simulation model at no pressure drop condition in the evaporator. The mass flow rate, evaporator temperature, and the evaporator pressure values were used along with Lockhard-Martinelli improved correlation to find the pressure drop. The evaporator dimensions were manipulated until the pressure drop in the evaporator is negligible so that the ideal condition is approached. It was found that square section of 30 x 30 mm and a depth of 20 μm would satisfy the design requirements.

**6.3 ANSYS Analysis of the JT Device**

The previous analysis shows that high inlet pressure is required for successful operation of a JT device. In the proposed design, maximum allowable inlet pressure is 6000 kPa (60 atm). This inlet pressure is significantly lower than inlet pressures used in
existing JT device. Existing JT devices use inlet nitrogen gas pressures ranging from 100 to 120 atm. It is important to examine whether the physical structure of the device can withstand such high-pressures. Therefore, a simple finite element analysis was done using the university edition of ANSYS commercial software package. In the analysis, actual dimensions of the device with a half cross section were used. The following sections present the simulation and discussion of finite element analysis of the proposed device.

The first analysis focused on the possibility of having a single wafer for the heat exchanger. In order to have a single wafer in the heat exchanger, hot fluid channels and cold fluid channels should be fabricated on either side of a both side polished silicon wafer. In such a heat exchanger, very high-pressure difference exists between the bottoms of hot and cold fluid channels. Because fluid channels are 6 cm long, there is a possibility that channel sagging may occur causing blockage of cold fluid channels. In addition, stress distribution within the packaged device should also be within acceptable limits.

Figure 6.6 shows a partial cross section of the packaged device taken along the width. All channels are evenly distributed on either side of the centerline. Hot or cold fluid channel separating walls are equally spaced with a distance of 50 μm whereas capillary channels are separated with a distance of 100 μm. The width of a hot and a cold fluid channel is 50 μm, and the depth is 20 μm whereas those dimensions in the capillary are 20 μm and 20 μm respectively. The evaporator has a cross-section of 30 X 30 mm and its depth is 20 μm. In the heat exchanger, vertical sidewalls of hot fluid channels are subjected to same pressure in opposite direction. Hence, there is no pressure difference
between different hot fluid channels. The separating walls will have same pressure on either side. Therefore, channel separating wall thickness is not critical and can be decided based on the ease of channel layout. However, thickness of the separating wall between hot fluid and cold fluid channels should be sufficiently thick enough to withstand the very high-pressure difference across separating walls. The stress induced by high-pressure difference in the separating wall should be within the allowable limit of stress of the silicon wafer and can be easily estimated in a stress analysis using the ANSYS software package.

Figure 6.6 Cross Section of the Packaged JT Device
In the device, all hot fluid channels are subjected to similar pressure loading conditions. All cold fluid channels, and all capillary channels also have similar pressure loading conditions with different intensities of loading. Therefore, a two-dimensional cross-section of a straight set of channels, shown in Figure 6.7, can be used to study possible channel sagging under pressure loading conditions. In the proposed JT device, pressure is uniformly distributed over the entire length of each channel. In addition, Figure 6.7 shows that the geometry of the packaged JT device is symmetric about the vertical central axis. Hence, in the finite element analysis, symmetry of the loading and geometry can be used to reduce computational time. The half geometry of Figure 6.7, used in the analysis, is presented in Figure 6.8.

Figure 6.7 A longitudinal Cross Section of Packaged JT Device with Heat Exchanger Channels Etched on the Same Wafer

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6.3.1 Pressure Loading

The pressure drop in the hot fluid channel is small (77 kPa), and therefore it is reasonable to assume inlet pressure (6000 kPa) throughout the channel in the stress analysis. The assumption would overestimate the stress slightly. Because the pressure is distributed uniformly throughout the channel, it can be applied along the bottom and the top surfaces of the hot fluid channel. Similarly, cold fluid pressure and capillary fluid pressure can be applied on top and bottom surfaces of respective channels. In Figure 6.8, those top and bottom surfaces are represented by lines, and therefore pressures should be converted to line pressures.

![Figure 6.8 Half Geometry of the Device Shown in Figure 6.7](image-url)
6.3.2 Constraints

Because A-A represents the axis through which half section was taken, this axis is constrained against horizontal movements. Therefore, all points lie on the axis A-A in Figure 6.8 can have upward or downward movements only. The channel ends are assumed to be fixed firmly to remaining portion of silicon wafers. In the case of the heat exchanger, the separating wall between the cold fluid and hot fluid channels behaves very similar to a cantilever. The bottom of the evaporator is assumed to be fixed because the evaporator bottom would rest on another supporting component. Vertical sides of the packaged device and the top surface of the glass cover plate are not fixed and free to undergo any deformation.

The ANSYS (university edition) finite element package was used to simulate the stress distribution and channel deflection with the heat exchanger fabricated on a single wafer. In the simulation, the “ANSYS Structural Solid Quadratic four node 42” element was used. Meshing was completed in such a way that critical areas, such as channel wall and channel wall support regions, have a fine mesh, but uncritical areas such as glass wafer areas consist of a coarse mesh. The number of elements produced in the meshing was 396. Figure 6.9 presents the stress distribution within the device.

The maximum stress that occurs within the device is about 3.3 MPa and at the fixed end. According to Petersen (1982), the maximum allowable stress is 7000 MPa. However, in a review completed by Jaddan et al. (2002) found that the maximum allowable stress depends on the type of support, type of silicon wafer, and its crystallographic orientation along the beam. Therefore, the maximum allowable stress...
may be as low as 1100 MPa in a cantilever support arrangement (Jaddan et al., 2002).

The maximum stress developed in the device is still well below the two limits discussed previously. Figure 6.9 clearly shows the deformation of heat exchanger under the given pressure loading conditions. The deformation was studied and the ANSYS solution was obtained for the deflection of channels. This solution is presented in Figure 6.10.

Figure 6.9 Stress Distribution within the Proposed JT Device with a Single Wafer in the Heat Exchanger

Figure 6.10 shows that the deflection of heat exchanger channels is significant and, the middle of heat exchanger channels lengths will have a deflection about 16 µm. Because the channel depth is 20 µm, this deflection will considerably block the middle of cold fluid channels. Therefore, a single wafer heat exchanger is ruled out of the design.
However, if the channel geometry is modified— for example if the channel layout is such that channel length is small—then the deflection may be small, and a single wafer may be used to fabricate the heat exchanger. Such a scenario should be investigated in future improvements of this proposed device.

One of the alternatives is to use two wafers to fabricate the heat exchanger and bond their backsides to have a single unit. In the analysis, half cross section shown in Figure 6.8 should be modified to include two wafers for the heat exchanger. The pressure loading remains the same as in the previous case. The modified half cross section is shown in Figure 6.11. Figure 6.12 represents the ANSYS stress simulation with two
wafers in the counter flow heat exchanger. In the analysis, pressure loading was applied similar to the case mentioned previously. The simulation shows that the maximum stress is 1.78 MPa in the heat exchanger, and it is slightly over half the maximum stress that would be produced with the single wafer for the heat exchanger. However, the location of the maximum stress has moved to the corner edge of the evaporator.

The results obtained for channel deflection when the heat exchanger consists of two silicon wafers are presented in Figure 6.13. Heat exchanger channel deflection is still there, but the maximum deflection is about 5.4 μm, a sharp drop from the maximum deflection observed from the single wafer heat exchanger case. Therefore, the heat exchanger with two silicon wafers would be desirable in the proposed JT device.
As shown in Figure 6.13, maximum deflection will occur at the middle of the section and it is about 1.6 μm. Therefore, the depth of the cold fluid channels at the middle of the section will be contracted by 1.6 μm whereas the similar point in the hot fluid channels will be expanded by the same amount. The simulation model developed using EES software does not include the effect of this kind of contraction and expansion on the thermal/fluid performance of the heat exchanger. Therefore, future developments should incorporate this possibility and predict the performance accordingly or include modifications to the design to avoid such channels sagging.

Figure 6.12 Stress Distribution within the JT Device with Two Silicon Wafers in the Heat Exchanger
6.4 Simulation of Temperature Distribution within the JT Device

The thermodynamic and heat transfer simulation model presented earlier was used to predict the optimum operating conditions. The EES model solution is presented in Appendix B. According to the model solution, the refrigeration occurs around 99 K. The maximum temperature of nitrogen refrigerant, 300 K, would be at the inlet. This temperature is reduced to 146 K in the heat exchanger when the device reaches steady state. 

Figure 6.13 Deflections of Heat Exchanger Channels under Maximum Inlet Pressure Conditions with Two Silicon Wafers in the Heat Exchanger

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state operating condition. It is desirable to investigate the temperature distribution
with in the system so that areas subjected to possible heat leaks can be identified.

This simulation was completed using ANSYS finite element package. In the
simulation, thermal solid element (Plane 55) was used with quadratic element shape. The
half section used to simulate the temperature distribution is shown in Figure 6.14 and it
included 17, 135 elements

6.4.1 Thermal Loading and
Boundary Conditions

The half section taken along the width consisted of 50 hot fluid and cold fluid
channels and 15 capillary channels. Hot fluid and cold fluid channels have been laid out
on the wafer so that each hot fluid or cold fluid channel is 50 μm apart from the other
one. The actual geometry, in the analysis, was created using graphical user interface
(GUI) of ANSYS. Different layers were glued before the meshing was completed. The
inlet temperature of refrigerant is 300 K and this temperature drops down to 146 K at the
outlet of the hot fluid. The average temperature 223.9 K was used as the hot fluid
temperature. This temperature was applied at every node on the surface of rectangular
channel. In cold fluid channels, temperature varies from about 100 K at the cold inlet to
297 K at the refrigerant exit. Therefore, a cold fluid channel average temperature of 196
K was applied to all nodes on the perimeter of channels. In the capillary, average
capillary temperature is about 120 K. This temperature was applied to all nodes on the
periphery of capillary channels. Evaporator temperature 99 K was applied on the
boundary lines of evaporator. The vertical side of the device that coincides with the half

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cross section line A-A was given zero convection. The top surface and the right hand vertical side are exposed to ambient condition. Therefore, convection loading was applied on the top glass surface and the vertical sidewall of the device. The convection coefficient of ambient depends on the nature of surrounding air. For air, the convection coefficient typically can vary between 5 to 25 W/(m².K) (Lienhard, 1987) depending on the air condition. In this analysis, convection coefficient of 10 W/(m².K) was used assuming that the air surrounding the device remain relatively still. The evaporator bottom is supposed to be exposed to heat flux of a heat source. in the event the device is

Figure 6.14 Part of the Half Section of the Proposed JT Device
used to cool a heat generating component. The refrigeration capacity of the proposed
device is about 250 mW at 99 K. Therefore, evaporator bottom was exposed to external
heat flux that would amount to 250 mW.

The steady state temperature distribution within the device is shown in Figure
6.15. The maximum temperature is concentrated around the top middle portion of the

![Figure 6.15 Steady State Temperature Distributions within the JT Device](image)

device with a maximum value of 223.9 K. The minimum temperature is about 99 K this
temperature seems to spread over the entire bottom silicon wafer. The right hand vertical
side of the device is subjected to temperature variation with the bottom right end having a
temperature of around 100 K. Because the right end vertical side temperature is lower than the ambient temperature, it is possible that heat leaks occur from this end into the device. Therefore, the device needs to be insulated properly.

The ANSYS analysis of the device provided some useful information for fabricating the JT device based on a layered configuration. Heat exchanger should consist of two silicon wafers to reduce the excessive channel sagging. Even though an inlet pressure of 6000 kPa is used, the stress developed in the device was within allowable limits. However, hot fluid channels would have excessive deflection when a single wafer is used for the heat exchanger. This ANSYS analysis shows that two wafers are required in the heat exchanger to reduce the channel deflection to acceptable limits. Table 6.2 presents a summary of the geometry obtained at the conclusion of simulation.

Table 6.2 Geometric Details of the JT Device

<table>
<thead>
<tr>
<th>Component</th>
<th>Length/(cm)</th>
<th>Width/(µm)</th>
<th>Depth/(µm)</th>
<th>No. of channels</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hot fluid channels</td>
<td>6</td>
<td>50</td>
<td>20</td>
<td>100</td>
</tr>
<tr>
<td>Cold fluid channels</td>
<td>6</td>
<td>50</td>
<td>20</td>
<td>100</td>
</tr>
<tr>
<td>Capillary channels</td>
<td>6</td>
<td>20</td>
<td>20</td>
<td>30</td>
</tr>
<tr>
<td>Evaporator</td>
<td>3</td>
<td>$3.10^4$</td>
<td>20</td>
<td>-</td>
</tr>
</tbody>
</table>

Maximum inlet pressure – 6000 kPa
Cooling temperature – 99 K
Refrigeration capacity – 257 mW
The temperature distribution within the device showed that the right vertical end of the device is at low temperatures relative to the ambient temperature. Therefore, exposure of this end to ambient might create paths of heat leak into the device. To prevent heat leaks, the device needs to be insulated properly. The insulation material and the thickness of insulation can be determined by extending the ANSYS analysis presented in this study.
CHAPTER 7

FABRICATION

Fabrication of the proposed JT device involved the photolithography process. Except for the length, the other geometrical parameters of the JT device fall into microscale. Therefore, photolithography methods were employed to fabricate the device. Figure 8.1 presented below is a schematic of the general procedure adopted in the fabrication. The complete procedure for each component is presented in this chapter.

7.1 Evaporator

The evaporator has a cross section of 30 x 30 mm, and it is 20 μm deep. Silicon <110> wafer was selected to etch the evaporator because <110> wafers produce vertical sidewalls in the wet chemical etching (with KOH). The wafer had a 2 μm thick SiO₂ layer on either side but only one face was fully polished. The fabrication procedure generally followed the procedure illustrated in Figure 7.1.

First, the wafer was preheated at 250°C for about 30 minutes to evaporate minute dust particles on the surface. After cleaning it down to room temperature, the wafer was cleaned in a nitrogen stream, and then a primer, HexamethylDisilazane (HMDS), which is a photoresist adhesion promoter, was applied. The Shiplay photoresist 1813 (see was selected as the photoresist. This is a positive photoresist with very good thermal and
Figure 7.1 Schematic Illustration of the Fabrication Procedure
physical properties. It also can produce a minimum feature size of 1 μm. After the priming has been completed, 1 μm thick photoresist was coated on the wafer. The spinning speed used in the coating is presented in Figure 7.13.

Because of cost effectiveness, a negative film mask of the evaporator was used. The exposure to UV light (λ = 360 nm) was completed in 6 seconds. This task was accomplished using an Electronic Vision Mask Aligner.

The exposed areas of the photoresist were dissolved in the developer MF 319 for 1 minute, rinsed in running deionized water, and then dried in a nitrogen gas stream. The pattern was examined under a microscope to ensure that the exposed areas were fully developed, and no photoresist was left on the pattern. If undeveloped areas were noticed, then the wafer was put back in the developer for another 30 seconds to complete the development.

After the development, the wafer was washed in de-ionized water and dried in a nitrogen stream. The pattern was then examined under a microscope. The microscopic examination was essential because it helps identify areas where photoresist was not properly developed. The wafer was then hard baked for 1 min at 120°C, cooled down to room temperature, and etched in Buffered Oxidant Etchant (BOE) solution at 20°C. The BOE solution was prepared mixing hydrogen fluoride (HF) and ammonium fluoride (NH₄F) together in 1:6 ratio. The etch rate of SiO₂ in BOE was about 0.05 μm/hr and therefore a total etching time of 40 minutes were needed to completely etch the exposed SiO₂ masking layer. The wafer was then washed in de-ionized water, dried in a nitrogen stream, and again washed in acetone to remove unexposed photoresist. The removal of remaining photoresist is recommended before wet chemical etching with KOH because:
any leftover photoresist would dissolve in KOH and roughen the etched pattern. The etching with 40% KOH was carried out at 80°C while the solution was stirred using a magnetic stirrer with a rotational speed of 250 revolutions per minute (rpm). The metal etching was completed in 18 minutes to obtain a depth of 20 μm.

Figure 7.2 is an SEM photograph of a section of the evaporator. The wall is clearly distinguishable from the evaporator bottom surface. The evaporator bottom seems to have peak and valleys caused by etching.

![Figure 7.2 SEM Photograph of a Section of the Evaporator](image)

### 7.2 Capillary Channels

Fabrication of capillary channels was similar to the lithographic procedure adopted to fabricate the evaporator. The capillary has 30 channels with a depth of 20 μm. However, the capillary has two holes close to the center of the wafer and that need to be etched first. The following section outlines the fabrication procedure.
Table 7.1 Step Heights Measured at Three Different Places on the Evaporator Bottom Surface

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Test 1</th>
<th>Test 2</th>
<th>Test 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Step Height (µm)</td>
<td>23.56</td>
<td>20.41</td>
<td>21.83</td>
</tr>
</tbody>
</table>

A silicon wafer <110> with both side polished, and a thickness of 2.5 µm of SiO₂ masking layer was selected for the fabrication of the capillary. Because the capillary needs to be bonded to two glass wafers separating the evaporator and the heat exchanger, both-side polished wafer was required.

The wafer was coated with photoresist according to the procedure outlined in the section 7.1. After soft baking, the coated wafer was exposed for 6 seconds through a mask to pattern two holes. After the development, the pattern was examined, and then the wafer was hot baked for about one minute in a nitrogen gas environment. The SiO₂ masking layer on the exposed areas was etched away in BOE solution in about 30-40 minutes time. Then the remaining photoresist coating was washed in acetone and de-ionized water. If any stains remained on the surface, they were washed away in isopropyl alcohol. After drying, a 1 µm thick PR 1813 photoresist coating was spun on the backside of the wafer. The mask pattern of holes was etched on the back side following a similar procedure described earlier. Then the wafer was mounted on a acrylic holder and immersed in 40% KOH solution. The KOH solution was stirred by a magnetic stirrer at a speed of 250 rpm. Before putting the wafer into KOH, the remaining photoresist was washed away with acetone. Any photoresist left on the wafer would quickly dissolve in KOH, but the dissolved photoresist will increase the surface roughness of the pattern. Therefore, it is a good practice to remove any photoresist remaining on the wafer before
KOH etching. During KOH etching, the pattern (holes) was etched from both sides because holes were patterned on both sides. The alignment of holes on both sides was accomplished using the primary cut of the wafer as the alignment edge. When making holes on silicon wafers using KOH etching, it is advantageous to etch from both sides so that the total etching time can be cut down by half. Moreover, the less time the wafer remains in KOH, the better the surface quality will be. The corrosive nature of KOH causes small pits on the wafer where the SiO₂ masking layer is weak and thin. In addition, KOH etching makes the wafer highly fragile. If the highest quality product is desired rather than the most economical one then KOH should be avoided and an alternative dry etching method should be used.

The holes on the capillary are closer to the center of the wafer. Therefore, it was difficult to mount the wafer on the spinner because the vacuum did not work and the spinner could not hold the wafer. This problem was overcome by pasting the wafer on another wafer with photoresist PR 1813 used as an adhesive. The patterning of capillary channels followed a similar procedure described previously. After BOE etching (patterning) wafers were put in an acetone bath for about 6-8 hrs to separate the wafer that was pasted to the capillary wafer. Then the capillary wafer was washed in de-ionized water stream and isopropyl alcohol and again in the de-ionized water stream. Then the wafer was put into 40% KOH and further etching was continued for 18 minutes at 80 °C with the solution stirred at 250 rpm by a magnetic stirrer. After KOH etching has been completed, the wafer was removed from its holder and rinsed in de-ionized water and dried at 110 °C for 30 minutes in a nitrogen environment.
Figure 7.4 is a photograph of the capillary wafer. The inlet and outlet ports and holes were purposely made larger to compensate for any misalignments in packaging the device.

The small white areas visible in Figure 7.4 are small pits caused by KOH etching. The figure also clearly shows damages at the wafer edge. This damage was a result of the excessive weakening of the wafer edges by KOH in the etching process. The two holes in the wafer are exposed to the evaporator. The hole etched on the outlet port let the expanding refrigerant flow into the evaporator. The other hole directs the refrigerant from the evaporator to cold fluid channels in the heat exchanger. Cross marks seen in the wafer are alignment marks used for aligning during patterning.

Figure 7.5 is an SEM photograph of the capillary channels. It shows that the channels are not exactly strait but are satisfactory for the present work. This problem could not be entirely eliminated because a film mask was used to pattern the capillary channels. In Figure 7.5 some undercutting at the entrance of channels is observed. This is a problem encountered in wet chemical etching. Figure 7.6 is an enlargement of the entrance. It clearly shows that the undercutting is limited to the entrance. It can also be observed that the channel bottoms are not flat. The sharp corners of a pattern usually get rounded after the exposure to UV light and this rounded shape exposed other etching planes to wet chemical etching causing undercutting. This problem could be avoided by introducing corner compensations to the pattern at the CAD drawing stage.

Table 7.2 presents the roughness and step height data for capillary channels. Data were obtained at three different locations on capillary channels and are presented in Appendix C.
Figure 7.4 Capillary Channel Wafer

Figure 7.5 SEM Photograph of Capillary Channels

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Figure 7.6 SEM Photograph of Capillary Channels Showing the Undercutting of Walls at the Channel Entrance

Table 7.2 Roughness Data at Three Different Locations in Capillary Channels

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Test 1</th>
<th>Test 2</th>
<th>Test 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capillary channel roughness</td>
<td>139.76 nm</td>
<td>181.03 nm</td>
<td>124.93 nm</td>
</tr>
<tr>
<td>Capillary channel step height</td>
<td>22.49 μm</td>
<td>22.97 μm</td>
<td>23.16 μm</td>
</tr>
</tbody>
</table>

The average roughness is 148.57 nm and the average channel depth is 22.87 μm. The three tests produced roughness data that vary in a wide range. But the step height data are quite satisfactory. The channel step heights include about 2 μm of SiO₂ masking layer. Therefore, the average step height should be about 20.87 μm where as the design step height (depth) is 20 μm.
7.3 **Hot Fluid Channels**

A both-side polished silicon wafer with a 2 µm SiO₂ masking layer was selected to fabricate the hot fluid channels. The hot fluid channel wafer needs to be bonded to the cold fluid channel wafer to make the counter flow heat exchanger of the JT device. Therefore, a both side polished wafer was required. Firstly, channels were fabricated on one side of the wafer by implementing a fabrication procedure similar to the one used to fabricate capillary channels. However, holes were etched from the backside using Inductively Coupled Plasma (ICP) etching. The exposure of holes on the backside of the wafer was very similar to the procedure adopted in the capillary to make holes.

Figure 7.7 shows the hot fluid channel wafer. Some damage on the wafer is very prominent, but the damaged region is well outside channel area. Therefore, this wafer was acceptable to be used in the heat exchanger. The damage was a result of weak SiO₂ masking areas affected by KOH and ICP etching. The hole on the outlet port will direct refrigerant to the top capillary inlet port and bypass the cold fluid channels. The other hole is exposed to the cold fluid outlet port so that the refrigerant can be released to ambience.

Figure 7.8 shows the SEM photograph of the hot fluid channels. In the hot fluid channels too, undercutting at the channel entrance can be observed. In addition, surface damage on the top surface of channel separating wall is visible. This damage was actually due to quality problems in the film mask printing and copied on to the pattern in the lithography process. Figure 7.9 clearly shows that the undercutting in hot fluid channels is limited to the channel entrance. It also shows that the channel bottom is not completely...
Figure 7.7 The Hot Fluid Channel Wafer

Figure 7.8 SEM Photograph of Hot Fluid Channels
Figure 7.9 SEM Photograph of Hot Fluid Channels
Showing Surface Damages on Channels
Separating Walls

flat. However, channel bottoms do not seem to have a grass like surface roughness variation. Therefore, the quality of the channel bottom is satisfactory despite some surface damage on the channel-separating wall.

The roughness of channels was measured using a Wyko surface profiler. Channel roughness data were collected from three different areas of channels. In addition, step heights were also obtained. This data is presented in table 7.3.

Table 7.3 Roughness Data at Three Different Locations in Hot Fluid Channels

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Test 1</th>
<th>Test 2</th>
<th>Test 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hot fluid channel</td>
<td>229.27 nm</td>
<td>197.40 nm</td>
<td>147.65 nm</td>
</tr>
<tr>
<td>roughness</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hot fluid channel</td>
<td>19.59 μm</td>
<td>20.68 μm</td>
<td>20.39 μm</td>
</tr>
<tr>
<td>step height</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

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The average roughness of hot fluid channels is 191.4 nm and the average step height (depth) is 20.22. These measurements were taken after completely removing SiO$_2$ masking layer. The fabricated depth is acceptable and close to the 20 µm design depth.

### 7.4 Cold Fluid Channels

Cold fluid channels in the heat exchanger have similar dimensions to hot fluid channels and same number of channels is used in hot fluid wafer and cold fluid wafer. Therefore, fabrication of cold fluid channels can be done following a similar procedure used to fabricate hot fluid channels. However, in this case, ICP etching was used to etch channels and holes.

First, after pre-cleaning and drying, both sides of the wafer were coated with photoresist PR 1813. Because the holes are required to be etched from the back side, back side SiO$_2$ masking layer should be protected. Therefore, a thin photoresist layer was coated on the backside to protect the SiO$_2$ masking layer. The coating of photoresist was done by following a similar procedure described previously. Patterning of cold fluid channels on the front side was done following the similar fabrication procedure for hot fluid channels. After patterning, the wafer was washed in acetone to remove remaining photoresist, and then isopropyl alcohol was used to remove any stains left on the wafer. Subsequently, the pattern was etched using ICP etching to make fluid flow channels.

Figure 7.10 is a photograph of the cold fluid channel wafer. In this patterning of the device, wafer would be flipped and the backside would be bonded to the backside of hot fluid channel wafer. The hole in the outlet port will coincide with the refrigerant exit in the hot fluid channel wafer while the inlet port would receive refrigerant coming from
the evaporator. The patches that can be noticed on the wafer are areas where SiO$_2$ layer has been stripped off. However, before bonding, the SiO$_2$ layer would be completely removed and therefore, the patches will not make any difference.

Figure 7.11 show an SEM photograph of cold fluid channels. The side walls are straight and channel bottoms are flat. Surface damage observed in the hot fluid channel walls can also be observed in cold fluid channel walls. This damage was caused by the imperfections of the film mask pattern.

In ICP etching, accelerated ions strike the surface vertically. When patterned channel edges are not straight and have imperfections, etching will produce surface imperfections on vertical sidewalls. The spines like features on vertical side walls were produced by channel edge imperfections. Figure 7.12 clearly shows the imperfections on sidewalls and top surfaces. However, it can be noticed that the channel bottom surface is

\begin{center}
\textbf{Figure 7.10 Cold Fluid Channel Wafer – The Wafer will be in Flipped Position in the Heat Exchanger}
\end{center}
flat and the surface seems to be smoother than the channel bottom surface obtained in KOH etching.

Table 7.4 presents the roughness and step height data for the cold fluid channels. The average roughness of cold fluid channels is 147.02 nm and is appreciably smoother than the average roughness of hot fluid channels. The average roughness of the capillary channels is 148.58 nm. However, capillary roughness varied over a wide range of values due to the inability of the surface profiler to capture a sufficiently large area to produce a better profile. The capillary channel bottoms are not flat, and the channel width is narrowing towards the bottom. The narrowing of channels towards the bottom compounds the problem of not capturing a sufficiently large area for profiling.
Table 7.4 Roughness Data at Three Different Locations in Cold Fluid Channels

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Test 1</th>
<th>Test 2</th>
<th>Test 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cold fluid channel roughness</td>
<td>152.68 nm</td>
<td>142.97 nm</td>
<td>145.71 nm</td>
</tr>
<tr>
<td>Cold fluid channel step height</td>
<td>23.46 μm</td>
<td>23.94 μm</td>
<td>23.81 μm</td>
</tr>
</tbody>
</table>

Figure 7.12 SEM Photograph Showing Damages on Vertical Side Walls

The average step height (depth) of cold fluid channels is 23.74 μm. This depth included the 2 μm of SiO₂ masking layer. Therefore, the actual channel depth would be around 21.74 μm. This depth represents a variation of about 9 percent from the design depth.
In this present work, several trials were carried out to establish the lithography procedure for the present work. The procedure did not require drastic changes in the fabrication procedure for other components of the JT device. The SiO₂ masking layer was completely removed from the hot fluid wafer before roughness measurements were taken. However, in fabricating cold fluid channels, capillary, and the evaporator, SiO₂ masking layer was not removed until before the cleaning process required in the bonding process.

To record the lithography process established to fabricate the JT device, a summary of the process is presented below.

7.5 Summary of the Fabrication Procedure

1. Heat the silicon wafer at 250 °C for 30 minutes to remove water molecules attached to the surface.
2. Cool the wafer to room temperature.
3. Spin primer – The spinner cycle is shown in Figure 7.13.
4. Coat 1 μm thick photoresist – PR 1813 positive photoresist is advisable. The spinner cycle is shown in Figure 7.14.
5. Soft bake for 1 minute at 120 °C in nitrogen environment.
6. Expose the pattern in a mask aligner to UV light (wave length = 360 nm) for 6 seconds.
7. Develop the pattern- if PR 1813 or PR 1811 was used as the photoresist, use MF 319 developer. Development time is1-2 minutes.
8. Wash the wafer in running de-ionized water and dry in a nitrogen stream. Examine the pattern under microscope using yellow light.
9. If the pattern is not properly developed, wash the wafer in acetone, then deionized water, and the isopropyl alcohol. Dry in a nitrogen stream and restart from step 1. If the pattern is properly developed and no photoresist residuals remain in the pattern area, proceed to step 10.

**Figure 7.13 Spinner Cycle for Applying Primer (Resist Adhesion Promoter)**

10. Hard bake for 1 minute in nitrogen environment at 120 °C.

11. Cool down to room temperature and etch in BOE solution to remove the SiO₂ masking layer in exposed areas. Etching time may vary depending on the thickness of SiO₂ layer. Usually, 2 µm thick SiO₂ layer can be etched in 40 minutes time. However, if the BOE solution is fresh, this time may go down even to 20 minutes.
12. Wash the wafer in running de-ionized water. Wash remaining photoresist in acetone until the photoresist layer is completely dissolved and washed away. Then wash in running de-ionized water and with isopropyl alcohol. Dry at 120 °C for 1-5 minutes.

13. Continue etching with 40% KOH or ICP etching to make channels and holes.

14. After etching, rinse in de ionized water, and wash the pattern with acetone, isopropyl alcohol, and again with water. Dry the wafer at 120 °C for 30 minutes. cool down and store.
CHAPTER 8

INTEGRATION OF THE JOULE-THOMSON DEVICE

The packaging of microdevices is a challenge in MEMS research and fabrication, and micro fluidic devices are no different. Devices that have fluid flow in micro channels should be properly sealed to avoid leakage. Packaging is especially challenging when high-pressures are involved. The bonding methods discussed in chapter 4 are usually used in the industry for packaging of micro devices. In the micro fluidic area, use of anodic bonding has become extremely popular. In the anodic bonding process, a piece of silicon wafer with a pattern on it is bonded to Pyrex glass wafer. This type of bonding produces hermetic seals of very high quality.

The fabrication of the JT device involved the photolithography process and is described in chapter 7. The integration or packaging the device is the next step in building proposed JT device. The proposed JT device consists of several layers of silicon wafers and glass wafers. The packaging of those layers should be such that each wafer is properly aligned with other wafers to assure the uninterrupted fluid flow. The proposed JT device is a novel design that aims to minimize possible heat leaks from surroundings to the working fluid. Current JT devices are prone to such external heat transfer.
In packaging the device anodic bonding and wafer fusion bonding were used. This chapter deals with the packaging of the proposed JT device using anodic bonding and fusion bonding. The process is illustrated in Figures 8.1 (a) and 8.1 (b).

### 8.1 Fixture for Anodic Bonding

The anodic bonding procedure was explained in chapter 4. As shown in Figure 8.1, the packaging of the present device involves several steps of anodic bonding. Very often, anodic bonding is done only with one silicon wafer and one glass wafer. However, micro fluidic devices sometimes require bonding of several layers. Commercial equipment is available to bond two silicon wafers to one glass wafer.

The present device involves several steps of anodic bonding as shown in Figures 8.1(a) and 8.1(b). Therefore, a fixture was built to accomplish the required anodic bonding steps.

The fixture consists of a steel plate which can slide into the base plate of a Thermolyne electric furnace which can heat up to 1200 °C and a top steel plate that consists of welded pins. Two small bolts were welded on to the other side of this top steel connected to negative terminal of a power supply. The base plate is fastened with another steel plate in order to place the silicon wafer. A thin steel rod was welded to the sliding base plate which can go through a hole on the backside of the furnace so that the rod can be connected to a power supply unit. Four guiding pins were welded to the base plate so that the glass wafer can sit in place without being dislocated. The top plate that consists of pins can be kept in place with the use of two 2-inch long ceramic rods that can go into circular recess on the base plate. Figures 8.2 and 8.3 illustrate the two units of the fixture.
Figure 8.1a Integration Procedure of the JT Device

1. Anodic bonding – Evaporator & Separating glass wafer.

2. Flip the previously bonded unit & then do anodic bonding – Capillary & glass wafer 1.

3. Flip the bonded unit in stage (2). Then anodically bond glass wafer (g2) to the Capillary.

4. Fusion bonding – Hot fluid channel wafer & Cold fluid channel wafer.
5. Flip the fusion bonded heat exchanger. Flip the bonded unit in stage 3 and anodically bond glass wafer (g2) to the cold fluid channel wafer.

6. Flip the bonded unit. Place the top glass wafer and anodically bond it to the heat exchanger.

Figure 8.1b Integration Procedure of the JT Device

Figure 8.2 Bottom Unit of the Anodic Bonding Fixture
8.2 Bonding of Evaporator and Glass Wafer

In anodic bonding, a Pyrex 7740 glass wafer is bonded to a silicon wafer. The coefficients of linear thermal expansion of these two materials are close to each other, and therefore the thermal stress does not exceed the allowable stress level for a silicon wafer. Table 8.1 provides some physical and thermal properties of these two materials.

The packaging process started with the anodic bonding of the evaporator wafer to the Pyrex 7740 glass wafer that separates the evaporator and the capillary. Before the anodic bonding process the evaporator was put into a BOE solution to completely remove the SiO₂ masking layer. The wafer was then rinsed in running de-ionized water, washed with acetone, isopropyl alcohol, and then rinsed with running de-ionized water. The wafer was then dried in a nitrogen stream and then at 120°C for 30 minutes. The glass wafer was
rinsed in de-ionized water, washed with acetone, isopropyl alcohol, and in running
de-ionized water. The glass wafer was then dried in a nitrogen gas stream and then at
120°C for 30 minutes. Then the evaporator was placed on the fixture especially designed
for the anodic bonding process, and the glass wafer was placed on top of the evaporator
wafer. The glass wafer was properly aligned with the guide pins on the fixture and a steel
plate with 12 welded protruded fins was placed on the glass wafer. This steel plate was
securely positioned in the fixture with two ceramic rods that go through two holes on the
steel plate and then fix into two corresponding circular recesses. A steel wire was
connected to a bolt welded on to the steel plate. The fixture was then placed in
Thermolyne electric capable of reaching 1200 °C. The furnace was switched on and the
temperature was gradually brought to 450 °C over a period of two hours. This
temperature was maintained for another 45 minutes, and then a dc voltage of 750 V was

<table>
<thead>
<tr>
<th>Table 8.1 Physical Properties of Silicon</th>
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<tbody>
<tr>
<td><strong>Young’s Modulus</strong></td>
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<tr>
<td><strong>Poisson’s Ratio</strong></td>
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<tr>
<td><strong>Density</strong></td>
</tr>
</tbody>
</table>

(Reference: Petersen, K.E. Silicon as a mechanical material. Proc. IEEE. Vol 70, No.5, 1982)
applied between the silicon wafer and the glass wafer. The base of the anodic bonding
structure was connected to the positive terminal whereas the glass wafer was connected
to the negative terminal. The voltage was applied until the anodic current dropped and
remained constant over a period of time. Then the furnace was switched off and the
temperature was brought down to room temperature over a period of 3 hours.
Figure 8.4 shows the bonded silicon wafer and the glass wafer. The evaporator is clearly seen underneath the glass wafer. A small void can be seen near the gas outlet. Voids are usually formed when a small dust particle trapped between the glass wafer or due to the release of gases during the bonding process. However, this air pocket is well away from the pattern.

The bonded area was examined under a standard CCD camera. In Figures 8.5a to 8.5b, dark areas show the bond between the wafer and the glass wafer. The light white spots seen in Figure 8.5 are the weakly bonded areas. The figures show that the bond is strong around the edge of the evaporator. Figure 8.6 shows the trapped air pocket near the edge of the glass wafer.

The variation of the anodic current with time is shown in Figure 8.5. The anodic current started with a high value of 3.4 mA and then dropped to a stable value of 0.3 mA over time period of 2 hrs. Initially, anodic current dropped very quickly and then the rate became slower as the time progressed.

The next step in the bonding process is to bond the capillary to the glass wafer. The process started with the removal of SiO₂ layer from the capillary. The process followed the similar procedure described in the previous section. Next the cleaning of the bonded unit was completed using acetone and isopropyl alcohol. Subsequent drying procedure was very similar to the procedure mentioned previously. The anodic bonding procedure was carried out at 450 °C. In the bonding process, anodic current started with 0.8 mA and then dropped to 0.1 mA in 22 minutes and remained unchanged for another 50 minutes approximately. The variation of the anodic current with time is presented in Figure 8.7.
Figure 8.4 Evaporator Anodically Bonded to Glass Wafer

Figure 8.8 presents the variation of anodic current with time for the bonding of the capillary wafer to the glass wafer that was previously bonded to evaporator.

In Figure 8.8, it can be noticed that the anodic current sometimes remained constant for some time and then suddenly dropped to a lower value (step drop). The least count (minimum readable value) of the ammeter on the furnace is 0.1 mA and therefore any variation smaller than 0.1 mA cannot be read. The other noticeable feature is that the anodic current is substantially less than the one observed in the previous bonding process. It was impossible to understand why the anodic current decreased significantly in the second bonding step. In bonding the capillary wafer to the glass wafer, bonded previously to the evaporator, negative voltage was applied to both the glass wafer and the evaporator. The capillary was on the base plate of the fixture and the positive voltage was
applied to the base. Figure 8.9 shows the bonded capillary wafer to the previously bonded unit. In the fabrication of capillary, KOH was used for deep etching including holes. The capillary wafer was severely weakened in the etching process and created small defects on the surface. Those defects can be clearly seen in Figure 8.9.

The bonding of the capillary should be followed by another bonding that bond the capillary to the glass wafer which separates the capillary from heat exchanger. The bonding procedure should begin with the cleaning of capillary and the glass wafer. Here, only the cleaning step is required, and it was done following the similar procedure described in previous sections. After cleaning and drying, the already bonded unit was placed on the base plate of the fixture such that evaporator was the bottom layer. The glass wafer was placed on the capillary and then properly aligned using the guide pins.

Figure 8.5a  CCD Picture of the Portion of Anodically Bonded Evaporator to a Glass Wafer
Figure 8.5b  CCD Picture of the Portion of Anodically Bonded Evaporator to a Glass Wafer

Figure 8.6  Air Pocket in the Bonding Area
available on the fixture. Then the fixture was placed in the furnace and switched on. The temperature was brought up to 450 °C over two hours. A voltage of 850 V was applied between the top glass plate and the capillary. Here, the positive voltage was tapped from the fixture base and then applied to capillary wafer using alligator clips. The anodic current was recorded with time as the bonding progressed.

The anodic bonding current started with a high value- 3.5 mA as in the very first bonding stage. The current did not drop as fast as observed in previous bonding. The bonding process continued for about 7 hours at the end of which the anodic current
dropped to 0.3 mA. Figure 8.10 shows the variation of anodic current with time. The glass wafer bonded to the capillary is shown in Figure 8.11. The figure shows that some damage was done to the glass wafer by the pins that applied the voltage. Small pits were created by the pins that rested on the wafer. In addition, the left hand edge of the glass wafer was slightly damaged on the surface but it was not clear how this damage occurred. The bond was examined with a CCD camera. Figure 8.12 shows a section near the top of capillary. In Figure 8.12, small light patches can be seen. These are actually weakly bonded areas. Figure 8.13 shows an area near capillary channels. This area seems to have many defects and may not have bonded strongly.
The next step in the fabrication process is to bond the cold fluid channel wafer and the hot fluid channels wafer together and form the heat exchanger. In chapter 4, wafer-to-wafer bonding methods were presented. In this case, fusion bonding of two wafers is preferred because, after bonding, the two bonded wafers have bulk like properties. Moreover, an intermediate layer between hot fluid channel wafer and cold fluid channel wafer may increase the thermal resistance. Fusion bonding requires stringent surface quality characteristics. The process adopted in this work is outlined below.

The cleaning procedure is known as RCA cleaning procedure. RCA stand for *Radio Corporation of America*. This procedure was developed by Werner Kern in 1965 while working for RCA. Since then this method has become the industry standard method.
for removing contaminants from wafers. The cleaning procedure has three major steps used sequentially.

1. **Organic Clean:** Removal of insoluble organic contaminants with a 5:1:1 H\textsubscript{2}O: H\textsubscript{2}O\textsubscript{2}: NH\textsubscript{4}OH solution.

2. **Oxide Strip:** Removal of a thin SiO\textsubscript{2} layer where metallic contaminants may accumulate as a result of the process 1.

3. **Ionic Clean:** Removal of ionic and heavy metal atomic contaminants using a solution of 6:1:1 H\textsubscript{2}O: H\textsubscript{2}O\textsubscript{2}: HCl.

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In the preparation of solution, 625 ml of de-ionized water was taken into a 1000 ml beaker. Then 125 ml of NH₄OH (30% concentration) was added into the beaker followed by 125 ml of H₂O₂ (30% concentration). The solution was stirred with clean Teflon rod and placed on a hot plate. A magnetic stirrer and temperature probe were placed in the solution. The temperature probe was set at 75°C and the solution was stirred at 250 rpm. When the temperature reached 75°C, the solution started bubbling violently. At this point, a wafer mounted on Teflon holders was immersed in the solution. The

Figure 8.11 Glass Wafer Bonded to Capillary Channel Wafer

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solution then started bubbling even more violently but subsided later. However, intense bubbling continued. *This procedure requires utmost adherence to relevant safety regulations because a slight mistake can cause fire or an explosion.* The wafer was
treated in the solution for 20 minutes and then removed. It was then rinsed in de-
ionized water and put into a BOE bath. After keeping the wafer 5 minutes in the BOE
bath, it was removed, rinsed in de-ionized water, and then dried in a nitrogen gas stream.
The next step is to clean the wafer in the ionic solution. This process is usually known as
RCA 2 cleaning, whereas the previously mentioned process is known as RCA 1 process.

In preparing RCA 2 solution, 625 ml of de-ionized water was taken into a 1000ml 
beaker, and 125 ml of HCl with a concentration of 33% was poured into the beaker with
de ionized water. Then 125 ml of H_2O_2 (concentration 30%) was added and the solution
was stirred with a Teflon rod. Then the beaker was placed on a hot plate and a
temperature measuring probe was installed. The probe temperature was set at 75°C and
the solution was stirred using a magnetic stirrer at 250 rpm. The solution started bubbling
vigorously around 75°C. This vigorous bubbling is an indication of the readiness of the
solution for cleaning.

After cleaning the wafers, water molecules attached to the surface should be
removed. This removal can be done by placing wafers under UV light for about 30
minutes. Then the wafers are mated in a clean room environment in such a way that the
bonding wave propagates forward from the initial point of contact. This is an important
step because it is important to eliminate voids. After that the wafers are placed in a
furnace and the annealing process is carried out at 1000°C for about 2 hours.

The author carried out several trials and was able to bond a wafer. The wafer
broke in the cooling process. The broken pieces revealed that the wafer was bonded in
some areas. It was found in a later trial that the breakage could be avoided heating and
cooling with a rate of 2°C/min. Some authors have suggested a cooling rate of 5°C/min.
After those trials have been completed, fusion bonding was carried out to bond two wafers that formed the heat exchanger. Subsequently, the heat exchanger was anodically bonded to the top glass wafer. Here, the heat exchanger should be connected to the positive terminal of the power supply, and the author succeeded in making the connection from the sides using alligator clips. The top glass wafer completely bonded to the heat exchanger.

The bonded unit held for a few hours and then separated from the capillary and heat exchanger. The separation was probably due to a poor bond resulting from unsatisfactory surface quality.
CHAPTER 9

CONCLUSIONS AND RECOMMENDATIONS

9.1 Conclusions

This dissertation deals with the design and fabrication of a microscale Joule-Thomson refrigerator based on a layered arrangement of silicon wafers and glass wafers. Firstly, a comprehensive literature survey was undertaken to fully understand the current state of knowledge in the development of JT devices. The literature survey revealed that there was no simulation model for JT devices to use as a design tool. In addition, existing JT devices are susceptible to external heat transfer which would inhibit heat exchanger performance. This dissertation attempted to provide a reasonable solution to these two problems. In the first stage of this work, a simple thermodynamic, heat transfer, and fluid flow model was developed for JT devices. In the development of the model, the conventional correlations were used because these equations were reasonably accurate enough for design purposes in microscale flow conditions. The model was developed by considering the simple Linde-Hampson system on which simple liquefaction systems are based. The governing equations were written for each component of the system and then solved using EES software. EES is a equation solver which has built in thermodynamic database for most refrigerants including nitrogen.
The model was used to simulate existing single layer JT devices. It was found that a refrigeration capacity of 250 mW could be obtained with 100 atm inlet pressure with a geometry that produced a flow rate of 15 ml/min approximately at STP. This result is in close agreement with the experimental results obtained by Little (1984) who reported that 250 mW refrigeration capacity with a flow rate of 18 ml/min approximately at STP.

The heat exchanger of JT device is unique because the same refrigerant is flowing through the counter flow heat exchanger on either side. The inlet temperature at the cold side is not independent of the inlet condition at the hot side and geometry of the device. Therefore, any changes at the inlet conditions of the hot side or the geometry will affect the performance of the device.

For a fixed geometry, the effect of the inlet pressure on the performance of a JT device was examined. As the inlet pressure increased, the device mass flow rate also increased. The variation is linear in the pressure range considered in this study.

The results also showed that the refrigeration capacity increased with the increase of inlet pressure. Refrigeration capacity depends on the mass flow rate and the enthalpy difference between the inlet at the hot side and outlet at the cold side. Both the mass flow rate and the enthalpy difference increase as the inlet pressure increases. Therefore, the refrigeration capacity continues to increase with the inlet pressure. The increase should occur up to a certain point, beyond which the refrigerant does not produce two-phase conditions after the expansion. Therefore, for a given inlet pressure there exist an optimum geometry that would provide the highest refrigeration capacity.

For a given geometry, the evaporator pressure increases as the inlet pressure increases at the hot side. Because two-phase conditions prevail in the evaporator, the
temperature is not independent of pressure. Therefore, an increase of evaporator temperature is observed as the pressure of the inlet refrigerant increases at the hot side. However, it was found that the refrigeration capacity increased with the increase of inlet pressure. Therefore, as the inlet pressure increases, refrigeration capacity increases but it should occur at increased evaporator temperatures.

In Chapter 6, the variation of the quality of the refrigerant with the inlet pressure was presented. As the inlet pressure increases, the refrigerant exiting the capillary becomes wetter helping to remove more heat by increasing the flow rate of liquid refrigerant for evaporation.

For a given inlet pressure, the effect of aspect ratio on the mass flow rate, effectiveness, and the refrigeration capacity was examined. As the aspect ratio increases, mass flow rate increases eventually reaching a maximum. Effectiveness increases with the increase of aspect ratio because the effective heat transfer area between hot fluid and cold fluid increase providing a larger $NTU$.

The simulation model was also used to obtain the geometry of a novel JT device based on a layered arrangement of silicon wafers and glass wafers. The design procedure was described in chapter 6. In the design process, the geometry, other than the number of channels in the heat exchanger, was fixed and the effect of inlet pressure on the refrigeration capacity was studied. The inlet pressures beginning 3500 kPa to 6000 kPa were considered. The refrigeration capacity increased with the number of heat exchanger channels and also with the inlet pressure. However, at 6000 kPa, 100 channels in the heat exchanger (on either side of the heat exchanger) were found to be the most desirable.
number of channels in the heat exchanger. Also, the maximum inlet pressure that would be used in the JT device was found to be 6000 kPa.

The effect of number of capillary channels on the refrigeration effect was examined while keeping the heat exchanger channels at 100 and the inlet pressure at 6000 kPa. As the number of capillary channels increased, the refrigeration capacity increased. The increase in mass flow rate, as the number of capillary increases, is a major factor that contributed to the increased refrigeration capacity.

The length of the heat exchanger channels has an effect on the refrigeration capacity. As the channel length increases, heat transfer area between the cold fluid and hot fluid increases resulting in an increase effectiveness and refrigeration capacity. In the present design, maximum channel length is limited to 6 cm because it produced higher refrigeration capacity and makes it easier to lay straight channels on the wafer. The main reason for choosing straight channel was that it was easier to lay 100 channels straight on the wafer in a layered design than with a winding flow arrangement.

The width and the depth of channels were made 50 µm and 20 µm respectively. The variation of refrigeration capacity with the width and the depth showed that those two dimensions were the best for the JT device.

The variation of capillary length also had an effect on the refrigeration capacity. As the length becomes shorter, the refrigeration capacity increased because of the improvement in the mass flow rate. However, with shorter capillary channels, the model predicted higher evaporator temperatures. In order to keep the evaporator temperature approximately around 100 K, the length was kept at 6 cm. Longer capillary length will
decrease the evaporator temperature but then straight channels could not be laid on the wafer.

The design of the evaporator posed a greater challenge. The end of the capillary is exposed to the evaporator and the refrigerant is assumed to be leaving the evaporator at saturated conditions. Therefore, in this ideal situation, the pressure drop within the evaporator can be considered negligible. However, in the actual situation, slow moving refrigerant cause a pressure drop. Moreover, two-phase conditions exist in the evaporator and heat transfer occurs into the evaporator. Under this situation, adiabatic conditions did not exist. Therefore, any pressure drop calculation should involve correction to the Lockhart-Martinelli correlation. However, it was extremely difficult to get the convergence of the model with those equations incorporated. This problem was overcome by solving the model for the ideal condition of no pressure drop in then evaporator and then using the ideal mass flow rate with Lockhart-Martinelli correlations to compute the pressure drop. The evaporator dimensions were adjusted until the pressure drop is negligible so that the ideal condition was approached.

An ANSYS analysis of the proposed layered arrangement showed that the heat exchanger needed to be fabricated in two wafers. A single wafer heat exchanger with straight channels would have deflection about 53 µm, which would block the flow in the cold fluid channels. The analysis also showed that stress level produced in the system is within the range of allowable stress. The maximum temperature within the system is about 223 K and occurs at the middle of the top glass cover plate.

In the fabrication process, silicon wafers, mostly <110>, with at least 2 µm SiO₂ masking layer were used. The process recipe for the fabrication of channels in the device
was established. This recipe was presented in chapter 7. The evaporator was completely etched using wet chemical etching and KOH was used to complete the deep etching to achieve 20 μm of depth. Total KOH etching time was in the range of 15-20 minutes. This etching time did not produce significant damage to the wafer but small pits could be seen. The capillary wafer as well was entirely fabricated using KOH etching. First, the two holes on the wafer were made. A both-side polished silicon wafer was used for the capillary because it needed to be bonded to two glass wafers on either side. In addition, hole patterns were made on either side of the wafer so that etching could occur from both sides. If the etching were done purely from one side, it would take more than 5 hours to complete the process. During this time considerable damage would occur to the wafer. Very often, longer duration KOH etching produced pits on the wafer surface, and it can severely weaken the wafer. Therefore, cutting down the process time by half was advantageous. In chapter 7, SEM photographs of capillary channels were presented. The quality of channels was satisfactory because, with film masks, it was extremely difficult to etch patterns smaller than 20 μm. There was some undercutting at the entrance of channels. This was because sharp corners were rounded off during the exposure, and in subsequent etching those corners exposed additional etching planes. However, etching rates are different along different planes and therefore cause undercutting. This problem can be avoided by introducing corner compensation at the CAD drawing stage. A special problem encountered in the fabrication of this wafer was that it could not be mounted on the spinner since a patterned hole in it prevented of holding the vacuum. This problem was overcome by temporarily bonding the wafer to another wafer and mounting it on the spinner. However, the photoresist coating near holes was not uniform and therefore the
procedure had to be repeated several times to obtain a reasonably good photoresist layer so that the channels could be etched.

The cold fluid channels were fabricated on a both-side polished silicon wafer that had a 2.5 μm SiO₂ masking layer. In the exposure, film mask for channels was exposed and it was followed by the exposure of holes. Subsequently, the holes were also patterned on the back side of the wafer. Inductively coupled Plasma etching (ICP) was used to etch the channels and holes. First, the channels were made and then the wafer was flipped and etched from the backside to complete the holes. An SEM picture of the cold fluid channels shows that the channel bottom is flat and the walls are very straight. There were some defects on the top surface of the channel wall and on the sidewall. These defects were originally on the film mask and then transferred to the substrate. The channel roughness values show that ICP etching produced smoother channels than wet etching. In fabricating the hot fluid channels, channels were etched in KOH and then ICP etching was used to make holes.

The packaging procedure started with the bonding of the glass wafer to the evaporator. A Pyrex 7740 glass wafer that was 3 mm thick and longer than a standard wafer was used. This is because, in later stages, voltage could be applied from sides of the glass wafer. The first bonding was done at 750 V and at a temperature of 450 °C. The bond area was examined under a microscope and the pictures were grabbed by a CCD camera mounted on the microscope. The pictures presented in chapter 8 show that bonding was very good.

The capillary wafer (backside) was anodically bonded to the other side of the glass wafer that was bonded previously to the evaporator. In this bonding, the capillary
was placed on the wafer support base and the previously bonded unit was flipped and then placed on the capillary wafer. Negative voltage was applied to the glass wafer from sides using alligator clips. However, the bonded area could not be examined with a microscope because light cannot penetrate the silicon wafer. An infrared camera should be used to examine the bonded area because it can penetrate through silicon wafer. The satisfactory bonding also suggests that voltage can be applied from sides to obtain the bonding. This kind of attempt has not been previously reported in the literature. So this attempt may be the first to explore such a possibility. The author was able to anodically bond the next glass wafer to the capillary (top side). However, the pictures obtained using a microscope and a CCD camera showed that the bonding was poor. In this bonding process, anodic current dropped to 0.3 mA and did not fall further. In a successful anodic bonding, the current should drop to 0.1 mA or below resulting in a strong bond. The bond held for about a week and then separated.

The heat exchanger consisted of two wafers. These two wafers should be fusion bonded to produce a good contact between backsides to assure proper heat transfer. In fusion bonding wafer cleaning is very important. The bonding surfaces should be free of metallic ions and other oxidizing elements. The standard cleaning procedure is known as RCA cleaning procedure and should be implemented with proper safety precautions. The author attempted several trials, and in one trial he was able to bond part of the wafer. However, it broke into pieces in subsequent cooling process. In this fusion bonding process, it is important to heat and cool wafers with a rate of about 2 °C per minute to avoid breakage. However, original heat exchanger wafers developed a temporary bond but did not produce fusion bonding. This incomplete bonding is probably due to the
inadequate pressure on the wafer surface during the bonding process. The heat exchanger should consist of two wafers if straight channels are employed in the heat exchanger. Single wafer heat exchanger produces excessive channels sagging with high inlet refrigerant pressure. The maximum temperature within the system is about 230 K while the minimum temperature is 98 K. The maximum temperature occurs at the top middle section of the device. The temperature distribution within the device showed that vertical sidewall temperatures of the packaged device were relatively low. Because those low temperature walls are exposed to the ambient, heat from the surrounding can flow into the device. Therefore, complete insulation of the packaged unit will be necessary.

The positive outcome of this research was the understanding that JT devices could be developed based on layered arrangements. Packaging is very challenging but can be achieved with careful testing and development of procedures. The attempt made in this study narrowly missed completing a successful device. However, the knowledge developed in the process would be very useful in subsequent work.

9.2 Recommendations

Based on the findings of this research work, the following specific recommendations are made.

9.2.1 The simulation model should be modified to include the heat transfer between hot fluid and cold fluid that occurs through the vertical sidewalls.

9.2.2 The model should be improved to predict the transient characteristics of the JT device so that cool down time, effect of pressure and temperature fluctuations can be predicted.
9.2.3 The model developed in this study does not provide complete information about the temperature and flow variations in the channels of the refrigerator. Two-phase transition occurs in the capillary channels and it would be necessary to identify the portion of the capillary length that has two-phase flow. Therefore, a fluid flow analysis in the JT device using FLOTRAN or other computational fluid flow software is recommended.

9.2.4 In future work, silicon wafers with both-side polished wafers should be used for capillary and heat exchanger. The masking layer (SiO₂) thickness of 2.5 µm is recommended.

9.2.5 ICP etching should be used to fabricate all channels and holes. This will prevent wafers getting weakened due to KOH etching. In addition, ICP etching produces better quality products.

9.2.6 A new fixture for anodic bonding of several layers at the same time would be useful in future work. The fixture used in the present study provides the voltage tapping but the alligator clips used to connect silicon and glass wafers become weakened in extended period of heating resulting poor voltage supply to wafers during the critical bonding process.

9.2.7 The heat exchanger should be redesigned so that a single wafer can be used for the heat exchanger. When two wafers are required in the heat exchanger, they need to be fusion bonded. Fusion bonding requires very well controlled wafer cleaning, initial hydrogen bonding, and annealing. This time consuming process always does not produce successful bonding. It is also important to eliminate possible channel sagging in the heat exchanger due to high inlet pressure.
Therefore, an optimum length that can withstand inlet pressure should first be determined before deciding on the actual lay out of channels on the wafer.

9.2.8 The possibility of using thinner Pyrex 7740 glass wafers in bonding should be examined. In this work, 3 mm thick glass wafers were used because the holes could be drilled without breaking them. However, with thick glass wafers it was difficult to apply voltage from the sides when the device is getting thicker in the packaging process. Small alligator clips were used to apply voltage from sides and those clips could not open up enough to hold the glass wafer. Because a small oven was used in the bonding process, bigger alligator clips could not be used due to space limitations.
APPENDIX A

SIMULATION MODEL FOR SINGLE LAYER
MICROSCALE JOULE-THOMSON
DEVICE
APPENDIX A

AP.A.1 Unformatted Equations of the Simulation Model for Single Layer Microscale Joule-Thomson Device

"Required input:

1. Geometry of the Heat Exchanger
2. Geometry of the Capillary
3. Inlet pressure and Temperature
4. Outlet pressure (atmospheric pressure)"

"With these inputs, the model will determine all intermediate thermodynamic states, the quality of the refrigerant after expansion, and the cooling effect"

FUNCTION f(Re)
    IF (Re<=2300) THEN f:=64/Re
    IF (Re>2300) THEN f:=0.316*Re^(-0.25)
END

FUNCTION counterflow(NTU, C_r)
    IF (C_r = 1) THEN
        counterflow:= NTU/(1+NTU)
    ELSE
        counterflow:=(1-exp(-NTU*(1-C_r)))/((1-C_r)*exp(-NTU*(1-C_r)))
    ENDIF
END

HEAT EXCHANGER EQUATIONS

\[
\begin{align*}
    \text{m dot} &= \rho_1 V_1 \text{Convert(l/min,m3/s)} \\
    D_h &= (4d_1w_1)/(2(d_1+w_1)) \\
    D_c &= (4d_2w_2)/(2(d_2+w_2)) \\
    f_h &= f(Re_h) \\
    f_c &= f(Re_c) \\
    Re_h &= \frac{\rho_h V_h D_h}{\mu_h} \\
    Re_c &= \frac{\rho_c V_c D_c}{\mu_c} \\
    V_h &= \frac{m \text{ dot}}{(\rho_1 V_h D_h)} \\
    V_c &= \frac{m \text{ dot}}{(\rho_2 V_c D_c)} \\
    \rho_h &= (\rho_1 + \rho_2)/2 \\
    \rho_c &= (\rho_4 + \rho_5)/2 \\
    T_h &= (T_{1+T_2})/2 \\
    T_c &= (T_{4+T_5})/2 \\
    \mu_h &= \text{VISCOSITY(Nitrogen,T=T_h,P=P_1)} \\
    \mu_c &= \text{VISCOSITY(Nitrogen,T=T_c,P=P_5)} \\
    cp_h &= \text{SPECHEAT(Nitrogen,T=T_h,P=P_1)}
\end{align*}
\]

"Mass flow rate"
"Hydraulic diameter of hot fluid channel"
"Hydraulic diameter of cold fluid channel"
"Friction factor-hot fluid channel"
"Friction factor-cold fluid channel"
"Reynolds number-hot fluid channel"
"Reynolds number-cold fluid channel"
"Velocity of hot fluid"
"Velocity of cold fluid"
"Density of hot fluid"
"Density of cold fluid"
"Hot fluid average temperature"
"Cold fluid average temperature"
"Viscosity-hot fluid"
"Viscosity-cold fluid"
"Sp.heat-hot fluid"
cp_c = SPECHEAT(Nitrogen, T=T_c, P=P_5)  "Sp.heat-cold fluid"
Pr_h = (mu_h*cp_h*Convert(kJ, J))/k_h  "Prandtl number-hot fluid"
Pr_c = (mu_c*cp_c*Convert(kJ, J))/k_c  "Prandtl number-cold fluid"
k_h = CONDUCTIVITY(Nitrogen, T=T_h, P=P_1)  "Thermal conductivity-hot fluid"
k_c = CONDUCTIVITY(Nitrogen, T=T_c, P=P_5)  "Thermal conductivity-cold fluid"
Nus_h = 0.023*Re_h^0.8*Pr_h^0.333  "Nussell's number-hot fluid"
Nus_c = 0.023*Re_c^0.8*Pr_c^0.333  "Nussell's number-cold fluid"
Nus_h = (h_h*D_h)/k_h  "Nusselt's number-hot fluid"
Nus_c = (h_c*D_c)/k_c  "Nusselt's number-cold fluid"
1/U_h = 1/(h_h + (w_1*N_chh)/(h_c*w_2*N_chc)+t/k_w)  "Overall heat transfer coefficient"
NTU = (U_h*A_h)/(C_min*Convert(kW, W))  "Number of transfer Units"
1/NTU = 1/A_h*(C_min*C_max)  "Heat transfer area - hot fluid"
C_min = min(m_dot*cp_h, m_dot*cp_c)  "Capacity-minimum fluid"
C_max = max(m_dot*cp_h, m_dot*cp_c)  "Capacity-maximum fluid"
(P_1-P_2)*Convert(kPa, Pa)=rho_h*f_h*L/D_h*V_h^2/2  "Pressure drop-hot channel"
(P_4-P_5)*Convert(kPa, Pa)=rho_c*f_c*L/D_c*V_c^2/2  "Pressure drop-cold channel"
h_1=ENTHALPY(Nitrogen, T=T_1, P=P_1)  "Enthalpy-hot fluid, inlet"
h_2=ENTHALPY(Nitrogen, T=T_2, P=P_2)  "Enthalpy-hot fluid, outlet"
h_5=ENTHALPY(Nitrogen, T=T_5, P=P_5)  "Enthalpy-cold fluid, at the exit of cold fluid channel"
h_55=ENTHALPY(Nitrogen, T=T_5, P=P_5)  "Enthalpy of cold fluid corresponding to inlet temperature at outlet pressure"
rho_1=DENSITY(Nitrogen, T=T_1, P=P_1)  "Density of hot fluid-inlet"
rho_2=DENSITY(Nitrogen, T=T_2, P=P_2)  "Density of hot fluid-outlet"
rho_5=DENSITY(Nitrogen, T=T_5, P=P_5)  "Density of cold fluid-outlet"
epsilon = (h_5-h_4)/(h_55-h_4)  "Effectiveness of the heat exchanger"
C_r = C_min/C_max  "Capacity ratio"
Ma_h = V_h/(1.4*8.314*1000/28.0*T_h)^0.5  "Mach number-hot fluid"
Ma_c = V_c/(1.4*8.314*1000/28.0*T_c)^0.5  "Mach number-cold fluid"
Aspr_h=d_1/w_1  "Aspect ratio-hot channel"
Aspr_c=d_2/w_2  "Aspect ratio-cold channel"
(h_1-h_2)=(h_5-h_4)  "Heat balance in the Xchanger"
Q_xchg=m_dot*(h_1-h_2)  "Heat transfer in the heat exchanger"

CAPILLARY SECTION

Dd_cap=(4*d_cap*w_cap)/(2*(d_cap+w_cap))  "Hydraulic diameter-capillary"
T_cap=(T_2+T_4)/2  "Average capillary fluid temperature"
P_cap=(P_2+P_4)/2  "Average capillary fluid pressure"
u_cap=VISCOSITY(Nitrogen, T=T_cap, P=P_cap)  "Capillary fluid viscosity"
 rho_cap=DENSITY(Nitrogen, T=T_cap, P=P_cap)  "Capillary fluid density"
V_cap=m_dot/(rho_cap*w_cap*d_cap*N_cap)  "Capillary fluid velocity"
 rho_cap=rho_4cap  "Average density-capillary fluid"
Re_cap=(rho_cap*V_cap*Dd_cap)/(mu_cap)  "Reynolds number-capillary fluid"
f_cap=f(Re_cap)  "Friction factor-capillary fluid"
(P_2-P_4)*Convert(kPa, Pa)=rho_cap*f_cap*L_cap/Dd_cap*V_cap^2/2  "Pressure drop-capillary" (No pressure drop in the evaporator)
Ma_cap=V_cap/(1.4*8.314*1000/28.0*T_cap)^0.5  "Mach number-capillary fluid"
h_f=Enthalpy(Nitrogen, T=T_4, x=0)  "Entalpy-saturated fluid"
{xx=(h_3-h_f)/(h_4-h_f)}  "Enthalpy-saturated fluid"
EVAPORATOR

\[ h_3 = h_2 \]
\[ h_4 = \text{ENTHALPY}(\text{Nitrogen}, T = T_4, x = 1) \]
\[ Q_{dot} = m_{dot} \cdot (h_5 - h_1) \cdot \text{Convert}(\text{kJ/s, J/s}) \]
\[ P_4 = \text{PRESSURE}(\text{Nitrogen}, T = T_4, x = 1) \]
\[ \rho_4 = \text{DENSITY}(\text{Nitrogen}, T = T_4, x = \text{quality}_x) \]
\[ \text{quality}_x = \text{quality}(\text{Nitrogen}, T = T_4, h = h_2) \]

\{(\text{Entropy calculation})\}

\[ S[1] = \text{entropy}(\text{Nitrogen}, T = T_1, P = P_1) \]
\[ S[2] = \text{entropy}(\text{Nitrogen}, T = T_2, P = P_2) \]
\[ S[3] = \text{entropy}(\text{Nitrogen}, T = T_4, x = \text{quality}_x) \]
\[ S[4] = \text{entropy}(\text{Nitrogen}, T = T_4, x = 1) \]
\[ S[5] = \text{entropy}(\text{Nitrogen}, T = T_5, P = P_5) \]
\[ T[1] = T_1 \]
\[ T[2] = T_2 \]
\[ T[3] = T_4 \]
\[ T[4] = T_4 \]
\[ T[5] = T_5 \]

\{(\text{Input Parameters})\}

\[ w_1 = 50e-6 \ [\text{m}] \]
\[ w_2 = 300e-6 \]
\[ L = 0.08 \]
\[ P_1 = 10000 \]
\[ P_5 = 101.325 \]
\[ T_1 = 300 \]
\[ t = 500e-6 \]
\[ k_w = 264 \]
\[ N_{chh} = 1 \]
\[ N_{chc} = 2 \]
\[ \text{Aspr}_h = 3 \]
\[ \text{Aspr}_c = 0.5 \]

"Following data are for the Capillary tube"

\[ d_{cap} = 50e-6 \]
\[ w_{cap} = 25e-6 \]
\[ N_{cap} = 1 \]
\[ L_{cap} = 0.04 \]
\[ P_{in} = (P_1)/(P_{cr} - P_4) \]
\[ QQ_{dot} = Q_{dot}/(m_{dot} \cdot (h_1 - h_4)) \]
\[ T_4 = (T_4)/(T_{cr} - T_4) \]
\[ P_{cr} = P_{Crit}(\text{Nitrogen}) \]
\[ T_{cr} = T_{Crit}(\text{Nitrogen}) \]
\[ T_{T_4} = (T_5)/(T_{cr} - T_4) \]
AP.A.2 Formatted Equations of the Simulation Model for Single Layer Microscale JT Device

SIMULATION MODEL OF THE SINGLE LAYER MICROSCALE JT REFRIGERATOR

Required input:
1. Geometry of the Heat Exchanger
2. Geometry of the Capillary
3. Inlet pressure and Temperature
4. Outlet pressure (atmospheric pressure)

With these inputs, the model will determine all intermediate thermodynamic states, the quality of the refrigerant after expansion, and the cooling effect.

Function $f(Re)$

$$f = \begin{cases} \frac{64}{Re} & \text{if } Re \leq 2300 \\ 0.316 \cdot Re^{-0.25} & \text{if } Re > 2300 \end{cases}$$

End $f$

Function $\text{counterflow}(NTU, C_r)$

$$\text{counterflow} := \begin{cases} \frac{NTU}{1 + NTU} & \text{if } C_r = 1 \\ \frac{1 - \exp(-NTU \cdot (1 - C_r))}{1 - C_r \cdot \exp(-NTU \cdot (1 - C_r))} & \text{else} \end{cases}$$

End $\text{counterflow}$

HEAT EXCHANGER EQUATIONS

$$m = \rho_1 \cdot \nu \cdot \left[ 0.000166667 \cdot \frac{m^3/s}{\nu \text{min}} \right] \quad \text{Mass flow rate}$$

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$D_h = \frac{4 \cdot d_1 \cdot w_1}{2 \cdot (d_1 + w_1)}$  Hydraulic diameter of hot fluid channel

$D_c = \frac{4 \cdot d_2 \cdot w_2}{2 \cdot (d_2 + w_2)}$  Hydraulic diameter of cold fluid channel

$f_h = f(Re_h)$  Friction factor-hot fluid channel

$f_c = f(Re_c)$  Friction factor-cold fluid channel

$Re_h = \frac{\rho_h \cdot V_h \cdot D_h}{\mu_h}$  Reynolds number-hot fluid channel

$Re_c = \frac{\rho_c \cdot V_c \cdot D_c}{\mu_c}$  Reynolds number-cold fluid channel

$V_h = \frac{m}{\rho_h \cdot w_1 \cdot d_1 \cdot N_{chn}}$  Velocity of hot fluid

$V_c = \frac{m}{\rho_c \cdot w_2 \cdot d_2 \cdot N_{chc}}$  Velocity of cold fluid

$\rho_h = \frac{\rho_1 + \rho_2}{2}$  Density of hot fluid

$\rho_c = \frac{\rho_4 + \rho_5}{2}$  Density of cold fluid

$T_h = \frac{T_1 + T_2}{2}$  Hot fluid average temperature

$T_c = \frac{T_4 + T_5}{2}$  Cold fluid average temperature

$\mu_h = \text{Visc('Nitrogen', T=T_h, P=P_1)}$  Viscosity-hot fluid

$\mu_c = \text{Visc('Nitrogen', T=T_c, P=P_5)}$  Viscosity-cold fluid

$\text{cp}_h = \text{Cp('Nitrogen', T=T_h, P=P_1)}$  Sp. heat-hot fluid

$\text{cp}_c = \text{Cp('Nitrogen', T=T_c, P=P_5)}$  Sp. heat-cold fluid
\[
Pr_h = \frac{\mu_h \cdot c_{ph} \cdot 1000 \cdot \frac{J}{kJ}}{k_h} \quad \text{Prandtl number-hot fluid}
\]

\[
Pr_c = \frac{\mu_c \cdot c_{pc} \cdot 1000 \cdot \frac{J}{kJ}}{k_c} \quad \text{Prandtl number-cold fluid}
\]

\[
k_h = \kappa \ ('\text{Nitrogen}', T = T_h, P = P_h) \quad \text{Thermal conductivity-hot fluid}
\]

\[
k_c = \kappa \ ('\text{Nitrogen}', T = T_c, P = P_c) \quad \text{Thermal conductivity-cold fluid}
\]

\[
Nu_{sh} = 0.023 \cdot Re_h^{0.8} \cdot Pr_h^{0.333} \quad \text{Nusselt's number-hot fluid}
\]

\[
Nu_{sc} = 0.023 \cdot Re_c^{0.8} \cdot Pr_c^{0.333} \quad \text{Nusselt's number-cold fluid}
\]

\[
Nu_{sh} = \frac{h_h \cdot D_h}{k_h} \quad \text{Nusselt's number-hot fluid}
\]

\[
Nu_{sc} = \frac{h_c \cdot D_c}{k_c} \quad \text{Nusselt's number-cold fluid}
\]

\[
\frac{1}{U_h} = \frac{1}{h_h} + \frac{w_1 \cdot N_{abh}}{h_c \cdot w_2 \cdot N_{ach}} + \frac{1}{k_w} \quad \text{Overall heat transfer coefficient}
\]

\[
\varepsilon = \frac{h_5 - h_4}{h_{ss} - h_4} \quad \text{Heat exchanger effectiveness}
\]

\[
NTU = \frac{U_h \cdot A_h}{C_{min} \left[ 1000 \cdot \frac{W}{kW} \right]} \quad \text{Number of transfer Units}
\]

\[
A_h = 2 \cdot d_h \cdot L \cdot N_{chh} \quad \text{Heat transfer area-hot fluid}
\]

\[
C_{min} = \min (m \cdot c_{ph}, m \cdot c_{pc}) \quad \text{Capacity-minimum fluid}
\]

\[
C_{max} = \max (m \cdot c_{ph}, m \cdot c_{pc}) \quad \text{Capacity-maximum fluid}
\]

\[
(P_1 - P_2) \left[ 1000 \frac{\rho_g}{kPa} \right] = \rho_h \cdot f_h \cdot \frac{L}{D_h} \cdot \frac{V_h^2}{2} \quad \text{Pressure drop-hot channel}
\]
\[ (P_4 - P_5) \cdot \left[ 1000 \cdot \frac{P_a}{kP_a} \right] = \rho_c \cdot f_c \cdot \frac{L}{D_c} \cdot \frac{V_c^2}{2} \]  

Pressure drop-cold channel

\[ h_1 = h'(\text{Nitrogen}, T=T_1, P= P_1) \]  

Enthalpy-hot fluid, inlet

\[ h_2 = h'(\text{Nitrogen}, T=T_2, P= P_2) \]  

Enthalpy-hot fluid, outlet

\[ h_3 = h_2 \]  

Isenthalpic expansion

\[ h_4 = h'(\text{Nitrogen}, T=T_4, x= 1) \]  

Enthalpy-state 4(saturated gas)

\[ h_5 = h'(\text{Nitrogen}, T=T_5, P= P_5) \]  

Enthalpy-cold fluid, at the exit of cold fluid channel

\[ h_{55} = h'(\text{Nitrogen}, T=T_1, P= P_5) \]  

Enthalpy of cold fluid corresponding to inlet temperature at outlet pressure

\[ \rho_1 = \rho'(\text{Nitrogen}, T=T_1, P= P_1) \]  

Density of hot fluid-inlet

\[ \rho_2 = \rho'(\text{Nitrogen}, T=T_2, P= P_2) \]  

Density of hot fluid-outlet

\[ \rho_5 = \rho'(\text{Nitrogen}, T=T_5, P= P_5) \]  

Density of cold fluid-outlet

\[ \rho_4 = \rho'(\text{Nitrogen}, T=T_4, x= \text{quality}) \]  

Density of fluid at saturation point - Quality of the refrigerant at the exit of the evaporator is \( x=1 \)

\[ P_4 = P'(\text{Nitrogen}, T=T_4, x= 1) \]  

Determines the saturation pressure. (Therefore, \( x=1 \) is ok)

\[ \varepsilon = \text{counterflow (NTU, } C_r) \]  

Effectiveness of the heat exchanger

\[ C_r = \frac{C_{mn}}{C_{max}} \]  

Capacity ratio

\[ M_{ah} = \frac{V_h}{1.4 \cdot 8.314 \cdot \frac{1000}{28} \cdot T_h} \]  

Mach number-hot fluid

\[ M_{ac} = \frac{V_c}{1.4 \cdot 8.314 \cdot \frac{1000}{28} \cdot T_c} \]  

Mach number-cold fluid
Aspr\textsubscript{h} = \frac{d_1}{w_1} \quad \text{Aspect ratio-hot channel}

Aspr\textsubscript{c} = \frac{d_2}{w_2} \quad \text{Aspect ratio-cold channel}

h_1 - h_2 = h_5 - h_4 \quad \text{Heat balance in the Xchanger}

\text{CAPILLARY SECTION}

Dd\textsubscript{cap} = \frac{4 \cdot d\text{cap} \cdot w\text{cap}}{2 \cdot (d\text{cap} + w\text{cap})} \quad \text{Hydraulic diameter-capillary}

T\text{cap} = \frac{T_2 + T_4}{2} \quad \text{Average capillary fluid temperature}

P\text{cap} = \frac{P_2 + P_4}{2} \quad \text{Average capillary fluid pressure}

\mu\text{cap} = \text{Viscosity (Nitrogen, T=T\text{cap}, P=P\text{cap})} \quad \text{Capillary fluid viscosity}

\rho\text{cap} = \rho('Nitrogen', T=T\text{cap}, P=P\text{cap}) \quad \text{Capillary fluid density}

V\text{cap} = \frac{m}{\rho\text{cap} \cdot w\text{cap} \cdot d\text{cap} \cdot N\text{cap}} \quad \text{Capillary fluid velocity}

\rho\text{cap} = \rho_4\text{cap} \quad \text{Average density-capillary fluid}

Re\textsubscript{cap} = \frac{\rho\text{cap} \cdot V\text{cap} \cdot Dd\text{cap}}{\mu\text{cap}} \quad \text{Reynolds number-capillary fluid}

f\text{cap} = f(Re\text{cap}) \quad \text{Friction factor-capillary fluid}

(P_2 - P_4) \cdot \left[ 1000 \frac{P_a}{kPa} \right] = \rho\text{cap} \cdot f\text{cap} \cdot \frac{L\text{cap}}{Dd\text{cap}} \cdot \frac{V\text{cap}^2}{2} \quad \text{Pressure drop-capillary}
Mach number-capillary fluid

\[ Ma_{cap} = \frac{V_{cap}}{\left[1.4 \cdot 8.314 \cdot \frac{1000}{28} \cdot T_{cap}\right]^{0.5}} \]

\[ h_t = h\left(\text{Nitrogen}', T=T_4, x=0\right) \quad \text{Entropy-saturated fluid} \]

quality* = \( x \left(\text{Nitrogen}', T=T_4, h=h_2\right) \)

\[ Q = m \cdot (h_5 - h_1) \cdot \left[1000 \cdot \frac{J_3}{kJ/s}\right] \quad \text{Cooling capacity at T4} \]

\[ Q_{xchg} = m \cdot (h_1 - h_2) \]

S_1 = s\left(\text{Nitrogen}', T=T_1, P=P_1\right) \]

S_2 = s\left(\text{Nitrogen}', T=T_2, P=P_2\right) \]

S_3 = s\left(\text{Nitrogen}', T=T_4, x=\text{quality*}\right) \]

S_4 = s\left(\text{Nitrogen}', T=T_4, x=1\right) \]

S_5 = s\left(\text{Nitrogen}', T=T_5, P=P_5\right) \]

\[ T_1 = T_1 \]

\[ T_2 = T_2 \]

\[ T_3 = T_4 \]

\[ T_4 = T_4 \]

\[ T_5 = T_5 \]

\[ \mu_1 = 0.00005 \, [m] \]

L = 0.08 \]

P_5 = 101.325 \]

T_1 = 300 \]

t = 0.0005 \]

k_w = 264 \]

N_chh = 1 \]

N_chc = 2 \]

Aspr_h = 3 \]

Aspr_c = 0.5
Following data are for the Capillary tube

d_{cap} = 0.00005
w_{cap} = 0.000025
N_{cap} = 1
L_{cap} = 0.04
vol = 1.2

\rho_{ho} = \rho('Nitrogen', T=273, P=101)

\nu_{mm} = \frac{1}{0.0000166667} \left(\frac{\text{m}^3}{\text{s}} \right)

\nu_{w} = \frac{m}{\rho_{ho}} \left(\frac{60000 \ \text{\nu_{min}}}{\text{m}^3/\text{s}} \right)

P_{in} = \frac{P_1}{P_{cr} - P_4}

Q_Q = \frac{\dot{Q}}{m \cdot (h_1 - h_4)}

T_T = \frac{T_4}{T_{cr} - T_4}

P_{cr} = P_{\text{Crit ('Nitrogen')}}

T_{cr} = T_{\text{Crit ('Nitrogen')}}

T_T = \frac{T_5}{T_{cr} - T_4}

NOMENCLATURE

m = total mass flow rate
w_1 = hot fluid channel width
w_2 = cold fluid channel width
L = length of a channel
P_1 = inlet gas pressure
P_5 = outlet gas pressure
\( T_1 \) = inlet gas temperature  
\( t \) = thickness of the separating plate  
\( k_v \) = thermal conductivity of the separating plate  
\( T_4 \) = saturation temperature  
\( N_{c h b} \) = No. of hot fluid channels  
\( N_{c h o} \) = No. of cold fluid channel  
\( A_{s p h} \) = aspect ratio of hot fluid channel  
\( d_{c a p} \) = depth of capillary tube  
\( w_{c a p} \) = width of capillary tube  
\( N_{c a p} \) = No. of capillary channels  

(not in alphabetical order)

**AP.A.3 Solution Window of Simulated Single Layer Microscale JT Device**

(Unit Symbols: [kW/K], [kg/(m²·s)], [kJ/kg], [W/(m²·K)], [W/(m²·K)]

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A_{s p h} )</td>
<td>0.85</td>
</tr>
<tr>
<td>( C_{m} )</td>
<td>0.000002773 [kW/K]</td>
</tr>
<tr>
<td>( d_{c} )</td>
<td>0.00469</td>
</tr>
<tr>
<td>( h_{1} )</td>
<td>217.5 [kJ/kg]</td>
</tr>
<tr>
<td>( h_{2} )</td>
<td>365.5 [kJ/kg]</td>
</tr>
<tr>
<td>( L )</td>
<td>0.008 [m]</td>
</tr>
<tr>
<td>( m )</td>
<td>0.00000215</td>
</tr>
<tr>
<td>( N_{c h b} )</td>
<td>0.000002124</td>
</tr>
<tr>
<td>( N_{c h o} )</td>
<td>0.00000214</td>
</tr>
<tr>
<td>( N_{c a p} )</td>
<td>0.00000547</td>
</tr>
<tr>
<td>( N_{u} )</td>
<td>0.000005647</td>
</tr>
<tr>
<td>( N_{u} )</td>
<td>0.000005647</td>
</tr>
<tr>
<td>( \rho_{p} )</td>
<td>101.3 [kPa]</td>
</tr>
<tr>
<td>( \rho_{g} )</td>
<td>101.3 [kPa]</td>
</tr>
<tr>
<td>( \rho_{w} )</td>
<td>1.247</td>
</tr>
<tr>
<td>( \rho_{v} )</td>
<td>283.2 [kg/m³]</td>
</tr>
<tr>
<td>( \rho_{w} )</td>
<td>0.00000547</td>
</tr>
<tr>
<td>( \nu_{t} )</td>
<td>0.00000547</td>
</tr>
<tr>
<td>( \nu_{w} )</td>
<td>0.00000547</td>
</tr>
</tbody>
</table>

Array variables are in the Arrays and follow

Table: A1.1 Simulated Performance Parameters of the Single Layer JT Device

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Table A1.1  Simulated Performance Parameters of the Single Layer JT Device

<table>
<thead>
<tr>
<th>Inlet Pressure (p_i/p_{cr-p_d})</th>
<th>Refrigeration Capacity (Q/m(h_{cr-h_d}))</th>
<th>Evaporator Temperature (T_y/(T_{cr-T_d}))</th>
<th>Exit Temperature (T_y/(T_{cr-T_d}))</th>
<th>Quality of the Refrigerant</th>
<th>Heat Exchanger Effectiveness</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.748</td>
<td>84.56</td>
<td>1.937</td>
<td>6.828</td>
<td>0.9071</td>
<td>0.9705</td>
</tr>
<tr>
<td>3.589</td>
<td>78.91</td>
<td>1.929</td>
<td>6.803</td>
<td>0.9130</td>
<td>0.9694</td>
</tr>
<tr>
<td>3.431</td>
<td>73.10</td>
<td>1.920</td>
<td>6.777</td>
<td>0.9192</td>
<td>0.9682</td>
</tr>
<tr>
<td>3.272</td>
<td>67.13</td>
<td>1.912</td>
<td>6.751</td>
<td>0.9255</td>
<td>0.9668</td>
</tr>
<tr>
<td>3.114</td>
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<td>1.903</td>
<td>6.723</td>
<td>0.9321</td>
<td>0.9654</td>
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<td>1.894</td>
<td>6.695</td>
<td>0.9388</td>
<td>0.9639</td>
</tr>
<tr>
<td>2.798</td>
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<td>1.885</td>
<td>6.666</td>
<td>0.9457</td>
<td>0.9623</td>
</tr>
<tr>
<td>2.641</td>
<td>41.97</td>
<td>1.876</td>
<td>6.636</td>
<td>0.9528</td>
<td>0.9607</td>
</tr>
<tr>
<td>2.483</td>
<td>35.35</td>
<td>1.866</td>
<td>6.604</td>
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<td>0.9589</td>
</tr>
<tr>
<td>2.326</td>
<td>28.57</td>
<td>1.856</td>
<td>6.571</td>
<td>0.9676</td>
<td>0.9570</td>
</tr>
<tr>
<td>2.169</td>
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<td>1.845</td>
<td>6.536</td>
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<td>0.9549</td>
</tr>
<tr>
<td>2.012</td>
<td>14.56</td>
<td>1.834</td>
<td>6.499</td>
<td>0.9834</td>
<td>0.9527</td>
</tr>
<tr>
<td>1.856</td>
<td>7.33</td>
<td>1.822</td>
<td>6.460</td>
<td>0.9916</td>
<td>0.9504</td>
</tr>
</tbody>
</table>

Table A1.2  Effect of Change of Aspect ratio of Hot Fluid Channel

<table>
<thead>
<tr>
<th>Aspect ratio of hot fluid channel</th>
<th>Corresponding aspect ratio of cold fluid channels</th>
<th>Mass flow rate (kg/s) x 10^5</th>
<th>Refrigeration capacity (Q/m(h_{cr-h_d}))</th>
<th>Heat exchanger effectiveness</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.3334</td>
<td>2.08</td>
<td>82.62</td>
<td>0.9679</td>
</tr>
<tr>
<td>3</td>
<td>0.5000</td>
<td>2.20</td>
<td>84.56</td>
<td>0.9705</td>
</tr>
<tr>
<td>4</td>
<td>0.6667</td>
<td>2.25</td>
<td>86.53</td>
<td>0.9728</td>
</tr>
<tr>
<td>5</td>
<td>0.8333</td>
<td>2.28</td>
<td>88.31</td>
<td>0.9747</td>
</tr>
<tr>
<td>6</td>
<td>1.0000</td>
<td>2.30</td>
<td>89.87</td>
<td>0.9763</td>
</tr>
<tr>
<td>7</td>
<td>1.1670</td>
<td>2.31</td>
<td>91.24</td>
<td>0.9776</td>
</tr>
<tr>
<td>8</td>
<td>1.3330</td>
<td>2.32</td>
<td>92.45</td>
<td>0.9788</td>
</tr>
<tr>
<td>9</td>
<td>1.5000</td>
<td>2.32</td>
<td>93.54</td>
<td>0.9799</td>
</tr>
<tr>
<td>10</td>
<td>1.6667</td>
<td>2.33</td>
<td>94.51</td>
<td>0.9808</td>
</tr>
<tr>
<td>11</td>
<td>1.8333</td>
<td>2.33</td>
<td>95.31</td>
<td>0.9816</td>
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<tr>
<td>12</td>
<td>2.0000</td>
<td>2.33</td>
<td>96.12</td>
<td>0.9823</td>
</tr>
<tr>
<td>13</td>
<td>1.1670</td>
<td>2.34</td>
<td>96.86</td>
<td>0.9830</td>
</tr>
</tbody>
</table>
APPENDIX B

SIMULATION MODEL OF THE PROPOSED MICROSCALE JOULE-THOMSON REFRIGERATOR
APPENDIX B

AP.B.1 Unformatted Equations

"Required input:
1. Geometry of the Heat Exchanger
2. Geometry of the Capillary
3. Inlet pressure and Temperature
4. Outlet pressure (atmospheric pressure)"

"With these inputs, the model will determine all intermediate thermodynamic states, the quality of the refrigerant after expansion, and the cooling effect"

{Functions in the Program}

FUNCTION f(Re)
    IF (Re<=2300) THEN f:=64/Re
    IF (Re>2300) THEN f:=0.316*Re^(-0.25)
END

FUNCTION counterflow(NTU,C_r)
    IF (C_r = 1) THEN
        counterflow:= NTU/(1+NTU)
    ELSE
        counterflow:=(1-exp(-NTU*(1-C_r)))/(1-C_r*exp(-NTU*(1-C_r)))
    ENDIF
END

"HEAT EXCHANGER EQUATIONS"

m_dot=rho_1*V_dot*Convert(l/min,m3/s)
D_h=(4*d_1*w_1)/(2*(d_1+w_1))
D_c=(4*d_2*w_2)/(2*(d_2+w_2))
f_h=f(Re_h)
f_c=f(Re_c)
Re_h=(rho_h*V_h*D_h)/mu_h
Re_c=(rho_c*V_c*D_c)/mu_c
V_h=m_dot/(rho_h*w_1*d_1*N_chh)
V_c=m_dot/(rho_c*w_2*d_2*N_chc)
rho_h=(rho_1+rho_2)/2

"Mass flow rate"
"Hydraulic diameter of hot fluid channel"
"Hydraulic diameter of cold fluid channel"
"Friction factor-hot fluid channel"
"Friction factor-cold fluid channel"
"Reynolds number-hot fluid channel"
"Reynolds number -cold fluid channel"
"Velocity of hot fluid"
"Velocity of cold fluid"
"Density of hot fluid"
\[
\rho_c = (\rho_4 + \rho_5) / 2
\]

\[
T_h = (T_1 + T_2) / 2
\]

\[
T_c = (T_4 + T_5) / 2
\]

\[
u_h = \text{VISCOSITY(Nitrogen, T=T_h, P=P_1)}
\]

\[
u_c = \text{VISCOSITY(Nitrogen, T=T_c, P=P_5)}
\]

\[
C_{ph} = \text{SPECHEAT(Nitrogen, T=T_h, P=P_1)}
\]

\[
C_{pc} = \text{SPECHEAT(Nitrogen, T=T_c, P=P_5)}
\]

\[
Pr_h = \frac{(nu_h * C_{ph} * \text{Convert(kJ,J) / k_h})}{k_h}
\]

\[
Pr_c = \frac{(nu_c * C_{pc} * \text{Convert(kJ,J) / k_c})}{k_c}
\]

\[
\rho_1 = \text{DENSITY(Nitrogen, T=T_1, P=P_1)}
\]

\[
\rho_2 = \text{DENSITY(Nitrogen, T=T_2, P=P_2)}
\]

\[
\rho_4 = \text{DENSITY(Nitrogen, T=T_4, x=quality_x)}
\]

\[
P_4 = \text{PRESSURE(Nitrogen, T=T_4, x=1)}
\]

\[
\rho_5 = \text{DENSITY(Nitrogen, T=T_5, P=P_5)}
\]

\[
\epsilon = \text{counterflow(NTU, C_r)}
\]

\[
\text{Re_h} = \frac{V_h}{1.4 \times 8.314 \times 1000 / 28.0 \times T_h} \times 0.5
\]

\[
\text{Re_c} = \frac{V_c}{1.4 \times 8.314 \times 1000 / 28.0 \times T_c} \times 0.5
\]

\[
\text{Ma_h} = \frac{V_h}{1.4 \times 8.314 \times 1000 / 28.0 \times T_h} \times 0.5
\]

\[
\text{Ma_c} = \frac{V_c}{1.4 \times 8.314 \times 1000 / 28.0 \times T_c} \times 0.5
\]

\[
\text{Aspr_h} = \frac{d_1}{w_1}
\]

\[
\text{Aspr_c} = \frac{d_2}{w_2}
\]

\[
\epsilon = \text{counterflow}(NTU, C_r)
\]

\[
\text{C_r} = \frac{C_{min}}{C_{max}}
\]

\[
\text{Mach number-hot fluid}
\]

\[
\text{Mach number-cold fluid}
\]

\[
\text{Aspect ratio-hot channel}
\]

\[
\text{Aspect ratio-cold channel}
\]

\[
\text{Heat balance in the Xchanger}
\]

\[
\text{CAPILLARY SECTION}
\]

\[
D_{d, cap} = \left(4 * d_{cap} * w_{cap} / (2 * (d_{cap} + w_{cap})) \right)
\]

\[
T_{cap} = (T_2 + T_4) / 2
\]

\[
P_{cap} = (P_2 + P_4) / 2
\]

\[
\mu_{cap} = \text{VISCOSITY(Nitrogen, T=T_{cap}, P=P_{cap})}
\]

\[
\rho_{cap} = \text{DENSITY(Nitrogen, T=T_{cap}, P=P_{cap})}
\]

\[
\epsilon = \text{counterflow}(NTU, C_r)
\]

\[
\text{C_r} = \frac{C_{min}}{C_{max}}
\]

\[
\text{Mach number-hot fluid}
\]

\[
\text{Mach number-cold fluid}
\]

\[
\text{Aspect ratio-hot channel}
\]

\[
\text{Aspect ratio-cold channel}
\]

\[
\text{Heat balance in the Xchanger}
\]

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$V_{\text{cap}} = \frac{m_{\text{dot}}}{(i + \frac{io}{\omega \tau \delta \zeta \eta})}$  "Capillary fluid velocity"

$\rho_{\text{cap}} = \frac{it_{io}}{\tau_{\omega \delta \zeta \eta}}$  "Average density-capillary fluid"

$Re_{\text{cap}} = \frac{(\rho_{\text{cap}}V_{\text{cap}}D_{d_{\text{cap}}})}{(\mu_{\text{cap}})}$  "Reynolds number-capillary fluid"

$f_{\text{cap}} = \frac{(Re_{\text{cap}})}{(P_{2} - P_{4})}$  "Friction factor-capillary fluid"

$(P_{2} - P_{4})^{\text{Convert(kPa,Pa)}} = \frac{\rho_{\text{cap}}f_{\text{cap}}L_{\text{cap}}D_{d_{\text{cap}}}}{V_{\text{cap}}A_{2}}$  "Pressure drop-capillary"

$Ma_{\text{cap}} = \frac{V_{\text{cap}}}{(1.4*8.314*1000/28.0*T_{\text{cap}})^{0.5}}$  "Mach number-capillary fluid"

$h_{x} = \text{Enthalpy(Nitrogen,T=T_{4},x=0)}$  "Entalpy-saturated fluid"

$\{xx=(h_{3-h_{f}})(h_{4-h_{f}})\}$

$quality_{x} = \text{quality(Nitrogen,T=T_{4},h=h_{2})}$

$Q_{\text{dot}} = m_{\text{dot}}(h_{5-h_{1}})$ Convert(kJ/s, J/s)  "Cooling capacity at T4"

$Q_{\text{xchg}} = m_{\text{dot}}(h_{1} - h_{2})$  "Cooling capacity at T4"

\begin{align*}
\text{Entropy calculation} \nonumber \\
S[1] = & \text{entropy(Nitrogen,T=T_{1},P=P_{1})} \\
S[2] = & \text{entropy(Nitrogen,T=T_{2},P=P_{2})} \\
S[3] = & \text{entropy(Nitrogen,T=T_{3},x=quality_{x})} \\
S[4] = & \text{entropy(Nitrogen,T=T_{4},x=1)} \\
S[5] = & \text{entropy(Nitrogen,T=T_{5},P=P_{5})} \\
\end{align*}

$T[1] = T_{1}$

$T[2] = T_{2}$

$T[3] = T_{4}$

$T[4] = T_{4}$

$T[5] = T_{5}$

\begin{align*}
\text{Input Parameters} \nonumber \\
w_{1} = & 50e^-6 \\
w_{2} = & 50e^-6 \\
L = & 0.06 \\
P_{1} = & 6000 \\
P_{5} = & 101.325 \\
T_{1} = & 300 \\
t = & 860e^-6 \\
k_{w} = & 264 \\
N_{chh} = & 100 \\
N_{chc} = & 100 \\
Aspr_{h} = & 0.4 \\
Aspr_{c} = & 0.4 \\
\end{align*}

"Following data are for the Capillary tube"

$d_{\text{cap}} = 20e^-6$

$w_{\text{cap}} = 20e^-6$

$N_{\text{cap}} = 30$

$L_{\text{cap}} = 0.06$
Results obtained for no pressure drop assumption in the evaporator.

\[
\begin{align*}
T_4 &= 99.3 \\
P_4 &= 235 \\
m_{dot} &= 0.00003459 \\
d_{evap} &= 20e-6 \\
w_{evap} &= 30e-3
\end{align*}
\]

Related equations for calculation of pressure drop in the evaporator. These equations were primarily obtained from "Cryogenic Systems" by Randall F. Barron. See the reference list.

\[
\begin{align*}
C_L &= 64 \\
C_G &= 64 \\
m &= 1 \\
n &= 1 \\
C &= 5 \\
DD_{evap} &= 4*d_{evap}w_{evap}/(2*(d_{evap}+w_{evap})) \\
X^2 &= (C_L*(Re_G)^{(m)*rho_G})*(C_G*(Re_L)^{(n)*rho_L})^((1-xx)/xx)^2 \\
 rho_G &= \text{density(Nitrogen, T = T_4, x = 1)} \\
 rho_L &= \text{density(Nitrogen, T = T_4, x = 0)} \\
Re_G &= \text{rho G}*V_G*DD_{evap}/mu_{evap G} \\
Re_L &= \text{rho L}*V_L*DD_{evap}/mu_{evap L} \\
V_G &= m_{dot}*xx/(rho_G*d_{evap}*w_{evap}) \\
V_L &= m_{dot}*(1-xx)/(rho_L*d_{evap}*w_{evap}) \\
mu_{evap G} &= \text{viscosity(Nitrogen, T = T_4, P = P_4)} \\
mu_{evap L} &= 119e-6 \\
vol &= \text{volume(Nitrogen, T = T_4, x = 0)} \\
phi_L &= ((X^2+c*X+1)^{0.5})/X \\
&\text{"Pressure drop per unit length if the liquid flowing along the length of the evaporator at the total mass flow rate (m_{dot})"} \\
DelP &= \text{pressure drop (kPa, Pa) = (rho_L*f_{evap}/DD_{evap})^*V_L^2/2} \\
f_{evap} &= f(Re_L) \\
L_{evap} &= 30e-3 \\
V_{evap} &= (m_{dot}/(rho_L))^1/((d_{evap}*w_{evap})) \\
Re_{evap} &= \text{rho evap*L_{evap}*V_{evap}/mu_{evap}} \\
PP &= (1-xx)^{(2-n)}*(phi_L)^2 \\
Delp_f &= \text{INTEGRAL(PP, xx, 0.9082, 0.99)} \\
\end{align*}
\]
{NOMENCLATURE}

(not in alphabetical order)

\( m_{\text{dot}} \) = total mass flow rate
\( w_1 \) = hot fluid channel width
\( w_2 \) = cold fluid channel width
\( L \) = length of a channel
\( P_1 \) = inlet gas pressure
\( P_5 \) = outlet gas pressure
\( T_1 \) = inlet gas temperature
\( t \) = thickness of the separating plate
\( k_w \) = thermal conductivity of the separating plate
\( T_4 \) = saturation temperature
\( N_{\text{chh}} \) = No. of hot fluid channels
\( N_{\text{chc}} \) = No. of cold fluid channel
\( \text{Aspr}_h \) = aspect ratio of hot fluid channel
\( \text{Aspr}_c \) = aspect ratio of cold fluid channel
\( d_{\text{cap}} \) = depth of capillary tube
\( w_{\text{cap}} \) = width of capillary tube
\( N_{\text{cap}} \) = No. of capillary channel
SIMULATION MODEL OF THE PROPOSED MICROSCALE JT REFRIGERATOR

Required input:

1. Geometry of the Heat Exchanger
2. Geometry of the Capillary
3. Inlet pressure and Temperature
4. Outlet pressure (atmospheric pressure)

With these inputs, the model will determine all intermediate thermodynamic states, the quality of the refrigerant after expansion, and the cooling effect.

Function $f(Re)$

If $(Re \leq 2300)$ Then

$$f := \frac{64}{Re}$$

If $(Re > 2300)$ Then

$$f := 0.316 \cdot Re^{-0.25}$$

End $f$

Function $\text{counterflow}(NTU, C_r)$

If $(C_r = 1)$ Then

$$\text{counterflow} := \frac{NTU}{1 + NTU}$$

 Else

$$\text{counterflow} := \frac{1 - \exp(\text{-NTU} \cdot (1 - C_r))}{1 - C_r \cdot \exp(\text{-NTU} \cdot (1 - C_r))}$$

EndIf

End $\text{counterflow}$

$$\dot{m} = \rho_1 \cdot \dot{V} \cdot \left[ \frac{0.0001666667}{\dot{V}_{\min}} \right]$$

$$D_h = \frac{4 \cdot d_1 \cdot w_1}{2 \cdot (d_1 + w_1)}$$

$$D_c = \frac{4 \cdot d_2 \cdot w_2}{2 \cdot (d_2 + w_2)}$$
\[ f_h = f(Re_h) \]
\[ f_c = f(Re_c) \]
\[ Re_h = \frac{\rho_h \cdot V_h \cdot D_h}{\mu_h} \]
\[ Re_c = \frac{\rho_c \cdot V_c \cdot D_c}{\mu_c} \]
\[ V_h = \frac{\dot{m}}{\rho_h \cdot w_1 \cdot d_1 \cdot N_{chh}} \]
\[ V_c = \frac{\dot{m}}{\rho_c \cdot w_2 \cdot d_2 \cdot N_{chc}} \]
\[ \rho_h = \frac{\rho_1 + \rho_2}{2} \]
\[ \rho_c = \frac{\rho_4 + \rho_5}{2} \]
\[ T_h = \frac{T_1 + T_2}{2} \]
\[ T_c = \frac{T_4 + T_5}{2} \]

\[ \omega_h = \text{Visc('Nitrogen', } T=T_h, P=P_1) \]
\[ \omega_c = \text{Visc('Nitrogen', } T=T_c, P=P_5) \]
\[ c_{p_h} = \text{Cp('Nitrogen', } T=T_h, P=P_1) \]
\[ c_{p_c} = \text{Cp('Nitrogen', } T=T_c, P=P_5) \]
\[ Pr_h = \frac{\mu_h \cdot c_{p_h} \cdot \left[ \frac{1000 \cdot J}{kJ} \right]}{k_h} \]
\[ P_{rc} = \frac{\mu_c \cdot c_{pc} \cdot \left( 1000 \cdot \frac{J}{kJ} \right)}{k_c} \]

\[ k_h = k('\text{Nitrogen}', T=T_h, P=P_1) \]
\[ k_c = k('\text{Nitrogen}', T=T_c, P=P_5) \]

\[ Nush = 0.023 \cdot Re_h^{0.8} \cdot Pr_h^{0.333} \]
\[ Nusc = 0.023 \cdot Re_c^{0.8} \cdot Pr_c^{0.333} \]

\[ Nush = \frac{h_h \cdot D_h}{k_h} \]
\[ Nusc = \frac{h_c \cdot D_c}{k_c} \]

\[ \frac{1}{U_h} = \frac{1}{h_h} + \frac{w_1 \cdot N_{chh}}{h_c \cdot w_2 \cdot N_{chc}} + \frac{t}{k_w} \]

\[ e = \frac{h_5 - h_4}{h_{55} - h_4} \]

\[ NTU = \frac{U_h \cdot A_h}{C_{\text{min}} \cdot \left( 1000 \cdot \frac{W}{kW} \right)} \]

\[ A_h = w_1 \cdot L \cdot N_{chh} \]
\[ C_{\text{min}} = \text{Min}(\dot{m} \cdot c_{ph}, \dot{m} \cdot c_{pc}) \]
\[ C_{\text{max}} = \text{Max}(\dot{m} \cdot c_{ph}, \dot{m} \cdot c_{pc}) \]

\[ (P_1 - P_2) \cdot \left[ 1000 \cdot \frac{P_a}{kP_a} \right] = \rho_n \cdot f_n \cdot \frac{L}{D_h} \cdot \frac{V_h^2}{2} \]

\[ (P_4 - P_5) \cdot \left[ 1000 \cdot \frac{P_a}{kP_a} \right] = \rho_c \cdot f_c \cdot \frac{L}{D_c} \cdot \frac{V_c^2}{2} \]
\[
\begin{align*}
    h_1 &= h('Nitrogen', T=T_1, P=P_1) \\
    h_2 &= h('Nitrogen', T=T_2, P=P_2) \\
    h_3 &= h_2 \\
    h_4 &= h('Nitrogen', T=T_4, x=1) \\
    h_5 &= h('Nitrogen', T=T_5, P=P_5) \\
    h_{55} &= h('Nitrogen', T=T_1, P=P_5) \\
    \rho_1 &= \rho('Nitrogen', T=T_1, P=P_1) \\
    \rho_2 &= \rho('Nitrogen', T=T_2, P=P_2) \\
    \rho_5 &= \rho('Nitrogen', T=T_5, P=P_5) \\
    \rho_4 &= \rho('Nitrogen', T=T_4, x=1) \\
    P_4 &= P('Nitrogen', T=T_4, x=1) \\
    \varepsilon &= \text{counterflow}(NTU, C_r) \\
    C_r &= \frac{C_{\text{min}}}{C_{\text{max}}} \\
    M_{a_h} &= \frac{V_h}{\left[1.4 \cdot 8.314 \cdot \frac{1000}{26} \cdot T_h \right]^{0.5}} \\
    M_{a_c} &= \frac{V_c}{\left[1.4 \cdot 8.314 \cdot \frac{1000}{26} \cdot T_c \right]^{0.5}} \\
    \text{Aspr}_h &= \frac{d_1}{w_1} \\
    \text{Aspr}_c &= \frac{d_2}{w_2} \\
    h_1 - h_2 &= h_5 - h_4
\end{align*}
\]
Capillary section

\[ D_{dcap} = \frac{4 \cdot d_{cap} \cdot w_{cap}}{2 \cdot (d_{cap} + w_{cap})} \]

\[ T_{cap} = \frac{T_2 + T_4}{2} \]

\[ P_{cap} = \frac{P_2 + P_4}{2} \]

\[ \mu_{cap} = \psi \text{visc}(\text{Nitrogen}, T=T_{cap}, P=P_{cap}) \]

\[ \rho_{4cap} = \rho \left[ \text{Nitrogen}, T=T_{cap}, x=\frac{1 + xx}{2} \right] \]

\[ V_{cap} = \frac{\dot{m}}{\rho_{4cap} \cdot w_{cap} \cdot d_{cap} \cdot N_{cap}} \]

\[ \rho_{cap} = \rho_{4cap} \]

\[ Re_{cap} = \frac{\rho_{cap} \cdot V_{cap} \cdot D_{dcap}}{\mu_{cap}} \]

\[ f_{cap} = f(Re_{cap}) \]

\[ (P_2 - P_4) \cdot \left[ 1000 \cdot \frac{\rho_\gamma}{kPa} \right] = \rho_{cap} \cdot f_{cap} \cdot \frac{L_{cap}}{D_{dcap}} \cdot \frac{V_{cap}^2}{2} \]

\[ Ma_{cap} = \frac{V_{cap}}{\left[ 1.4 \cdot 8.314 \cdot \frac{1000}{28} \cdot T_{cap} \right]^{0.5}} \]

\[ h_\gamma = h(\text{Nitrogen}, T=T_4, x=0) \]

\[ xx = \frac{h_3 - h_\gamma}{h_4 - h_\gamma} \]

\[ \dot{Q} = \dot{m} \cdot (h_5 - h_1) \cdot \left[ 1000 \cdot \frac{J/s}{kJ/s} \right] \]

\[ Q_{xchg} = \dot{m} \cdot (h_1 - h_2) \]

\[ \dot{Q}_1 = \dot{Q} \]

\[ s_1 = s(\text{Nitrogen}, T=T_1, P=P_1) \]

\[ s_2 = s(\text{Nitrogen}, T=T_2, P=P_2) \]

\[ s_3 = s(\text{Nitrogen}, T=T_4, x=xx) \]
\[ S_4 = s4 \]
\[ S_5 = s5 \]
\[ SS_j = 1 \cdot S_j \text{ for } j = 1 \text{ to } 5 \]
\[ T_1 = T_1 \]
\[ T_2 = T_2 \]
\[ T_3 = T_4 \]
\[ T_4 = T_4 \]
\[ T_5 = T_5 \]
\[ TT_j = 1 \cdot T_j \text{ for } j = 1 \text{ to } 5 \]
\[ w_1 = 0.00005 \]
\[ w_2 = 0.00005 \]
\[ L = 0.06 \]
\[ P_1 = 6000 \]
\[ P_5 = 101.325 \]
\[ T_1 = 300 \]
\[ t = 0.00086 \]
\[ k_w = 264 \]
\[ N_{chh} = 100 \]
\[ N_{chc} = 100 \]
\[ Aspr_{nh} = 0.4 \]
\[ Aspr_{nc} = 0.4 \]

Following data are for the Capillary tube
\[ d_{cap} = 0.00002 \]
\[ w_{cap} = 0.00002 \]
\[ N_{cap} = 30 \]
Evaporator

\[ T_4 = 99.3 \]

\[ P_4 = 741.7 \]

\[ \dot{m} = 0.00003459 \]

\[ d_{\text{evap}} = 0.00002 \]

\[ w_{\text{evap}} = 0.03 \]

\[ C_L = 64 \]

\[ C_G = 64 \]

\[ m = 1 \]

\[ n = 1 \]

\[ C = 5 \]

\[ D_{\text{devap}} = 4 \cdot d_{\text{evap}} \cdot \frac{w_{\text{evap}}}{2 \cdot (d_{\text{evap}} + w_{\text{evap}})} \]

\[ x^2 = \frac{C_L \cdot R_{G m} \cdot \rho_G \cdot \left( \frac{1 - xx}{xx} \right)^2}{C_G \cdot R_{G n} \cdot \rho_L} \]

\[ \rho_G = \rho(\text{'Nitrogen'}, T=T_4, x=1) \]

\[ \rho_L = \rho(\text{'Nitrogen'}, T=T_4, x=0) \]

\[ R_{G} = \rho_G \cdot V_G \cdot \frac{D_{\text{devap}}}{\mu_{\text{evap}, G}} \]

\[ R_{L} = \rho_L \cdot V_L \cdot \frac{D_{\text{devap}}}{\mu_{\text{evap}, L}} \]

\[ V_G = \frac{\dot{m} \cdot xx}{\rho_G \cdot d_{\text{evap}} \cdot w_{\text{evap}}} \]

\[ V_L = \frac{\dot{m} \cdot \left( \frac{1 - xx}{\rho_L \cdot d_{\text{evap}} \cdot w_{\text{evap}}} \right)}{\mu_{\text{evap}, G} = Visc(\text{'Nitrogen'}, T=T_4, P=P_4)} \]
\[ k_{\text{evap},L} = 0.000119 \]

\[ \nu_l = \psi(\text{Nitrogen}', T=T_d, x=0) \]

\[ \phi_l = \frac{(X^2 + C \cdot X + 1)^{0.5}}{X} \]

Pressure drop per unit length if the liquid flowing along the length of the evaporator at the total mass flow rate \( \dot{m} \)

\[ \Delta P_l = \left[ 1000 \cdot \frac{\rho_l}{kPa} \right] = \rho_l \cdot \frac{f_{\text{evap}}}{D_{D_{\text{evap}}}} \cdot \frac{V_L}{2} \]

\[ f_{\text{evap}} = f(Re_L) \]

\[ L_{\text{evap}} = 0.03 \]

\[ PP = ((1 - xx)(2 - n)) \cdot \phi_l^2 \]

\[ \Delta P_l = \int_{0.99}^{0.9933} (PP) \, dxx \]
### AP.B.3 Solution Window of Proposed Microscale Joule-Thomson Refrigerator

#### Unit Settings: [KJ/(Kg*K)]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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<td>Asprₐ</td>
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</tr>
<tr>
<td>Cₐ</td>
<td>0.00000361 [kJ/K]</td>
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<tr>
<td>dₐ</td>
<td>0.00002657 [m]</td>
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<tr>
<td>tₐ</td>
<td>0.05143</td>
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<tr>
<td>hₐ</td>
<td>223.1 [kJ/kg]</td>
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<tr>
<td>hₐ</td>
<td>322.4 [kJ/(m²*K)]</td>
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<td>Tₑ₀</td>
<td>196.9 [K]</td>
</tr>
<tr>
<td>Vₑ₀</td>
<td>18.13 [m/s]</td>
</tr>
<tr>
<td>wₑ₀</td>
<td>0.00002 [m]</td>
</tr>
</tbody>
</table>

Array variables are in the Arrays window.
Table A.2.1 Variation of Refrigeration Capacity with Inlet Pressure and Number of Heat Exchanger Channels

<table>
<thead>
<tr>
<th>Number of heat exchanger channels</th>
<th>Refrigeration Capacity / (mW)</th>
<th>Inlet Pressure / (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3500</td>
<td>4000</td>
</tr>
<tr>
<td>75</td>
<td>13.94</td>
<td>30.78</td>
</tr>
<tr>
<td>100</td>
<td>18.40</td>
<td>36.84</td>
</tr>
<tr>
<td>150</td>
<td>23.07</td>
<td>42.83</td>
</tr>
<tr>
<td>200</td>
<td>25.46</td>
<td>45.67</td>
</tr>
<tr>
<td>250</td>
<td>26.89</td>
<td>47.27</td>
</tr>
<tr>
<td>300</td>
<td>27.85</td>
<td>48.25</td>
</tr>
<tr>
<td>350</td>
<td>28.53</td>
<td>48.91</td>
</tr>
<tr>
<td>400</td>
<td>29.04</td>
<td>49.37</td>
</tr>
<tr>
<td>500</td>
<td>29.75</td>
<td>49.94</td>
</tr>
</tbody>
</table>

Table A.2.2 Variation of Refrigeration Capacity and Mass Flow Rate with Number of Capillary Channels (Inlet Pressure 6000 kPa, Number of Heat Exchanger Channels 100)

<table>
<thead>
<tr>
<th>Number of Capillary Channels</th>
<th>Refrigeration Capacity / (mW)</th>
<th>Mass Flow Rate/ (kg/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>80.47</td>
<td>7.64E-06</td>
</tr>
<tr>
<td>15</td>
<td>125.90</td>
<td>1.31E-05</td>
</tr>
<tr>
<td>20</td>
<td>171.90</td>
<td>1.95E-05</td>
</tr>
<tr>
<td>25</td>
<td>217.70</td>
<td>2.69E-05</td>
</tr>
<tr>
<td>30</td>
<td>262.90</td>
<td>3.54E-05</td>
</tr>
</tbody>
</table>
APPENDIX C

ROUGHNESS AND SURFACE PROFILE DATA
APPENDIX C

AP.C.1 Hot fluid channels

Roughness profiles – unit of measurement is micrometers (μm)

Roughness data - 1

Roughness data - 2
Roughness data -3

Step height profiles - unit of measurement is micrometers (μm)

Step height profile - 1
Step height profile - 2

Step height profile - 3
Step height profile – 4

Mode: VSI
Mag: 40.0 X

2D Profiles

03/08/92
1702.94

Size: 268 X 236

Title:
Note:
AP.C.2 Cold fluid channels

Roughness profiles - unit of measurement is micrometers (µm)

Roughness profile - 1

Roughness profile - 2
Roughness profile -3

Step heights - unit of measurement is micrometers (μm)

Step height – 1

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Step height – 2

Step height – 3

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AP.C.3 Capillary channels

Roughness profiles - unit of measurement is micrometers (µm)

Roughness profile -1

Surface Data

Date: 07/02/02
Time: 14:15:14

Surface Statistics:
Ra: 139.76 nm
Rq: 170.01 nm
Rz: 600.48 nm
Rt: 1.04 µm

Set-up Parameters:
Size: 18 x 31
Sampling: 4500 nm

Processed Options:
Terms Removed
Tilt
Filtering
None

Title: Subregion
Note: X offset: 162 Y offset: 100

Roughness profile - 2

Surface Data

Date: 07/02/02
Time: 14:13:21

Surface Statistics:
Ra: 181.03 nm
Rq: 219.25 nm
Rz: 793.38 nm
Rt: 1.24 µm

Set-up Parameters:
Size: 14 x 43
Sampling: 4320 nm

Processed Options:
Terms Removed
Tilt
Filtering
None

Title: Subregion
Note: X offset: 174 Y offset: 190

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Roughness profile -3

Step heights- unit of measurement is micrometers (μm)

Step height profile – 1-

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Step height profile -2

Step height profile -3
Step height profile – 4

Step height profile – 5
3D plot of a capillary channel

Another 3D plot of capillary channels
**NOMENCLATURE**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>heat transfer area of side 1 (m²)</td>
</tr>
<tr>
<td>$A_2$</td>
<td>heat transfer area of side 2 (m²)</td>
</tr>
<tr>
<td>$A_{fl}$</td>
<td>fin surface area on side 1 (m²)</td>
</tr>
<tr>
<td>$A_{f2}$</td>
<td>fin surface area on side 2 (m²)</td>
</tr>
<tr>
<td>$A_w$</td>
<td>base area of heat exchanger (m²)</td>
</tr>
<tr>
<td>$c_p$</td>
<td>specific heat capacity at constant pressure (kJ/kg.K)</td>
</tr>
<tr>
<td>$C$</td>
<td>factor in Lockhart-Martinelli equation</td>
</tr>
<tr>
<td>$C_G$</td>
<td>factor in Lockhart-Martinelli equation</td>
</tr>
<tr>
<td>$C_L$</td>
<td>factor in Lockhart-Martinelli equation</td>
</tr>
<tr>
<td>$C_{max}$</td>
<td>maximum capacity rate (kJ/kg)</td>
</tr>
<tr>
<td>$C_{min}$</td>
<td>minimum capacity rate (kJ/kg)</td>
</tr>
<tr>
<td>$d$</td>
<td>diameter (m)</td>
</tr>
<tr>
<td>$d_1$</td>
<td>hot fluid channel depth (m)</td>
</tr>
<tr>
<td>$D_h$</td>
<td>hydraulic diameter (m)</td>
</tr>
<tr>
<td>$f$</td>
<td>friction factor</td>
</tr>
<tr>
<td>$g$</td>
<td>gravitational acceleration (m/s²)</td>
</tr>
<tr>
<td>$h$</td>
<td>convective heat transfer coefficient (W/m².K)</td>
</tr>
<tr>
<td>$h_i$</td>
<td>enthalpy (kJ/kg.K)</td>
</tr>
<tr>
<td>$h_f$</td>
<td>friction head loss (m)</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>( k )</td>
<td>thermal conductivity (kJ/m·K)</td>
</tr>
<tr>
<td>( L )</td>
<td>length (m)</td>
</tr>
<tr>
<td>( m )</td>
<td>mass flow rate (kg/s)</td>
</tr>
<tr>
<td>( N )</td>
<td>number of fluid flow channels</td>
</tr>
<tr>
<td>( Ntu )</td>
<td>Number of Transfer Units</td>
</tr>
<tr>
<td>( Nu )</td>
<td>Nusselt's number</td>
</tr>
<tr>
<td>( p )</td>
<td>pressure (N/m(^2))</td>
</tr>
<tr>
<td>( p_{in} )</td>
<td>non dimensional inlet pressure ((p/(p_{cr}-p_i)))</td>
</tr>
<tr>
<td>( Q )</td>
<td>refrigeration capacity (J/s)</td>
</tr>
<tr>
<td>( QQ_{ref} )</td>
<td>non dimensional refrigeration capacity ((Q/m(h_{cr}-h_i)))</td>
</tr>
<tr>
<td>( TT )</td>
<td>non dimensional temperature ((T/(T_{cr}-T_i)))</td>
</tr>
<tr>
<td>( Pr )</td>
<td>Prandtl number</td>
</tr>
<tr>
<td>( Re )</td>
<td>Reynolds number</td>
</tr>
<tr>
<td>( t )</td>
<td>wall thickness (m)</td>
</tr>
<tr>
<td>( T )</td>
<td>temperature (K)</td>
</tr>
<tr>
<td>( U )</td>
<td>overall heat transfer coefficient (W/m(^2)·K)</td>
</tr>
<tr>
<td>( V )</td>
<td>velocity</td>
</tr>
<tr>
<td>( V )</td>
<td>volume flow rate (m(^3)/s)</td>
</tr>
<tr>
<td>( w )</td>
<td>width (m)</td>
</tr>
</tbody>
</table>
$x$ - quality of refrigerant

$X$ - factor in Lockhart-Martinelli equation

**Subscripts**

- $c$ - cold
- $cap$ - capillary
- $chc$ - channel-cold
- $chh$ - channel-hot
- $cr$ - critical
- $f$ - fluid
- $g$ - gas
- $h$ - hot
- $i$ - thermodynamic state point $i = 1, 2, 3, 4, 5$
- $max$ - maximum
- $min$ - minimum

**Greek letters**

- $\Delta p$ - pressure drop (Pa/m)
- $\epsilon$ - effectiveness
- $\varphi_L$ - factor in Lockhart-Martinelli equation
- $\eta_1$ - fin effectiveness side 1
- $\eta_2$ - fin effectiveness side 2
- $\eta_{1l}$ - fin effectiveness of side 1
\eta_2 \quad \text{fin effectiveness of side 2} \\
\mu \quad \text{viscosity (kg.m/s}^2\text{)} \\
\rho \quad \text{density (kg/m}^3\text{)}
REFERENCES


**Internet sites**

1. [http://www.sensormag.com](http://www.sensormag.com)
4. [http://www.tau.ac.il/chemistry/OnlineCourse/physchem1.doc](http://www.tau.ac.il/chemistry/OnlineCourse/physchem1.doc)
VITA

Lionel Gamini Hewavitharana obtained his Bachelor of Science (B.S) degree in Mechanical Engineering from University of Peradeniya, Sri Lanka. After his graduation, he joined the Rice Processing Research and Development Center (RPRDC) of Sri Lanka and engaged in research and development activities in rice processing. Subsequently, he joined the Department of Mechanical Engineering of University of Peradeniya, Sri Lanka as a junior academic staff member and worked towards a Master of Philosophy (M.Phil.) degree. In September 1994, he enrolled in the Mechanical Engineering program of Louisiana Tech University and graduated with an M.S. in summer 1997. In September 1997, he enrolled in the Doctor of Engineering program at Louisiana Tech University, and later switched to the Doctor of Philosophy (Ph.D.) program.