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Optimal lot-sizing, pricing, and product intergenerational lifestyle decisions for the case of disruptive innovations in fashion

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OPTIMAL LOT-SIZING, PRICING, AND PRODUCT INTERGENERATIONAL LIFESTYLE DECISIONS FOR THE CASE OF DISRUPTIVE INNOVATIONS IN FASHION

by

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ABSTRACT

The objective of this dissertation is to determine production schedules, production quantities, selling prices, and new product introduction timing to fulfill deterministic price-dependent demand for a series of products in such a way as to maximize profit per period.

In order to accomplish the above task, some main assumptions are made. First, it is assumed that the series of products being considered are associated with sequential non-disruptive innovations in technology as well as disruptive innovations in fashion. That is to say, the products represent subsequent generations in the same family of products in an industry that experiences repeated minor technological innovations and in which product success is due in part to fashionability (Fisher, 1997). Second, it is assumed that the planning horizon is sufficiently long and product lifecycles are sufficiently short that several generations of the product family are planned. Third, it is assumed that the producer is following a solo-product roll strategy (Billington, Lee, & Tang, 1998). This means that the inventory of one product iteration is exhausted at the same time that the next product iteration is introduced and ready for sale. Fourth, it is assumed that demand for each product iteration is governed by a modified version of the Bass (1969) diffusion model that incorporates price. Fifth, it is assumed that the various demand and cost characteristics being considered do not change from one product iteration to the next. Sixth, it is assumed that no backlog of demand is maintained and that any unmet demand is lost. Seventh, it is assumed that the manufacturer is a monopolist or at least the dominant member of a market that is made up of it and smaller competitors that are not large enough to affect the market in a meaningful way.

The formulated profit maximization problem uses the Thomas (1970) model which in turn depends in its solution on theorems first presented by Wagner and Whitin (1958a). An extensive numerical study that aims at examining the sensitivity of the planned product lifecycle length and profit per period to changes in model parameters is performed using software developed especially for that purpose. The results of the analysis reveal that the above two measures are more sensitive to changes in market-oriented parameters than to changes in operations-oriented parameters. Managerial implications of the research findings are discussed.

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CHAPTER 1

INTRODUCTION

Fisher (1997) presents a framework that divides products into functional and innovative categories. Clothing is certainly a functional product; however, Fisher uses the fashion apparel industry as a prime example of innovative product manufacturers. Demand in the fashion apparel industry is driven by innovations in fashion, not by the basic functionality of the clothing. Technological innovations abound in electronic products, but in the boutique electronics industry innovations in fashion are also very important. High levels of demand for a product may be due to some technological innovation such as enhanced functionality of the product, but increased demand is often attributed to fashion.

One of Fisher's (1997) main points is that innovative products have higher gross profit margins than functional products but have demand that is more difficult to predict. If a company seeks the higher margins of innovative products, they must spend the money that is required to become responsive in order to meet market demand. Most boutique electronics are based on technology that was state-of-the-art not very long ago but has since become relatively easy to replicate. If one ignores fashion, as the technology becomes more commonplace the product's margin should decrease. However a firm may be able to maintain high margins by leveraging fashion appeal. A prime example of this is the Motorola RAZR. When the Motorola RAZR was first introduced, it sold for around $500 despite the fact that it had functionality similar to other phones which sold for much less (Cuneo, 2006).

This scenario is not limited to mobile phones. The MP3 format has been around since the mid-nineties and portable MP3 players have been readily available since 1999. Apple introduced the iPod in late 2001. Over the ensuing six years iPod sales totaled over 100 million units with sales of over 50 million units in 2007 alone (Apple Computer Inc, 2007). From its inception in 2001, around 20 different models have been developed not counting multiple colors of otherwise identical iPods. These 20 models fall into essentially 5 types of iPod. The rest of this product diversity is due in large part to fashion.

This study focuses on products in an industry experiencing incremental technological innovations and disruptive fashion innovations. It is assumed that product lifecycles are short enough that repeat purchases of a single product generation by the same customer do not occur.

Objective

The objective of this dissertation is to determine production schedules, production quantities, selling prices, and new product introduction timing to fulfill a deterministic price-dependent demand for multiple generations of an innovative product in such a way as to maximize profit per period. In order to accomplish this, some main assumptions are made. First, it is assumed that the series of products being considered are associated with sequential non-disruptive innovations in technology as well as disruptive innovations in fashion. That is to say, the products represent subsequent generations in the same family of products in an industry that experiences repeated minor technological innovations and which product success is due in part to fashionability (Fisher, 1997). Second, it is assumed that the planning horizon is sufficiently long and product lifecycles are sufficiently short that several generations of the product family are planned. Third, it is assumed that the producer is following a solo-product roll strategy (Billington, Lee, and Tang, 1998). This means that the inventory of one product iteration is exhausted at the same time that the next product iteration is introduced and ready for sale. Fourth, it is assumed that demand for each product iteration is governed by a modified version of the Bass (1969) diffusion model that incorporates price. Fifth, it is assumed that the various demand and cost characteristics being considered do not change from one product iteration to the next. Sixth, it is assumed that no backlog of demand is maintained and that any unmet demand is lost. Seventh, it is assumed that the manufacturer is a monopolist or at least the dominant member of a market that is made up solely of the dominant member and smaller competitors that are not large enough to affect the market in a meaningful way.

Research Motivation

A review of the operations management literature shows many heuristic and exact methodologies that are appropriate for selecting lot-sizes for functional products; however, these studies are generally not appropriate for new or innovative products. The marketing literature contains many articles that discuss new product introduction; however these studies generally do not consider the impact of operational considerations such as production lot-sizes. The motivation of this study is to bridge the gap between the existing operations management and marketing literature by creating a model that considers both lot sizing and new product introduction timing as well as other important considerations while maximizing profits for the manufacturer of innovative products such as personal electronic devices by controlling price, production, and product introduction timing in the presence of incremental technological innovation and disruptive fashion innovation.

From an operations management standpoint, the current study is an extension of Wagner and Whitin's (1958b) classic dynamic lot-sizing model. Wagner and Whitin assume that demand in each period is known and constant *a priori*. This leads to an optimal production schedule that defines the timing of production. The Wagner and Whitin model focuses on cost minimization. Implicit in this is the assumption that price is fixed and thus can be ignored. In other words, profit can be maximized by minimizing cost because revenue is a fixed constant. The model presented in this study differs from Wagner and Whitin's model in several important ways. First, similar to the work of Thomas (1970), the current model relaxes the assumption that price is a fixed constant. The current model allows for the fact that changing prices can have an impact on demand and thus revenue. In doing so, the nature of the resulting model changes from cost minimization to profit maximization. Second, the current model incorporates new product introduction timing decisions. After a thorough investigation of the available literature, it appears that the current study is the only one that includes a model incorporating new product introduction into a lot-sizing decision framework. Finally, unlike many exact lot-sizing methods presented in the next chapter, the current study thoroughly grounds parameter values in the literature rather than relying on general functional forms without consideration for the values of the various models' parameters.

From a marketing standpoint, the current study is an extension of Bass's (1969) classic diffusion model. The Bass model is a very robust model for product growth and diffusion. It has been found empirically to exhibit a very good fit with data on initial purchases of a wide variety of products. Although many applications of the original model exist, only a few consider the impact of price. Those that do consider price do not consider other operational variables such as production schedules and production quantities.

With respect to these two streams of literature, the goal of the current study is to bridge the gap between operations management and marketing by simultaneously considering important aspects of the problem at hand from both fields into a single model.

Contribution and Applicability

To the best knowledge of the author, the current study is the first to consider optimum production schedules, production quantities, selling prices, and new product introduction timing in a multi-generational product scenario. This research is thought to be applicable to sequential non-disruptive innovations in technology as well as disruptive innovations in fashion. Product lifecycles are relatively short and not limited to coincide with technological breakthroughs. Instead, it is assumed that new products are introduced in a regular pattern. Notable examples include, but are not limited to, consumer electronic devices such as iPods and high-end cell phones.

Overview of the Remaining Chapters

This chapter focuses on the conceptual underpinnings of the current study. Chapter 2 focuses on reviewing the relevant literature. Chapter 3 focuses on developing the model. Chapter 4 focuses on presenting the optimal solution methodology with perfect information. Chapter 5 focuses on the sensitivity of profit and new product introduction timing to both known and unknown changes in the values of the input parameters. Chapter 6, the last chapter, is concerned with summarizing the research, presenting concluding remarks, and discussing implications for managerial practice avenues for future research.

CHAPTER 2

SURVEY OF LITERATURE

The existing literature relevant to this study is presented in this chapter. It is divided into three broad categories: the lot-sizing literature, the diffusion of innovation literature, and the new product introduction timing literature. Much literature that could be classified into one of these three categories is not relevant to the study at hand and is thus omitted. For example, this chapter does not include any literature concerned with materials requirement planning despite the fact that this is a major subcategory of the lot-sizing literature.

Lot-Sizing Literature

Over the past 100 years countless lot-sizing models and methods have been developed that are based to a greater or lesser extent on the EOQ model. These lot-sizing methods can be divided into two categories: exact methods that produce guaranteed optimal solutions and heuristic methods that produce solutions that are not guaranteed optimal.

Optimal Lot-Sizing Methods

The Wilson economic order quantity (EOQ) model is so called because of analysis by Wilson (1934). However, the model is quite a bit older. Its original development is generally credited to Harris (1913); however, there are some anecdotal references that place its origin even earlier. Regardless of who is credited with the creation of the EOQ model, the EOQ formula, $Q = \sqrt{\frac{2DS}{H}}$ where D is annual demand, S is setup or ordering cost, and H is holding cost per unit per year, is well known. The EOQ formula assumes that setup and holding costs are the only pertinent costs. Variable costs are not considered pertinent because they do not depend on the quantity ordered. In order for the EOQ to be optimal, demand must occur at a fixed and constant rate.

The next major advance in optimal lot-sizing methods is Wagner and Whitin (1958b) which allows demand to vary from one period to the next but requires that demand in each period be fixed and constant. Wagner and Whitin present four important theorems. First, it is not optimal to carry inventory into a period in which production occurs. Second, at optimality production in every period is either zero or the sum of some integer number of sequential periods beginning in the period in question. Third, if at optimality beginning inventory in some period is zero then the problem can be partitioned into two sub-problems where one includes all periods prior to the period in question and the other includes the period in question and all subsequent periods. Fourth, if for a given planning horizon it is optimal for the "last" production run to occur in a given period, then when the planning horizon is lengthened the new optimal "last" production run will occur no sooner than the period previously indicated. These theorems are discussed in more detail in Chapter 4.

Hadley and Whitin (1963) present the next major advance in optimal lot-sizing methods. Hadley and Whitin reformulate the EOQ model to apply when quantity discounts are offered. The total annual cost equation used in the development of both the original EOQ and Hadley and Whitin's model is $TAC = \frac{SD}{Q} + VD + \frac{HQ}{2}$ where the first term is total setup costs incurred throughout the year, the middle term is total variable costs incurred throughout the year, and the last term is the total holding costs incurred throughout the year. Harris (1913) and Wilson (1934) treat variable cost per unit, V, as a constant. The EOQ formula is then found by taking the derivative of this equation with respect to Q, setting this equal to zero, and solving the resulting equation for Q. When V is treated as a constant, the derivative of the middle term is equal to zero and the term can be ignored. When V is a function of Q, as is the case when there are quantity discounts, no closed form solution is possible. Despite the absence of a closed form solution, Hadley and Whitin present a method that is guaranteed to find the optimal purchase quantity. Rubin, Dilts, and Barron (1983) present an improved version of Hadley and Whitin's method. It is nearly identical on a conceptual level but requires fewer computations to find the optimal solution.

Heuristic Lot-Sizing Methods

Each of the lot-sizing methods discussed in the previous section produce guaranteed optimal solutions. The generation of a solution that is guaranteed optimal requires not only a methodical process, but also assumptions that are often very strict. When these assumptions are violated, the old methods are no longer guaranteed to generate optimal solutions. Sometimes the relaxation of assumptions requires a more complex method to be adopted in order to continue to assure optimality. An example of this is Wagner and Whitin's (1958b) relaxation of Wilson's (1934) assumption that demand occurs at a fixed constant rate. At other times the relaxation of assumptions requires the use of a heuristic solution methodology because appropriate exact methods become intractable.

Many heuristic lot-sizing methods exist. Two of the simplest are derived directly from the EOQ formula. The EOQ formula returns an order quantity that is optimum when all the applicable assumptions hold. The first heuristic is to use the fixed order quantity (FOQ) suggested by the EOQ formula. When demand varies from period to period, the FOQ method is not guaranteed optimal. This is due in part to residual inventory. Residual inventory is inventory held over into a period when a new lot arrives. When demand is constant, the EOQ method results in regular periodic orders. The periodic order quantity (POQ) heuristic also has regular periodic orders. Order sizes vary based on demand levels in order to eliminate the residual inventory problem that occurs with the FOQ heuristic.

Several lot-sizing heuristics are based conceptually on the EOQ method even if they are not derived directly from it. Silver and Meal (1973) developed a very well known heuristic of this type. When all of the applicable assumptions hold, the EOQ method results in holding costs equal to setup costs. Every lot requires a setup with an associated cost of S. The Silver-Meal heuristic seeks to make the holding costs associated with each lot as close to S as possible. The decision maker begins by tentatively sizing an order so that it will meet demand for exactly one period. This results in setup costs of S and no holding costs. Because the holding costs are less than the setup cost for this order, the decision maker then tentatively sizes the order so that it will meet demand for exactly two periods. This new tentative lot-size results in the same setup cost and a holding cost greater than or equal to that of the last tentative schedule. Again the decision maker compares the setup cost to the cumulative holding cost. If the holding cost is still less than the setup cost, the decision maker adds an additional period to the lot. This process continues until the total holding costs for a lot is greater than the setup cost for that lot. Once this occurs, a new lot begins in the next period.

Many other lot-sizing heuristics exist. Wemmerlov and Whybark (1984) examine the performance of fourteen lot-sizing heuristics including all of the ones discussed in this section. They find that when demand is uncertain, few of these heuristics have significantly different cost performance.

Some relevant lot-sizing literature exists in the style goods literature. Hausman and Peterson (1972) present a lot-sizing model applicable to style goods that also considers product mix. They enforce capacity limitations in later periods that require some production to occur in earlier periods when forecasts are still poor. The authors state that the model can be formulated as a dynamic program but cannot actually be solved as such when there are two or more products. Instead, they solve it using three different heuristic approaches and evaluate the performance of these heuristics using numerical examples.

Hartung (1973) presents a style goods model that is solved using dynamic programming. An innovative aspect of this model is that unlike previous dynamic programming models that are defined in terms of two state variables (one for inventory levels and one for demand), this model is defined using a single state variable without loss of information.

Bitran, Haas, and Matsuo (1986) present a model that is similar to that of Hausman and Peterson (1972). It differs from their model in that it is set up so that products are grouped into product families where setup costs between families are substantial but setup costs between items in a family are negligible. Another difference is that demand for an entire family is known and only demand for individual items is stochastic. It is interesting to note that almost a decade before the phrase accurate response was coined, one of its key elements was presented: "Intuitively, it seemed sensible for the company to load their plants early in the year with products with flat patterns of forecast errors and postpone until later in the year loading their plants with products with rapidly decreasing forecast error" (Bitran et al, 1986, p227). This is conceptually very similar to Fisher, Hammond, Obermeyer, and Raman (1994, p84) who state that a key point of accurate response is that predictable products "should be made the furthest in advance in order to reserve greater manufacturing capacity for making unpredictable items closer to the selling season." Matsuo (1990) presents a slight modification of the model that treats time as continuous rather than discrete.

Lot-Sizing Models with Pricing

None of the literature discussed in the previous two sections consider price as a decision variable. This is the case for the majority, but certainly not all, lot-sizing literature. This section discusses that subset of the literature that simultaneously considers lot-sizing and pricing decisions. It includes only that literature that is most closely related to the current study or of the greatest overall significance. The interested reader is directed to Chan, Shen, Simchi-Levi, and Swann (2004) for a more thorough review of the lot-sizing literature that includes pricing.

Possibly the oldest lot-sizing model that includes pricing decisions is Whitin (1955). Two basic models are presented. In the first model, the classic EOQ model, and all of its various assumptions, is used to determine order quantities and the resultant cost is calculated. Profit is then maximized by representing demand as a linear function of price, taking the derivative of the profit function with respect to price, and solving for price. The second model presented is very similar to the newsboy problem (Hax and Candea, 1984, p146). The expected profit and expected loss for one additional unit of inventory are calculated and set equal to each other in order to find the target stocking level.

Similar to Whitin (1955), Kunreuther and Richard (1971) investigate the impact of centralized versus decentralized decision-making on the joint pricing and lot-sizing decision. The authors use a variant of the EOQ model where price is a decision variable and assume price does not change across periods. They model decentralized decision making as a two step process where the price is optimized while ignoring inventory and setup costs and then the optimal lot-size for this given price is determined. They model centralized decision making as the simultaneous determination of price and lot-sizes. They find that decentralized decision making is relatively costly when compared to centralized decision making, particularly when the product of setup and holding costs is high or when the EOQ generated by sequential decision making is low.

Wagner and Whitin (1958a) approach the problem of lot-sizing and pricing from an economic perspective. They present two cases, one with constant demand and one with dynamic demand. In the constant demand case, they describe a three-dimensional graph where demand rate, lot-size, and cost are the three axes. They explain that a cross section of this graph for any given demand rate will show a U-shaped relationship between order quantity and cost. Though it is not stated explicitly in the article, these cross sections are each instances of the U-shaped curve described by Harris (1913) and

Wilson (1934) in the development of the EOQ equation. The authors then describe plotting the minima of these curves as another curve describing average cost as a function of demand rate (for optimal lot-sizes). From the average cost curve, the marginal cost curve can be developed and set equal to the marginal revenue curve to find the optimal demand rate. The authors then show that such economic/calculus based solution methodologies do not work when there are multiple periods in which demand is not identical if there is a fixed charge, or setup cost, that is non-zero. At this point the authors suggest that a dynamic procedure should prove fruitful. They outline the basic concepts and theorems that are required to solve a problem very similar to the one addressed in the current study, but as Eliashberg and Steinberg (1993, p866) point out, they "do not explicitly provide such a formulation." The solution methodology is instead included in Wagner and Whitin (1958b), where it is used to solve a simplified formulation that no longer considers demand a function of price. Twelve years later the two halves are put together in an article by Thomas (1970). Thomas demonstrates that the four theorems of Wagner and Whitin (1958b) for their cost minimization problem are also applicable in the profit maximizing problem where demand is dependent on price. Thomas also demonstrates that prices in the various periods can be optimized one at a time. The current study is an extension of Thomas that also includes new product introduction timing.

Kunreuther and Schrage (1973) present another model that simultaneously incorporates pricing and lot-sizing decisions. Although this model does include price as a decision variable, it requires that price not vary across periods. Their problem is to determine the optimal fixed price and the periods in which to place orders.

One of the first models to jointly consider production timing and pricing decisions in the presence of demand that decays over time is presented by Cohen (1977). Cohen assumes that demand decreases over time following a negative exponential distribution whereas the current study assumes that demand follows the Bass (1969) model. Besides differences in the functional nature of the demand curve, the reasoning behind the reduction in demand is very different between Cohen and the current study. Cohen models the reduction in demand as a function of inventory on hand. The logic behind this is that the model is appropriate for perishable items such as produce. This leads to very different behavior from that found in the current study where the demand pattern is based on product lifecycle for non-perishable items.

Monahan (1984) presents a major extension of Hadley and Whitin's (1963) work on quantity discounts. As stated erlier, Hadley and Whitin present a method of selecting the optimal quantity to order given a discount schedule offered by a supplier. Monahan (1984), instead, presents a method of determining the optimal quantity discount schedule for a manufacturer to offer to its customer. Monahan describes the object of a quantity discount as an enticement to customers to place larger, less frequent orders. When all of the appropriate assumptions are satisfied, a customer should place orders in accordance with their EOQ, and any deviation from this quantity results in a cost increase. In order to entice a customer to increase their order size, the discount should at least make up for the increased cost associated with deviating from the EOQ. Monahan presents closed form solutions for the order size and discount price that maximize profit for the supplier while allowing the customer to maintain profits identical to those they would receive if there were no quantity discount. This landmark article spawned an entire body of literature including Rosenblatt and Lee (1985), Lee and Rosenblatt (1986), Banerjee (1986), Goyal (1987), Joglekar (1988), and more recently Rubin and Benton (2003). This list is not exhaustive. These articles are similar in that they all modify the problem originally presented by Monahan to some extent and then present solutions that are generated in a similar manner. Each presents an optimal solution given the particular set of assumptions presented.

The interested reader is directed to Eliashberg and Steinberg (1993), which reviews much of the literature discussed above in greater detail. Section 4.2, "Dynamic-price models" of their review is of particular relevance to the current study.

Diffusion of Innovation Literature

Although it is not the oldest, the Bass (1969) model is probably the most important model of the diffusion of innovation. Although it has been modified and applied in a wide range of areas, the Bass model was originally developed to model first purchase incidents for product categories. The article in which the Bass model first appeared was named one of the ten most influential articles in the history of *Management Science* (Bass, 2004). The Bass model is based on two fundamental behavioral forces: innovation and imitation. The portion of demand that occurs independently of the cumulative demand is represented by the coefficient of innovation, p. The portion of demand that varies with cumulative demand is represented by the coefficient of imitation, q. When a product is first introduced, only innovators purchase the product. In subsequent periods, some portion of the total demand is caused by additional innovators purchasing the product and some portion is caused by imitators purchasing the product. Thus, the likelihood of purchase at time t given that no purchase has yet been made is

$\frac{f(t)}{1-F(t)} = p + q * F(t)$, where $f(t)$ is the likelihood of purchase at time t and $F(t)$ is the integral of $f(t)$ with respect to t. Solving this differential equation when $F(0) = 0$ yields $F(t) = \frac{1-e^{-bt}}{1+ae^{-bt}}$,where a is defined as the ratio of q to p and b is defined as the sum of p and q. Cumulative sales up to time t, or $Q(t)$ can be found by multiplying $F(t)$ by m, or market potential. Taking the derivative of $f(t)$ with respect to t, setting the result equal to zero, and solving for t yields $\frac{\ln a}{b}$, or the time of peak demand. Many applications of the Bass model exist but most will not be addressed here. This literature review is concerned with two subsets of the literature related to the Bass model: the subset that extends the original model to include price and the subset that considers multiple generations of products. For a more thorough investigation of the literature the interested reader is directed to Mahajan, Muller, and Bass (1990), Parker (1994), Mahajan, Muller, and Wind (2000), and Meade and Islam (2006).

Diffusion Models Incorporating Price

The diffusion of innovation literature contains two basic approaches for the incorporation of price as a decision variable. The first approach is to model the current demand rate, $q(t)$, as a function of the current price, or P_t, and $Q(t)$ which is in turn a function of previous prices. Examples of this approach include Robinson and Lakhani, (1975), Dolan and Jeuland (1981), Mahajan and Peterson (1978; 1982), Kalish (1985), and Kalish and Lilien (1986). Jain and Rao (1990) find empirically that models employing a demand rate structure of the form $f(Q(t)) * g(P_t)$, where f is a non-negative function that first increases and then decreases in $Q(t)$ and g is a function that decreases in P_t tend to exhibit better fit than other model structures.

The second approach is to model the demand rate as a function of time and price as follows: $q(t, P_t) = f(t)*g(P_t)$, where f is an exogenous lifecycle curve such as the Bass (1969) model as a function of time and g is the same type of price response function described in the previous paragraph. This approach was introduced by Bass (1980) and followed by Bass and Bultez (1982) and Mesak (1990).

Two main forms of g are present in the diffusion of innovation literature. The first is of the form e^{-B*P}, where B is a constant price sensitivity parameter. This type of formulation has been employed by Robinson and Lakhani (1975), Dolan and Jeuland (1981), and Thompson and Teng (1984). This formulation suggests that price elasticity of demand is a linearly increasing function of price. The second form of g present in the literature is $\left(\frac{P_t}{P_0}\right)^{-\eta}$, where η is a constant price elasticity parameter and P_0 is a fixed base price. This type of price response function has been used by Bass (1980), Bass and Bultez (1982), Mesak (1990), and Jain and Rao (1990). Bass, Krishnan, and Jain (1994) find empirically that models employing this price response function tend to exhibit better fit than models employing the exponential prices response function described above.

<u>Diffusion Models Incorporating Successive Generations</u>

Norton and Bass (1987; 1992) extend the original Bass model to consider multiple generations within a product family where repeat purchases across generations may occur. They point out that often a new product will not replace an old product immediately and entirely but instead that the two generations may compete for a time prior to the complete discontinuation of the old product. As such they include substitution in their model. Norton and Bass (1987) also empirically test their model using data on microchips and find that it has a good fit. Norton and Bass (1992) extend the model to include repeatedly purchased products such as pharmaceuticals and disposable diapers. The authors conclude that the coefficients of innovation and imitation for a given product do not change significantly from generation to generation.

Wilson and Norton (1989) also consider both diffusion and substitution through an extension of the model presented by Kalish (1985). Specifically, they address the question of timing for the introduction of a product extension where the goal is to maximize profit for the original product and its extension. One of Wilson and Norton's main findings is that when the planning horizon is sufficiently long, it is optimal for a monopolist to either introduce a second generation as soon as possible or not to introduce it at all.

Mahajan and Muller (1996) propose a model that simultaneously captures the adoption and substitution patterns for successive generations of a durable technological innovation. They find that it is optimal to either introduce a new product as soon as possible or to delay introduction until the growth phase of the current generation has ended. This decision depends on a number of factors including the relative size of the market potentials, gross profit margins, and the diffusion and substitution parameters.

The models discussed so far in this section do not consider marketing mix variables. Both normative (e.g., Bayus, 1992; Padmanabhan and Bass, 1993) and empirical (e.g., Speece and MacLachlan, 1992; 1995; Danaher, Hardie, and Putsis, 2001) research has considered the pricing decision in a product substitution setting. For more on this subject the interested reader is directed to Bayus, Kim, and Shocker (2000).

New Product Introduction Timing Literature

This section contains a review of the new product introduction timing literature. It is subdivided into three sections. The first section concerns the relationship between introduction timing and business performance. The second section concerns strategic considerations in introducing new product generations, the phasing out of older product generations, and the organization of product development efforts. The first two sections include only deterministic models. The third section presents two stochastic models.

Introduction Timing and Business Performance

Robinson (1990) empirically investigates the impact of product innovation on market share. The basic research question being addressed is whether the additional design costs and time to market required by innovative products will tend to be offset by revenue gains. Robinson uses market share as a surrogate for revenue gains and finds that product innovation has a strong positive relationship with market share and that incompatibility with previous products does not have a significant relationship with market share.

Huff and Robinson (1994) empirically investigate the relationship between market entry timing, new product introduction frequency, and market leadership. They find that among surviving firms, the first firm in a market usually develops a leadership position with respect to market share. They find that as the length of the competitive rivalry between the first and second firm in the market increases that this leadership tends to erode and that this erosion occurs faster in markets with short lifecycle products.

Kessler and Chakrabarti (1996) examine new product introduction pacing and present recommendations. They recommend that increased innovation speed is most appropriate in environments that have high levels of competition and dynamism as well as low levels of regulatory restrictions.

Hua and Wemmerlov (2006) empirically examine the relationship between the frequency of new product introductions and market performance in the PC industry. Their findings suggest that product advantage mediates this relationship. The model also suggests that product change orientation and technology competence moderate this relationship; however, the data does not support this.

Strategic Considerations and
Introduction Timing

Cohen, Eliashberg, and Ho (1996) introduce a multistage model of new product development and derive expressions for optimal time-to-market. They assume that investments in product quality occur at a constant rate over the planning horizon. This leads to the conclusion that a firm should slow its product development activities to create a superior product if product margins are high and demand levels for the new product are expected to be high.

Billington et al (1998) discuss successful new product introduction strategies. Discussion centers on the timing of product introductions and describes two basic strategies: solo roll and dual roll. In a solo roll strategy an attempt is made to sell out of the old product at the same time that the new product becomes available. In a dual roll strategy the two are sold simultaneously for a time before the old product is discontinued completely.

Krishnan, Singh, and Tirupati (1999) present a model that concerns the introduction of multiple products in a product family in the presence of a product platform. The basic idea here is that the development of an underlying platform leads to reduced introduction costs for all products on that platform but that the development of the platform itself is associated with a fixed cost.

Carrillo (2005) investigates the pace of product developments in a single firm and relates these to the clockspeed of the firm's industry. She models diffusion using the generalized Bass model (Bass et al, 1994) and assumes that only a single generation can exist in the market at any one time and that the fixed cost for product introduction are independent of introduction timing.

Stochastic Introduction
Timing Models

Kurawarwala and Matsuo (1996) present a model that simultaneously considers forecasting and production of short lifecycle products. Their model is similar to the current study in that it seeks to incorporate concepts from the marketing and inventory management literature. Differences between the two models are discussed below.

Krankel, Duenyas, and Kapuscinski (2006) present a model that seeks to optimize the timing of the introduction of successive products when demand is based in part on the technology of a product and technological advance is stochastic.

Placement of the Current Study

Placement Relative to the Lot-Sizing Literature

The lot-sizing literature consists of heuristic methods and exact methods. Both types of approach have their strengths and weaknesses. The major strength of heuristic methods is that they do not require the use of as many simplifying assumptions. The major weakness is that they do not ensure optimality. This weakness has two major facets. From a practical standpoint, when optimality is not assured decision makers cannot be confident that they are making the best decisions. From an academic standpoint, when optimality is not assured the analyst cannot get a clear view of the relationship between input parameters and performance. Exact methods insure optimality but require the use of somewhat stylized problems.

A thorough review of the literature reveals few lot-sizing methodologies that are appropriate for innovative products: Hausman and Peterson (1972), Hartung (1973), Bitran et al, (1986), and Matsuo (1990). All of these articles differ from the current study in that they present heuristics rather than exact solutions. Hausman and Peterson (1972) present three solution heuristics. These heuristics treat each product similarly to the classic newsvendor problem then use Lagrange multipliers to allocate capacity. Hartung (1973) presents a heuristic that is appropriate when demand is stochastic. Birtran et al, (1986) and Matsuo (1990) both use two-phase heuristics to solve the problem. In the first phase, an aggregate plan is produced where aggregation is at the product family level. In the second phase, the plan is disaggregated to individual products.

Chan et al (2004) present a thorough review of the literature concerned with the coordination of price decisions with manufacturing and distribution decisions. They point out that the integration of pricing, production, and distribution decisions is still in its early stages in most industries but that it has the potential to "radically improve supply chain efficiencies in much the same way as revenue management has changed airline, hotel and car rental companies" (p 336). Of all of the models discussed in this chapter, the model presented in Chapter 3 is most closely related to that of Thomas (1970). It extends the work of Thomas in several important ways. Chief among these extensions are the use of a well-defined demand function (Bass, 1969), the use of a realistic price response function (Bass, 1980), and the incorporation of new product introduction timing into the model.

Placement Relative to the Diffusion of Innovation Literature

The diffusion of innovation literature cited above falls into two categories: those articles that consider multiple generations, and those articles that consider the effect of price on demand. The current study sits at the intersection of these two bodies of literature in that it simultaneously considers both multiple generations and the relationship between price and demand. In addition to these considerations, the formulated profit maximization model also considers the production and inventory costs together with product development costs. In addition, the current study investigates the impact of various diffusion and cost parameters on the optimal introduction timing of successive generations and the related optimal profits.

Placement Relative to the New Products Introduction Timing Literature

Much of the new products introduction literature is concerned either with the relationship between decision antecedents of introduction and the business outcomes of introduction. This portion of the literature is omitted from the discussion above because it is largely irrelevant to the current study. A smaller portion of the literature is concerned with the relationship between introduction timing and business performance. This literature is reviewed above.

Of all the articles discussed in this chapter, Carrillo (2005) is one of the most similar to the current study. Both models assume that generations of products have equal planned produce life cycle lengths, that only a single generation is present in the market at any one time, and that product development and introduction costs for different generations are equal. Carrillo differs from the current study in that it does not consider optimal pricing or production scheduling decisions.

A second similar article is Kurawarwala and Matsuo (1996). Like the model presented in Chapter 3, it incorporates many inventory management concepts with a forecasting model based on the Bass model. Major differences between the two models include the absence of price from the model presented by Kurawarwala and Matsuo as well as their assumption that procurement leadtimes are greater than product lifecycles.

A third article that is akin to the current study is Krankel et al (2006). Both present models that seek to optimize new product introduction timing in the presence of incremental technological innovations. One major difference is that Krankel et al explicitly model these innovations using a stochastic process whereas the current model captures innovation implicitly. Another major difference is that the current study also simultaneously optimizes production scheduling and pricing while these issues are not considered by Krankel et al.

In sum, although there is significant research in lot-sizing, diffusion of innovations, and new product introduction timing, to the best knowledge of the author, this dissertation is the first to consider optimum product introduction timing, pricing, and production scheduling decisions in a multi-generational product scenario.

CHAPTER 3

MODEL DEVELOPMENT

Problem Framework

This chapter considers the situation of a monopolistic manufacturer planning to introduce G successive generations of a product. The manufacturer aims to determine the lifecycle, T, of each generation together with the related production schedule, production quantities, and selling prices to meet a deterministic price-dependent demand so as to maximize profit over a sufficiently long planning horizon.

In addition to the assumptions made in the first chapter, the first generation is assumed to be ready for introduction at time zero at which time the initial inventory is also zero. After the first generation is introduced, product development efforts for the second generation, scheduled to be introduced to the market at time T plus one, begin. By time T, all units of the first generation are sold and its ending inventory at time T is zero. This sequence of events repeats itself until the last generation is introduced at time $(G - 1) * T + 1$ and all of the last generation's units are sold by time GT so that ending inventory is again zero. Production is assumed to be instantaneous at the beginning of the period and assumed to precede demand for the period. Production is not required to take place in every period, but if production does occur it is allowed to meet demand in the period in which it occurs and possibly in some ensuing periods. Demand for each generation is assumed to be governed by a modified version of the Bass (1969) model in which market potential is a function of price. Furthermore, the discount rate is assumed to be zero. The weakness of such an assumption is substantially mitigated due to the periodic nature of the problem.

Demand Curve

The first step of problem development is the definition and discussion of the demand pattern observed by the organization. In this study, demand is based on the Bass model (Bass, 1969), one of the ten most influential articles in the history of *Management Science* (Bass, 2004). Because the Bass model does not consider repeat purchases, it is more appropriate for durable products that are purchased infrequently relative to the products' lifecycles. The type of products being considered in the current model are not durable goods per se; however, they have lifecycles short enough that repurchase within a single product iteration is unlikely. The Bass model is designed to predict demand for entire product categories; however, it may also be used for individual products (Krishnan, Bass, & Kumar, 2000).

The Bass model is based on two fundamental behavioral forces: innovation and imitation. The portion of demand that occurs independently of the cumulative demand is represented by the coefficient of innovation. The portion of demand that varies with cumulative demand is represented by the coefficient of imitation. When a product is first introduced only innovators purchase the product. In subsequent periods, some portion of the total demand is caused by additional innovators purchasing the product and some portion is caused by imitators purchasing the product. Thus, the likelihood of purchase at time T given that no purchase has yet been made is $\frac{f(t)}{1-F(t)} = p + q * F(t)$, where $f(t)$ is the likelihood of purchase at time t, $F(t)$ is the integral of $f(t)$ with respect to t, p is the coefficient of innovation, and q is the coefficient of imitation. Note that the importance of innovators is great at first but diminishes monotonically over time. Also note that the values of p and q are related to the units used to measure time. Often when using the Bass model to estimate the demand pattern of a product, the ratio of q to p, which is not dependent upon the time units used, is a more appropriate measure to consider (Bass, 1969; Non, Franses, Laheij, & Rokers, 2003). Norton and Bass (1987) solve the differential equation shown above for $F(t)$ after defining a as the ratio of q to p and defining b as the sum of p and q. This is shown in Equation 1 below.

$$F(t) = \frac{1 - e^{-bt}}{1 + ae^{-bt}} \tag{1}$$

Demand occurring between time t minus one and time t is found by taking the difference between Equation 1 and itself for the values of t and t minus one and multiplying the result by the size of the market. Thus absent of any price impact, which is discussed below, demand in period t is defined by Equation 2 where m is the total size of the potential market.

$$D_\beta(t) = m * [F(t) - F(t-1)] \tag{2}$$

The subscript beta represents the fact that this is basic demand prior to consideration of a price impact. A constant elasticity price response function of the form $\left(\frac{P_t}{P_0}\right)^{-\eta}$ is used in the current model. This is the same price response function used by Bass (1980) and it has been found to result in superior model fit when compared to the exponential types of price response function discussed in Chapter 2 (Jain & Rao, 1990;

Bass et al, 1994). Thus, Equation 3 represents demand after the impact of price has been considered.

$$D_t = D_\beta * \left(\frac{P_t}{P_0} \right)^{-\eta} \quad (3)$$

The multiplicative demand function shown in Equation 3 is consistent with that employed by Bass (1980), Bass and Bultez (1982), and Mesak (1990). Demand in period t is a function of four model parameters and one decision variable: the coefficient of innovation, the coefficient of imitation, the size of the market, the price elasticity parameter, and the price at time t. Of these, only P_t is a decision variable. The rest of the variables are model input parameters.

Production Environment

The purpose of the demand curve is to generate the demand pattern observed by the organization. The production environment is the collection of considerations that have an impact on the organization's response to this demand. Specifically, this section deals with model parameters that define the cost structure faced by the organization. This cost structure, along with the demand pattern discussed in the previous section, defines the dynamic lot-sizing with price elasticity and new product introduction, or DLPENP, model. In Chapter 4 an optimal solution methodology that assumes perfect information is presented and an in-depth example problem instance is generated and solved. In Chapter 5 sensitivity analysis is performed.

The production environment is defined by four cost parameters. Holding cost, H, is the cost associated with holding one unit for one period. It captures not only materials handling costs but also the cost of capital, or the opportunity cost associated with having money tied up as inventory instead of being available for investment purposes. The setup cost, *S*, is the cost of initiating a production run. These two parameters are also included in the classic dynamic lot-sizing, or DLS, problem. The current problem also includes two other parameters not included in the classic DLS problem. The DLS problem is generally formulated in terms of cost minimization instead of profit maximization. Because of this the variable cost, *V*, or cost per unit is not required in the DLS problem whereas it is included in the current problem. There is also a one-time cost associated with introducing new products. This includes not only costs associated with product design but also any marketing costs such as pre-launch advertising expenditures. Product introduction costs are captured by the variable *I*. The input parameters for both the demand curve and the production environment are listed in Table 1.

Table 1 Input parameters.

Parameter	Description
a	Ratio of imitation to innovation
b	Sum of innovation and imitation
m	Market size
η	Price elasticity parameter
H	Holding cost per unit per period
S	Setup cost for a production run
V	Variable cost of product
I	New product introduction cost

Decision Variables

Four groups of decision variables are associated with this problem. The first is the price to charge in each period. The second concerns when to produce. The third concerns the quantity to produce when production occurs. The fourth decision variable concerns when to introduce the next product generation.

Problem as a Mixed Integer Programming Model

The DLPENP model is an extension of the classic DLS model. It differs from the classic DLS model in two important ways, both of which concern the demand function. In the DLS problem, demand varies from period to period but the demand levels are fixed and known *a priori*. In the DLPENP problem, demand in period t is not fixed but instead is deterministically related to the various input parameters and price in period t. The second major difference between the DLPENP and DLS problems concerns the number of periods in a problem instance. In the classic DLS problem the number of periods, or T, is fixed *a priori*. In the DLPENP problem, T is a decision variable.

The purpose of this section is to present a formal formulation of the DLPENP model. The topic of solving the DLPENP model is reserved for Chapter 4. The DLPENP model can be formulated as a nonlinear Mixed Integer Program (MIP). The MIP model is formulated with an objective function that is to be maximized or minimized, a list of decision variables that can be adjusted in order to optimize the objective function, and constraint equations that limit the possible values of the decision variables. The DLPENP problem does not include capacity restrictions or backorders. The objective of the DLPENP model is the maximization of average profit per period. One version of the objective function for of this MIP is given by Expression 4. Alternate versions of this objective function are discussed below.

$$Max: \frac{\sum_{t=1}^{T}\left((P_t - V)*D_t - S*\sigma_t - H*Inv_t\right) - I}{T} \tag{4}$$

Before addressing the constraints on this objective, each element of this objective function should be discussed. The index value t takes on integer values from one to T and appears in the subscript of several variables. Periods are equal to one week so P_1 is the value of P in week one, P_2 is the value of P in week two, etc. The decision variable T represents the length of a product's lifecycle. Another decision variable is P_t, which represents the price charged in period t. The parameter V is the variable cost, which remains constant across periods. Demand in period t is represented by D_t, which is defined by Equation 3. It is a function of the various input parameters and the decision variable P_t. An alternate version of Expression 4 that includes only model parameters and decision variables instead of D_t can be found by substitution of Equations 1 through 3 into Expression 4. The parameter S is the setup cost, which remains constant across periods. The binary decision variable σ_t takes on a value of one when production occurs in period t and a value of zero when no production occurs. The parameter H is the inventory holding cost, which remains constant across periods. Inventory on hand at the end of period t is represented by Inv_t, which is defined by Equation 5.

$$Inv_t = Inv_{t-1} + X_t - D_t \text{ for } t = 1 \text{ to } T \tag{5}$$

Note that X_t is a decision variable that represents the production quantity in period t. Also note that Inv_0 is equal to zero. An alternate version of Expression 4 that includes only model parameters and decision variables can be found by repeated substitution of Equation 5 into Expression 4. Finally, the parameter I is the new product introduction cost.

Expression 4 is subject to three different types of constraints. The inventory balance constraints, shown in Equation 5, represent the first type of constraint. These constraints ensure that in a given period, beginning inventory plus production minus demand is equal to ending inventory. Note that ending inventory in one period is equal to beginning inventory in the next period.

The second type of constraint upon expression four ensures that production is sufficient to meet demand. The production constraints are shown in Equation 6.

$$\sum_{t=1}^{i} X_t \geq \sum_{t=1}^{i} D_t \text{ for } i = 1 \text{ to } T \tag{6}$$

These constraints ensure that the cumulative production up to a given period is sufficient to meet the cumulative demand up to that period.

The final type of constraint upon Expression 4 ensures that setups occur in each period where the production quantity is positive. The setup constraints are shown in Equation 7.

$$m * \sigma_t - X_t \geq 0 \text{ for } t = 1 \text{ to } T \tag{7}$$

Recall that sigma is a binary variable. These constraints ensure that setups occur in every period in which production occurs.

It should be noted that throughout this formulation interaction between periods occurs only in the inventory balance constraints. This is a critical point, the importance of which will be shown in Chapter 4.

Parameter Levels for Example Problems

In practice, decision makers will base the input parameter values on a combination of historical data, business process data, and any other applicable sources of data. In order to make the example as useful as possible, input parameters should be set at levels that are likely to be found in practice. The purpose of this section is to determine what parameter levels are most likely to be reasonable.

The first parameter to be considered is *a*, which is the ratio of the coefficient of imitation to the coefficient of innovation, or q/p. Bass (1969) finds that *a* ranges from 9.0 to 82.4. Pae and Lehmann (2003) find empirical evidence that *a* is positively related to the length of a product's lifecycle. In a study related to the diffusion of short life-cycle products, Kurawarwala and Matsuo (1996) reports values of *a* as small as 1.7. The products being considered in the current study have relatively short lifecycles; therefore, the range of values for *a* considered in the current study is 1.5 to 20.

The second parameter to be considered is *b*, which is the sum of the coefficients of innovation and imitation, or $p + q$. Bass (1969) finds that *b* ranges from 0.19 to 0.68. As seen in the expression for time of peak demand, $1/b \ln a$, *b* is negatively related to product lifecycle length (Bass, 1969). Note that Bass (1969) uses annual data while the current study uses weekly data. In order to make a *b* value derived using annual data appropriate for use with weekly data, it must be divided by the number of weeks in a year, or 52 (Non et al, 2003). This would result in a range of values from 0.004 to 0.013. In their study of short lifecycle products, Kurawarwala and Matsuo (1996) report values of *b* as large as 1.296 when using monthly data. This is equivalent to a value of 0.3 after converting it to a weekly value. This value is significantly higher than any other values that were found in a search of the literature. Therefore the range of values for *b* considered in the current study is 0.01 to 0.1 and is associated with weekly data.

The third parameter to be considered is *m*, which is the total population of potential customers. In this regard, the upper bound on *m* is set near 10% of the population of the United States. Such a percentage appears plausible in practice. For example, when Sony introduced the PlayStation 2, the highly popular original

PlayStation that was launched only four years earlier had an installed base of 30 million in the United States (Peterson, 2004). The lower bound on m is set near 1.5% of the population of the United States. Therefore the range of values for m considered in the current study is 5,000,000 to 30,000,000.

The fourth parameter to be considered is η, which is the price elasticity parameter. As will be shown in Chapter 4, η must be greater than one. Bass (1980) finds values of η as large as 8.02. Therefore the range of values for η considered in the current study is 1.5 to 8.

The fifth parameter to be considered is H, which is holding costs measured in dollars per unit per week. The operations management literature does not normally discuss holding costs in dollars, but in terms of a percentage of the product's selling price. Many sources in the production management literature suggest that holding costs are between 15% and 40% of price per unit per year (Rubin et al, 1983; Jordan, 1989; Raman & Kim, 2002). The products being considered in the current study are expected to sell for around \$80 to \$150. Therefore the range of values for H considered in the current study is \$0.20 to \$1 per unit per week.

The sixth parameter to be considered is S, which is setup cost. The production management literature does not normally discuss setup costs in dollars because it is highly context specific. The literature suggests two ways of addressing this difficulty. Some authors, such as Berry (1972), set S indirectly by considering the economic Time Between Orders (TBO). One of the most commonly used values for economic TBO is two weeks (Benton & Whybark, 1982; Lin, Krajewski, Leong, & Benton, 1994). Economic TBO, measured in years, is equal to the ratio of the EOQ to annual demand.

When using the values of *H* mentioned in the previous paragraph, an expected annual demand of around one million units, and a target TBO of around two weeks, this method suggests a range of values for *S* of around \$5,000 to \$30,000. A simpler approach that does not rely on demand is proposed by Wemmerlov (1982). Wemmerlov suggests that the ratio of *S* to *H* be used and employs a range of values from 25 to 600 (p469). Note that Wemmerlov's *H* is the annual holding cost whereas the *H* used in the current study is weekly holding cost. When using the values of *H* mentioned in the previous paragraph, Wemmerlov's range of ratios suggest a range of values for *S* of \$250 to \$30,000. Therefore the range of values for *S* considered in the current study is \$1,000 to \$30,000 per unit per week.

The seventh parameter to be considered is *V*, which is variable cost per unit. According to Fisher (1997) innovative products have gross profit margins (GPM) between 20% and 60%. Based on this and selling prices in the range from \$80 to \$150, the range of values for *V* considered in the current study is from \$50 to \$100 per unit.

The eighth and final parameter to be considered is *I*, which includes research and development as well as other new product introduction costs. Ulrich and Eppinger (2004) suggest that research and development costs for new products tend to be less than 5% of total revenue. As stated previously, expected annual demand is around one million units and prices are expected to be \$80 to \$150 per unit. Also product lifecycles are expected to be around three years. Therefore the range of values for *I* considered in the current study is \$5,000,000 to \$25,000,000. The factor ranges discussed above are summarized in Table 2.

Table 2 Derivative parameter ranges.

Parameter	Factor Level Range
a	1.5 to 20
b	0.01 to 0.1
m	5,000,000 to 30,000,000
η	1.5 to 8
H	0.2 to 1
S	1,000 to 30,000
V	50 to 100
I	5,000,000 to 25,000,000

CHAPTER 4

MODEL SOLUTION

Analytical Solution Methodology

Chapter 3 presents a framework for modeling pricing, production timing, production quantity, and new product introduction timing decisions with the objective of maximizing profit per period. In this chapter a method is presented which finds the guaranteed optimal solution to the DLPENP problem in polynomial time.

Wagner and Whitin's Methodology

The methodology presented is an extension of the methodology presented by Wagner and Whitin (1958b), which was first outlined by Wagner and Whitin (1958a) and later synthesized by Thomas (1970). As stated in Chapter 2, Wagner and Whitin (1958b) present four theorems. First, the optimal solution will be a dominant production sequence. A dominant production sequence is one in which $Inv_{t-1} * X_t = 0$ for all t (Manne, 1958). Stated another way, a dominant production sequence is one in which either starting inventory or production quantities, or both, are equal to zero in every period. This theorem is key to the development of Wagner and Whitin's other three theorems as well as the viability of the solution presented later in this chapter so it is important to discuss the assumptions that are required for it to be valid. Wagner and Whitin's first theorem is based on two assumptions. The first is that beginning inventory in the first period is zero and the second is that production costs are linearly related to production volumes regardless of production timing or production quantities. As explained by Lundin and Morton (1975, p713) both of these assumptions can be relaxed to a certain extent without invalidating the theorem. The inclusion of inventory at the beginning of the first period requires a trivial modification to the theorem and adding non-linearity to the production cost requires no modification so long as the cost curve remains concave. Note that the current study makes both of these assumptions. Thus the theorem holds true for the current study as well. Also note that Thomas (1970) demonstrates that Wagner and Whitin's theorems concerning the cost minimization problem are also applicable to the profit maximization problem.

Wagner and Whitin's second theorem can be stated mathematically as follows: $X_t = \begin{cases} 0 & k < t \\ \sum_{j=t}^{k} D_k & t \le k \le T \end{cases}$ for all t. Stated another way, production in each period is equal to zero or the sum of demand for some number of sequential periods beginning in that period and continuing for some integer number of periods into the future. This theorem follows from the first finding and from the assumption that all demand must be met. Note that the current study makes this assumption as well, thus this theorem holds true for the current study as well.

Wagner and Whitin's third theorem is that whenever $Inv_t = 0$ is optimal for a particular value of t then periods 1 through t can be considered independently of periods t+1 through T. This is not due to any additional assumptions but is instead due to the nature of the problem being solved. Specifically, it is due to the fact that the inventory balance equations, $Inv_{t-1} + X_t - D_t = Inv_t$, are the only place where different periods interact. When there is no inventory held over from one period to the next the linkages between those two periods break and the planning horizon can be subdivided into two segments which when solved independently will result in the same solution as the solution to the original undivided problem. Once again, this theorem holds true for the current study as well.

Wagner and Whitin's fourth theorem, known as the planning horizon theorem, relates to the following recursive function:

$f(t) = \min\left\{ \begin{matrix} \min \\ 1 \le j < t \end{matrix} \left[S + \sum_{i=j}^{t-1} \sum_{k=i+1}^{t} HD_k + f(j-1) \right], S + f(t-1) \right\}$, where $f(t)$ is the cost of producing to meet demand in period one through period t, $f(0) = 0$, j indicates the last period in which production occurs in order to meet demand up through period t, and $\begin{matrix} \min \\ 1 \le j < t \end{matrix}$ indicates that j is selected to minimize the terms inside the square brackets. Note that while j is defined over the range 1 to $t - 1$ for use inside the square brackets it takes on a value of t if the terms after the square brackets are the minimum. If demand is to occur in period t, the variable j is not used and instead the terms the right of, as opposed to inside of, the square brackets are used to determine costs. Because of the recursive nature of the function, $f(t-1)$ must be calculated prior to $f(t)$ for $t > 0$ while $f(0) = 0$. When t is equal to one, j is also equal to one and production occurs in the first period. This is associated with a cost of $S + f(0)$, or S. When $t = 2$ and $j = 1$, the terms inside the square brackets are $S + \sum_{i=1}^{1} \sum_{k=i+1}^{2} HD_k + f(0)$, or $S + HD_2$. This is associated with producing in period one to meet demand in both period one and period two. When $t = 2$ and $j = 2$, demand in period two is to be met by production in period two. Recall from the first theorem that if production occurs in period two then ending inventory in the period one equals 0. Also recall from the third theorem that when ending inventory in period one equals 0 that the problem can be partitioned between period one and period two. It is already known that $f(1) = S$ represents the minimum cost solution through period one and so if production occurs in period two then $f(2) = S + f(1)$, or $2S$. Thus, the minimization operator outside of the outer brackets indicate that $f(2)$ equals the minimum of $S + HD_2$ and $2S$. The planning horizon theorem states that if $f(B)$ is minimized by $j = A \leq B$, then in any period $t > B$ it is sufficient to consider only $A \leq j \leq t$. Suppose that $f(2)$ is minimized by $j = 2$ with an associated cost of $2S$. The planning horizon theorem states that $f(3)$ is NOT minimized by $j = 1$. This theorem requires no additional assumptions. If $f(2)$ is minimized by $j = 2$ then $S + HD_2$ is greater than $2S$. It follows then that $S + HD_2 + 2HD_3$ must also be greater than $2S + HD_3$.

The planning horizon theorem also holds true for the current study. It is used in the example problem but is not used in the computer solution methodology presented below. The reason for this is that the solution methodology presented below is capable of optimally solving a much wider range of problems than just the DLPENP problem as defined in Chapter 3. Wagner (1960) demonstrates that if one ignores the planning horizon theorem, then Wagner and Whitin's solution methodology can be used to solve a very general class of problems while a method that relies on the planning horizon theorem is only appropriate for a relatively small subset of these problems. Although the current formulation does fall into this subset, it was decided by the author not to use the planning horizon theorem in the solution methodology presented below in order to assure that this generalizability of the method is not compromised.

Conditional Optimal Solutions

The DLPENP problem contains four types of decision variables. The solution methodology described in this chapter finds the optimal values of P_t, σ_t, and X_t while holding T fixed at various levels. Throughout this chapter, this is referred to as finding the optimal solution conditional upon the value of T. First, T is set equal to one and the conditional optimal solution is found. Next, T is set equal to two and the conditional optimal solution is found. This process continues for subsequent values of T. The nature of the demand curve, as well as the fact that T appears in the denominator of the objective function, ensures that beyond a certain point the objective function decreases as T increases. This point will be explained in greater detail below. The solution procedure terminates after comparing each conditional optimal solution value and selecting the value of T associated with the global optimal solution.

Solution Conditional upon T Equal to One

The solution process begins by setting the decision variables T and σ_1 equal to one. Note that the only circumstance where σ_1 is not equal to one is the degenerate case where the total size of the potential market is equal to zero. The gross profit function in period one is given by Expression 8.

$$(P_1 - V) * D_\beta * \left(\frac{P_1}{P_0}\right)^{-\eta} \tag{8}$$

Expression 8 is found by substituting Equation 3 into the first term of Expression 4. The optimal value of P_1 conditional upon T equal to one is found by taking the derivative of expression 8 with respect to P_1, setting the result equal to zero, and solving for P_1. This conditional optimal value of P_1 is given by Equation 9.

$$P_1 = \frac{V * \eta}{\eta - 1} \tag{9}$$

Similarly to Bass (1980), P_0 is equal to price in the first period. Therefore Equation 9 also defines P_0. Note that as stated in the Chapter 3 η is constrained in this study to be strictly greater than one. The value of D_1 associated with the previously determined value of P_1 is then calculated by substitution of Equations 1, 2, and 9 into Equation 3. Next, X_1, the final decision variable, is set equal to D_1. Now that the optimal values of all of the decision variables conditional on T equal to one have been determined, Expression 4 is used to find the optimal value conditional upon T equal to one.

Solution Conditional upon *T* Equal to Two

After the optimum solution to the DLPENP problem conditional upon T equal to one has been determined, the optimum solution conditional upon T equal to two is determined. When T is equal to one, there is only one possible conditional optimal solution. When T is equal to two, there are two possible conditional optimal solutions and both must be investigated. The first potential solution is associated with σ_1 and σ_2 both equal to one. When production occurs in period two, Equation 5 simplifies to $Inv_2 = X_2 - D_2$ and the problem can be partitioned into two sub-problems. Each of the two sub-problems can be solved by the method described in the previous section. In the first sub-problem, P_1 and D_1 are calculated and X_1 is set equal to D_1. In the second sub-problem P_2 and D_2 are calculated and X_2 is set equal to D_2. Recall that σ_1 and σ_2 have both already been set equal to one. This represents one of two solutions that may potentially be the optimal solution conditional upon T equal to two.

The other potential solution is associated with σ_1 equal to one and σ_2 equal to zero. When production does not occur in period two, beginning inventory in period two is not equal to zero and the problem cannot be partitioned into two sub-problems. When units are held in inventory before being sold, holding costs are accrued. In the current scenario, products sold in period two are produced in period one and held for one period. Gross profit is given by Expression 10 rather than Expression 8.

$$(P_2 - V - H) * D_\beta * \left(\frac{P_2}{P_1}\right)^{-\eta} \tag{10}$$

The optimal value of P_2 conditional upon T equal to two is found by taking the derivative of Expression 10 with respect to P_2 and solving for P_2. This is given by Equation 11.

$$P_2 = \frac{(V + H) * \eta}{\eta - 1} \tag{11}$$

When products are held for N_t periods before being sold in period t, gross profit is given by Expression 12. Note that both Expression 8 and 10 are instances of Expression 12.

$$(P_t - V - HN_t) * D_\beta * \left(\frac{P_t}{P_1}\right)^{-\eta} \tag{12}$$

The optimal value of P_t is found by taking the derivative of Expression 12 with respect to P_t and solving for P_t. This is given by Equation 13.

$$P_t = \frac{(V + HN_t) * \eta}{\eta - 1} \tag{13}$$

Once P_1 and P_2 have been calculated, D_1 and D_2 are calculated. Finally, X_1 is set equal to the sum of D_1 and D_2 and X_2 is set equal to zero. Recall that σ_1 has already been set equal to one and σ_2 has already been set equal to zero. This represents the second of two solutions that may potentially be the optimal solution conditional upon T equal to two.

One of the two solutions discussed in the previous two paragraphs is optimal conditional upon T equal to two. These two solutions are associated with different production schedules, in this case different values of σ_2. This leads to different profit margins in the second period and thus different optimal values of P_2. These different values of P_2 lead to different values of D_2. In the DLS problem, two different values of D_2 implies that two separate problem instances are being considered. However, in the DLPENP problem, the values of D_t are not determined *a priori*. They are deterministically related to the various input parameters and P_t. As such, despite the differences in D_2, these two solutions represent alternate potentially optimal solutions to the same problem instance. As such, the solution with the greatest profit per period is the optimal solution conditional upon T equal to two.

Solution Conditional upon
Subsequent values of T

It is known *a priori* that production will occur in period one. Stated another way, it is known that σ_1 must be equal to one. It is not known *a priori* whether or not production will occur in any other period in the global optimal solution. Thus, there are 2^{T-1} potential combinations of values for σ_t. The main purpose of this section is to show how some of these solutions can be disregarded as not potentially optimal.

There is one potentially optimal solution conditional upon T equal to one. There are two potentially optimal solutions conditional upon T equal to two. There are three, not four, potentially optimal solutions conditional upon T equal to three. When T is equal to three, the last period in which production occurs is period one, period two or period three.

If the last period in which production occurs is period one, it is known that σ_1, σ_2, and σ_3 are equal to one, zero, and zero respectively. For this scenario, N_1 equals zero, N_2 equals one, and N_3 equals two. It is further known that there is one optimal set of P_t and X_t conditional upon T equal to three and this set of σ_t. If the last period in which production occurs is period two, it is known that σ_1, σ_2, and σ_3 are equal to one, one, and zero respectively. For this scenario, N_1 and N_2 both equal zero and N_3 equals one. It is further known that there is one optimal set of P_t and X_t conditional upon T equal to three and this set of σ_t. If the last period in which production occurs is period three, it is known that σ_1 and σ_3 are both equal to one; however, it is unknown whether σ_2 is equal to zero or one.

When T is equal to three and the last period in which production occurs is period three, it is initially uncertain whether σ_2 is equal to zero or one. Regardless of the value of σ_2, it is know that σ_3 is equal to one. Further, it is known that the optimal beginning inventory in period three is equal to zero. This causes Equation 5 to simplify to $Inv_3 = X_3 - D_3$. This allows the problem to be partitioned into two sub-problems: one including the first two periods and the other including only period three. The partition between periods two and three causes the sub-problem concerning the first two periods to be identical to the determination of the optimal solution conditional upon T equal to two.

The use of the optimal solution conditional upon T equal to two reduces the number of potentially optimal solutions to the problem conditional upon T equal to three from four to three. It is known that the optimal values of σ_1 and σ_2, conditional upon T equal to two are equal to the optimal values of σ_1 and σ_2 conditional upon T equal to three and σ_3 equal to one. The partitioning problems into sub-problems, some of which are equivalent to previously solved problems, ensures that no more than T solutions are potentially optimal conditional upon a particular value of T. This solution methodology uses the exact same logic as that of Wagner and Whitin (1958).

The Determination of T

Demand for a product has growth, maturity, and decline phases. Beyond the value of t associated with peak demand, Equation 2 decreases asymptotically towards zero as t increases. As demand approaches zero, revenue and profit also decrease. Thus as T increases, the numerator of Expression 4 asymptotically approaches some constant value. As T increases, the denominator of Expression 4 increases. Therefore it is known *a priori* that beyond a certain value of T, Expression 4 decreases asymptotically towards zero as T increases. The optimal value of T is found by evaluating expression 4 for subsequent values of T until this asymptotic behavior appears and then selecting the optimal value of T from among those values already considered.

Computational Effort

Computational effort is a measure used to rate the relative difficulty of solving various mathematical problems using various methodologies. Suppose that there is a solution methodology that requires T individual calculations in order to determine the optimal value of Expression 4 conditional upon T. In other words, suppose one calculation must be made to maximize expression 4 conditional upon T equal to one, two must be made to optimize conditional upon T equal to two, etc. This hypothetical solution methodology requires $\sum_{t=1}^{T} t$ calculations to insure optimality. This is also equal to $\frac{T^2+T}{2}$ calculations. When discussing computational effort, all constant coefficients and lower terms are ignored. Thus, this hypothetical methodology would be said to be of order T^2.

This is written as O(T^2). Computational effort is used to sort mathematical problems and solutions into difficulty classes. The problem described above is an example of a polynomial time problem because there is a polynomial relationship between T and the required computational effort. In reality, many calculations must be performed in order to determine the optimal value of Expression 4 conditional upon T. However, this only impacts the constant coefficients, which are ignored when determining computational effort. Thus, the method described above is O(T^2).

The solution methodology requires a number of computations proportional to T raised to a power. This means that the DLPENP problem can be solved optimally in polynomial time and is therefore a member of the easiest class of problems in complexity theory.

An Example DLPENP Problem

The purpose of this section is to present an illustrative example problem. Table 3 contains the information known by the decision maker for an arbitrary problem. The values in Table 3 are selected from the interior of the ranges of values listed in Table 2. It is assumed that the decision maker has perfect information concerning these parameter values. This assumption will be explored further in Chapter 5.

Table 3 Parameter levels for an arbitrary problem instance.

Parameter	Factor Levels
a	15
b	0.05
m	10,000,000
η	3.5
H	0.3
S	20,000
V	70
I	20,000,000

When solving the DLPENP problem, first T is set equal to one and profit in period one is calculated. The only viable production schedule when T is one is to produce in period one. From Equation 13, the optimal value of P_1 is $\frac{70*3.5}{3.5-1}$, or \$98 per unit. Note that 98 is also the value of P_0. From Equations 1 through 3, D_1 can be calculated. It is equal to $10000000*\left[\frac{1-e^{-0.05}}{1+15*e^{-0.05}}-0\right]*\left(\frac{98}{98}\right)^{-3.5}$, or 31,942 units. From Expression 4, profit in period one is equal to $(98-70)*31942-20000-20000000$, or negative \$19,125,622. This is the maximum profit attainable when T is equal to one. It is also the maximum profit per period attainable when T is equal to one. Note that intermediate values such as 31,942 units may be rounded when presented but that the non-rounded versions of these numbers are used in any subsequent calculations. This pattern is continued throughout the example problem.

Here it is convenient to reformulate the profit equation as a recursive equation in a manner similar to Wagner and Whitin (1958b). Maximum profit for a given value of T is defined by Equation 14.

$$f(T)=\max\left\{\max_{1\le j<T}\left[\sum_{i=j}^{T}(P_i-V)*D_i-S-\sum_{i=j}^{T-1}\sum_{k=i+1}^{T}HD_k+f(j-1)\right],(P_T-V)*D_T-S+f(T-1)\right\} \quad 14$$

Equation 14 is only slightly different from the $f(t)$ equation used by Wagner and Whitin and can be derived in the same manner (Bellman, 1957). The difference is that it maximizes profit instead of minimizing cost. The reader is invited to verify that, when $T = 1$ and $j = 1$ in the current problem, Equation 14 also evaluates to negative \$19,125,622. Note that $f(0)$, the profit associated with meeting all demand through period zero, is negative I. Also note that when $j = T$ the terms outside the square brackets are used to evaluate Equation 14.

When T is equal to two, the two viable production schedules are to produce only in period one, and to produce in both period one and period two. When T is equal to two and production occurs only in the first period, the optimal price in period one is \$98 per unit as shown above and the optimal price in period two is $\frac{(70+0.3)*3.5}{3.5-1}$, or \$98.42 per unit. Demand in period one is equal to 31,942 units as shown above and demand in period two is equal to $10000000*\left[\frac{1-e^{-0.05*2}}{1+15*e^{-0.05*2}}-\frac{1-e^{-0.05}}{1+15*e^{-0.05}}\right]*\left(\frac{98.42}{98}\right)^{-3.5}$, or 32,865 units. Note that X_2 and Inv_2 are both equal to zero for the current plan. Therefore Equation 5 shows that Inv_1 is equal to D_2. Now that all of the elements of Expression 4 are known, it can be used to determine the average profit per period associated with this solution as shown: $\frac{((P_1-V)*D_1-S-H*D_2)+((P_2-V)*D_2)-I}{2}$, or negative \$9,100,731. The reader is invited to verify that, when $T = 2$ and $j = 1$ in the current problem, Equation 14 divided by T also evaluates to negative \$9,100,731.

When T is equal to two and production occurs in both periods, the optimal price is \$98 per unit in both periods as shown above. Demand in period one is equal to 31,942 units as shown above and demand in period two is equal to

$10000000*\left[\frac{1-e^{-0.05*2}}{1+15*e^{-0.05*2}}-\frac{1-e^{-0.05}}{1+15*e^{-0.05}}\right]*\left(\frac{98}{98}\right)^{-3.5}$, or 33,360 units. Note that according to Wagner and Whitin's (1958) first theorem that Inv_1 and Inv_2 are both equal to zero for the current plan. Now that all of the elements of Expression 4 are known, it can be used to determine the average profit per period associated with this solution as shown: $\frac{((P_1 - V)*D_1 - S) + ((P_2 - V)*D_2 - S) - I}{2}$, or negative \$9,105,764. The reader is invited to verify that, when T = 2 and j = 2 in the current problem, Equation 14 divided by T also evaluates to negative \$9,105,764. The maximum profit attainable when T is equal to two is the maximum of negative \$9,100,731 and negative \$9,105,764, or negative \$9,100,731. This is greater than negative \$19,125,622, the maximum profit per period attainable when T is equal to one. This means that the optimal value of T is greater than one.

When T is equal to three, up to three policies are potentially optimal. One of these policies is associated with production only during the first period. With the production schedule temporarily confined to period one only, Equation 13 can be used to determine the optimal price in each period. With prices determined, Equations 1 through 3 can be used to determine demand in each period. Once these values are known, Equation 14 can be used to determine average profit per period. Note that production only during period one is associated with j equal to one in Equation 14. The reader is invited to verify that when T = 3 and j = 1 in the current problem, Equation 14 divided by T evaluates to negative \$5,748,927. When T is equal to 3 and j is equal to two, production occurs during the first and second periods. The reader is invited to verify that when T = 3 and j = 2 in the current problem, Equation 14 divided by T evaluate to negative \$5,748,877. Once again, Equations 1 through 3 and 12 must be used before using Equation 14. When T is equal to 3 and j is equal to 3, production occurs during the first and third periods. The reader is invited to verify that when T = 3 and j = 3 in the current problem, Equation 14 divided by T evaluates to negative \$5,748,731. The maximum profit per period attainable when $T = 3$ is the maximum of negative \$5,748,927, negative \$5,748,877, and negative \$5,748,731, or negative \$5,748,731. This is greater than negative \$9,100,731, the maximum profit per period attainable when T is equal to two. This means that the optimal value of T is greater than two.

Note that in this example problem, production during all of the first three periods is not considered. Recall that according to Wagner and Whitin's (1958b) first and third theorems that if production occurs in the third period that the first two periods can be considered independently of the third and all subsequent periods. During the calculation of $f(2)$, it was found that profit over the first two periods is maximized by producing in period one and not producing in period two. Therefore, profit over the first three periods will not be maximized by production in all three periods.

When T is equal to four, up to four policies are potentially optimal. However, only two policies are potentially optimal for the current problem. As is shown above, $f(3)$ is maximized by $j = 3$. Therefore according to the planning horizon theorem, in any period greater than three it is sufficient to consider only j greater than or equal to three (Wagner and Whitin, 1958b). When T is equal to four and j is equal to three, production occurs during the first and third periods. The reader is invited to verify that when $T = 4$ and $j = 3$ in the current problem, Equation 14 divided by T evaluates to negative \$4,059,773. When T and j are both equal to four, production occurs during the first, third, and fourth periods. The reader is invited to verify that when $T = 4$ and $j = 4$ in the current problem, Equation 14 divided by T evaluates to negative \$4,062,067. The maximum profit per period attainable when $T = 4$ is negative \$4,059,773, which is greater than negative $5,748,731, the maximum profit per period associated with $T = 3$. This means that the optimal value of T is greater than 3.

The finding of $f(5)/5$ and each subsequent $f(T)/T$ is conceptually identical to the finding of the values of $f(1)$ through $f(4)/4$ above. The reader is invited to verify that $f(T)/T$ is at its maximum when $T = 81$. Further investigation of the current problem when $T = 81$ shows profit is maximized by producing in odd numbered periods from one to 19 as well as in all periods from 20 to 81. The profit maximizing prices are $89 per unit in periods in which production occurs and $98.42 per unit in other periods. Given this information it is trivial to determine the optimum production quantities schedule; however, this is omitted for space considerations. This globally optimal set of policies results in average profits per period of approximately $2,426,573 per period.

<u>Computer Solution Methodology</u>

The example problem described above as well as any other instance of this type of problem can be solved automatically using software. This section describes one such piece of software. The presentation includes a description of how the software goes about finding a solution as well as highlights of various software features designed to aid the decision maker. The software is written using visual basic and has been extensively validated by comparing its output to hand calculations for hundreds of problem instances. Figure 1 shows an image of the software's user interface after solving the example problem described above.

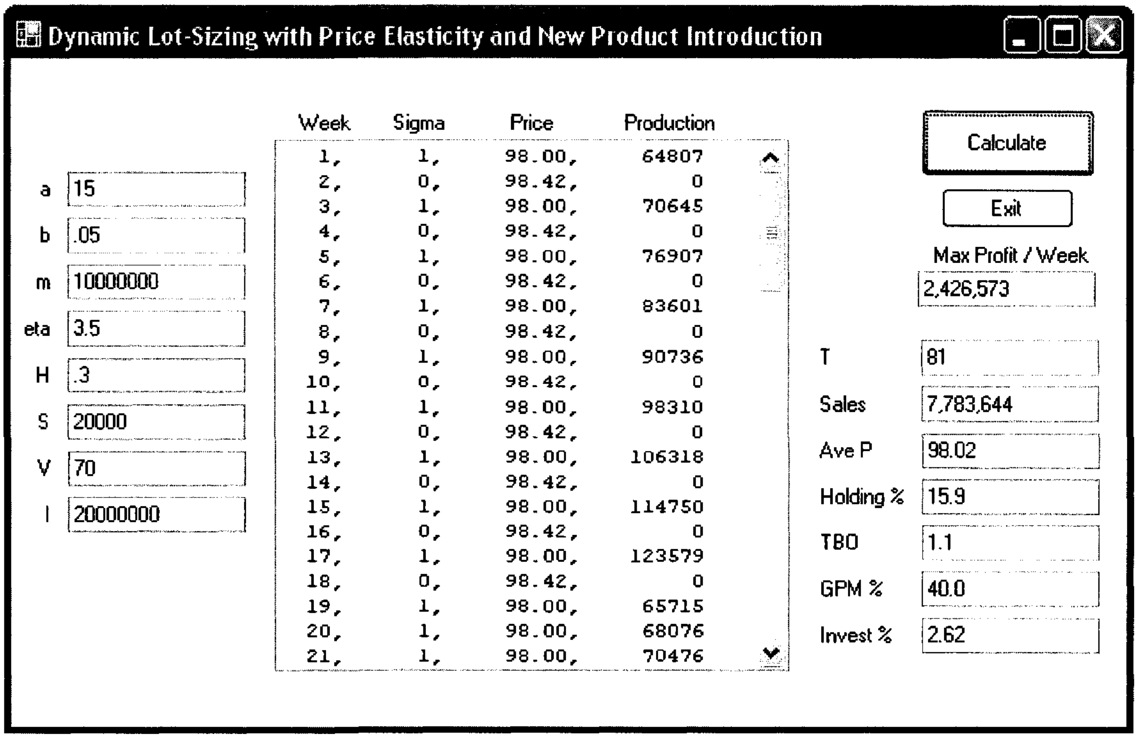

Figure 1 Image of the user interface.

When the user enters values for the various input variables and clicks *Calculate*, the software first checks to see that only numeric values are entered. If any non-numeric values are entered, the software returns an appropriate error message to the user instead of beginning the solution process. Once it is determined that only numeric values are entered for all input parameters, the main solution process begins.

The software uses several arrays, or vectors, in order to find the optimal solution to any given problem instance. Each element of these arrays is associated with a one-week time period. In order for the software to perform correctly, the number of elements in each array must be greater than the number of periods in the product's lifecycle. As the number of elements in each array increases the time required to solve the problem increases at an increasing rate. As such, there is motivation to keep the arrays as short as possible while still making them long enough to perform adequately. Fisher (1997) suggests that product lifecycles are between three months and one year while empirical evidence from the computer industry suggests that lifecycles tend to be around 3.7 years (Bayus, 1998). The software is currently set to solve problems with lifecycles of up to 300 weeks because it is expected that most of the problem instances generated during the sensitivity analysis in Chapter 5 will have lifecycles of less than 300 weeks. If a set of input parameters that will result in a lifecycle of over 300 weeks, the software returns an appropriate error message rather than attempting to solve the problem. Although the software requires that the number of elements in each array be determined *a priori*, it is designed so that it is easy to change this number. Therefore any problem instances that are not solved due to lifecycles exceeding the allowable range can be flagged and reexamined after increasing the allowable range.

The software uses a forward recursion dynamic programming algorithm. This means that the first solution generated by the software maximizes profit in the first period assuming that there will be no second or any other periods. Next the solution that maximizes profit during the first two periods assuming there will be no third period is determined. This iterative process is repeated generating solutions with successive "last" periods. The process terminates after determining the solution that maximizes average profit $T^{max} + 10$ where T^{max} is the maximum lifecycle that the software is designed to accept. The maximum profit per period for all values of T from one to $T^{max} + 10$ are compared. If the maximum of these maximum is associated with a value of T greater than T^{max}, an error message is returned as described in the previous paragraph. As described previously in this chapter, profit per period asymptotically approaches zero beyond a certain point. The stopping procedure described here insures that if this asymptotic behavior does not occur prior to T^{max} that the user is notified. Figure 2 is a flowchart describing the logic used by the software to find the optimal solution.

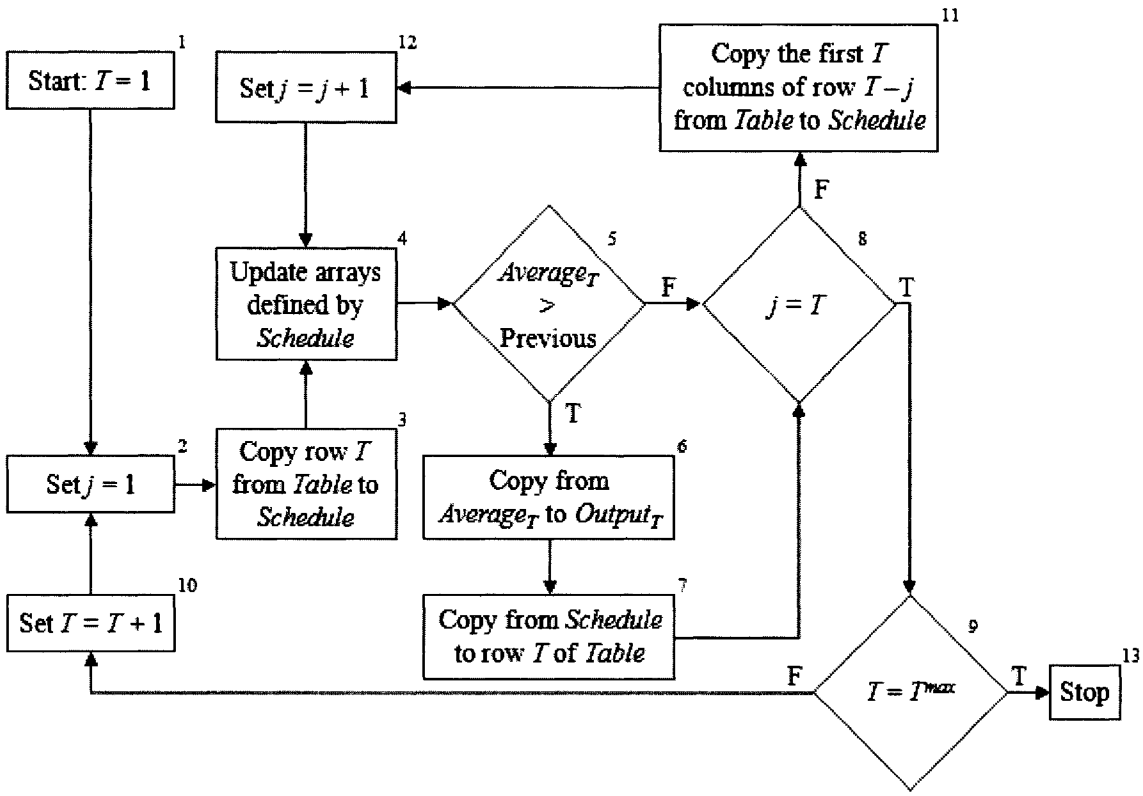

Figure 2 Flowchart of software logic.

As stated previously, the software begins by initializing T to one. This is shown in the first box in Figure 2. All other variables used in the software are also reinitialized to their starting values at this time. In the second box in Figure 2, j is set equal to one. The third box in Figure 2 copies a set of *Sigma* values from a two-dimensional array called *Table*. Each row of *Table* contains the values of *Sigma* associated with a production schedule. At startup, *Table* is initialized so that every row contains a one in the first column and zeros in every other column. *Schedule* is a one-dimensional array that contains the values of *Sigma* that is currently being used.

The arrays mentioned in the fourth box relate to the values of *N*, *P*, *D*, *Inventory*, and *Profit*, which are described in Chapter 3. Note that new values are calculated for each of these arrays every time a new set of *Sigma* values are accessed in the third box and that these are the optimal values conditional upon both the current value of *T* and the currently active values for *Sigma*.

The fifth box compares $Average_T$ to the previous value of $Average_T$. *Average* is a one-dimensional array that contains the average profits per period. $Average_T$ is the average profit per period over the first *T* periods. At initialization, every element of *Average* is set to negative infinity. The actual value of $Average_1$ is always greater than negative infinity. Therefore, the first time the fifth box is reached the statement is true so the lower path is followed to the sixth box.

In this sixth box, $Output_T$ is replaced by the current value of $Average_T$. *Output* is a one-dimensional array that is initialized to contain all zero values at startup and which contains the maximum possible values of *Average* when the program terminates. At termination, $Output_T$ is the maximum profit per period conditional upon the value of *T*. The largest value element of *Output* is the maximum profit per period, or the optimal value being sought. In the seventh box, the currently row of *Table* associated with the current value of *T* is replaced by the current contents of *Schedule* which is the best schedule found so far and the one associated with the new value of $Output_T$ from the previous box.

The eighth box is used to determine whether all of the potentially optimal solutions have been examined for each value of T. Recall that for a particular value of T, at most T solutions are potentially optimal (Wagner and Whitin, 1958b). As stated previously, the software initializes both T and j to one and only one solution is potentially optimal conditional upon T equal to one. This means that the first time the eighth box is reached the statement is true so the right path is followed to the ninth box. The ninth box is used to determine whether the maximum lifecycle stopping conditions have been met. The first time that the ninth box is reached the statement is false so the left path is followed to the tenth box.

The tenth box is used to increment the value of T. The first time the tenth box is reached T is incremented to two and the process described above is repeated until the eighth box is reached. When the eighth box is reached this second time T is equal to two while j is still equal to one. Thus the statement is false so the upper path is followed to the eleventh box.

The eleventh box is used to incorporate the appropriate value of $f(j - 1)$ into the system as described by Equation 14. As stated previously, $f(j - 1)$ is the profit associated with meeting all demand up through period j minus one. The software does not actually store the profit values associated with a particular solution but instead stores the values of sigma. When the profit is required, the sigma values are used to recalculate the other arrays as described above.

The twelfth box is used to increment the value of j. The first time the twelfth box is reached j is incremented to two and the process described above is repeated until the fifth box is reached. When the fifth box is reached this third time the value of $Average_2$

may or may not be greater than the previous value of $Average_2$. If the new value is greater than the old value, the path that leads through the sixth and seventh boxes is taken and the new optimal solution conditional upon T equal to two is recorded. If the new value is less than or equal to the old value, the path that leads directly to the eighth box is taken and the previous solution is maintained. In this case it is now known that the previous solution is optimal conditional upon T equal to two.

Regardless of which solution is maintained, when the eighth box is reached this third time T and j are both equal to two. Thus the statement is true so the right path is followed to the ninth, tenth, and second boxes. The value of T is incremented to three and j is set equal to one. At this point three passes are made through the programming logic with j incremented between each pass and the best of these three solutions is maintained. Then T is incremented to four and four passes are made through the programming logic with j set equal to one, two, three, and four and the best of these solutions is maintained. This process is continued until T reaches T^{max} at which point the software terminates.

Prior to termination, the software makes several additional passes through the program logic and additional values of $Average_T$ are calculated and compared to those generated during the main program sequence. If any of these are superior to the best solution generated prior to $T = T^{max}$ then it is known that the optimal value of T exceeds the bounds acceptable to the software and an error message is generated. As stated previously, the software is designed so that it is easy to set T^{max} to an arbitrarily large value. The only problem with doing so is that larger values of T^{max} require additional processing time.

CHAPTER 5

SENSITIVITY ANALYSIS

Sensitivity analysis is concerned with the response of a system "under various scenarios related to perturbations in different problem parameters" (Bazaraa, Sherali, & Shetty, 1993, p22). This chapter focuses on both the relationships between the profit per period and the various input parameters and the relationships between T and the various input parameters. The sensitivity of the objective value to changes in the input values is analyzed under both perfect and imperfect information cases; the sensitivity of T to changes in the input values is analyzed under perfect information. The model presented in Chapter 3 is completely deterministic; thus, all statistical inference methods are inappropriate. This is because those methods rely on the analysis of error variance over multiple replications. The deterministic nature of the model means that it does not contain any error variance and multiple replications will always result in identical outcomes.

The fact that the model is deterministic led the author to attempt to analyze the relationships between the various input parameters and profit analytically, by solving the profit function for the various parameters. Unfortunately, due to the complexity of the profit function and the non-linearities associated with the σ decision variables, this approach is intractable. Despite the intractability of solving the profit function for a particular parameter, it is relatively straightforward to determine the average profit per period associated with any particular set of input parameter values. Therefore, even though the equation of the curve describing the relationship between profit and a parameter cannot be determined, the shape of the curve is relatively easy to find.

Interaction Effects under Perfect Information

In this section, the relationships between various pairs of input parameters and both profit per period and optimal product lifecycle length are investigated in the presence of perfect information. Each data point in this section is associated with the optimal solution to a problem instance as generated by the method described in Chapter 4.

Equation 1 contains two input parameters, a and b. The form of Equation 1 suggests that there may be an interaction between a and b. Equation 2 contains only a single input parameter, m. However it also includes $F(t)$, which is a function of both a and b. This suggests that m may have an interaction with either a or b. Similarly, Equation 3 includes input parameter η and suggests the possibility of interactions among a, b, m, and η. Expression 4 includes the remaining input parameters. The relationships shown in Expression 4 do not suggest the presence of interactions among the remaining input parameters or between this set of parameters and the previous set.

Equations 1 through 3 suggest the potential for a four-way interaction among a, b, m, and η. It is graphically impossible to display a four-way interaction so instead a series of two-way interactions are displayed in this section. This requires the caveat that the entirety of the interrelationships between the various input parameters may not be shown.

Note that the vertical axes of Figures 3, 5, 7, 9, 11, and 13 have the same scale even though the curves on these figures have quite different ranges. The reason for this is that it eases comparison between the various figures, thus allowing the reader to get a better idea of the relative differences in the sensitivity of profit to the various input parameters. The vertical axes of Figures 4, 6, 8, 10, 12, and 14 have the same scale to show the sensitivity of the product introduction interval to the various input parameters.

Interaction Between *a* and *b*

Table 3 from Chapter 4 lists parameter levels for a problem instance. Figures 3 and 4 show the impact of varying the values of the input parameters *a* and *b* from the values found in Table 3. The values of the remaining parameters are equal to the values found in Table 3. Figure 3 shows that when profit per period is the dependent variable, there is a slight interaction between *a* and *b*.

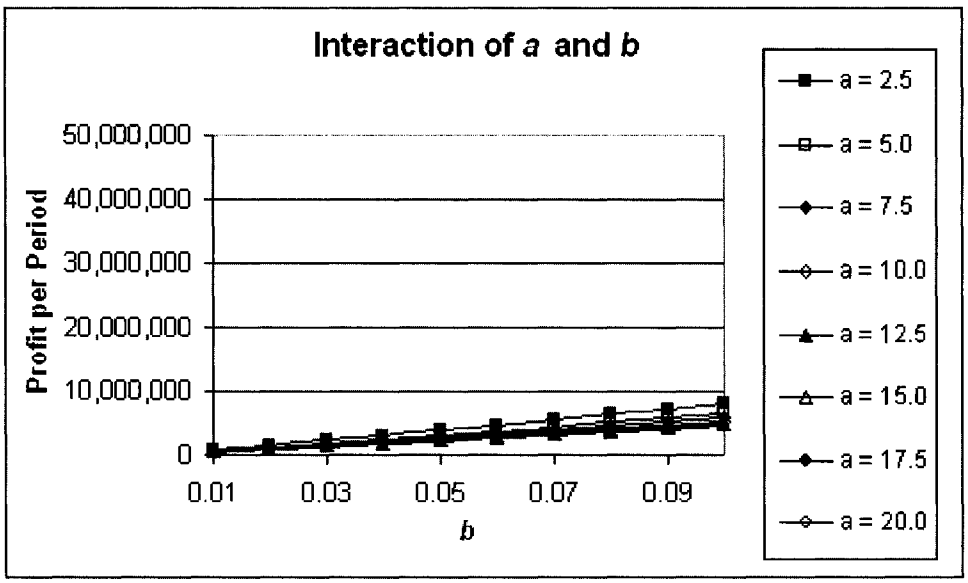

Figure 3 Interaction of *a*, *b*, and profit.

Figure 4 shows that when product introduction interval is the dependent variable, there is also a slight interaction between *a* and *b*.

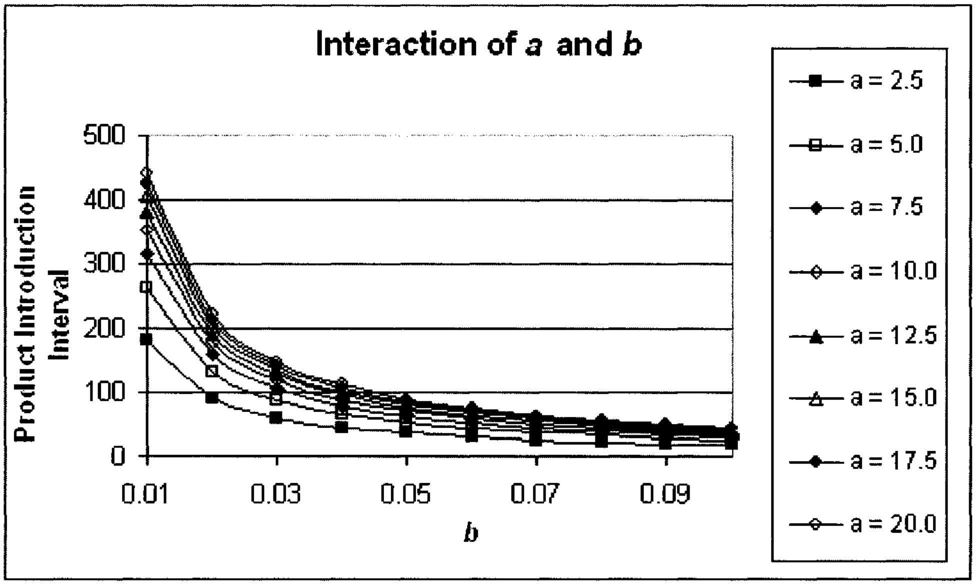

Figure 4 Interaction of *a*, *b*, and *T*.

Interaction Between *a* and *m*

Figures 5 and 6 show the impact of varying the values of the input parameters *a* and *m* from the values found in Table 3. The values of the remaining parameters are equal to the values found in Table 3. Figure 5 shows that when profit per period is the dependent variable, there is a slight interaction between *a* and *m*.

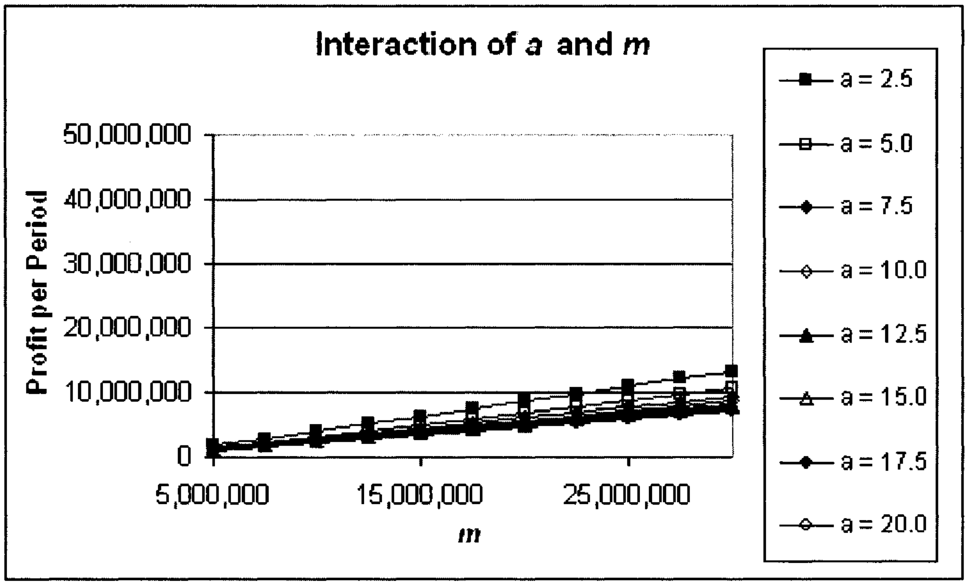

Figure 5 Interaction of *a*, *m*, and profit.

Figure 6 shows that when product introduction interval is the dependent variable, there is no noticeable interaction between *a* and *m*.

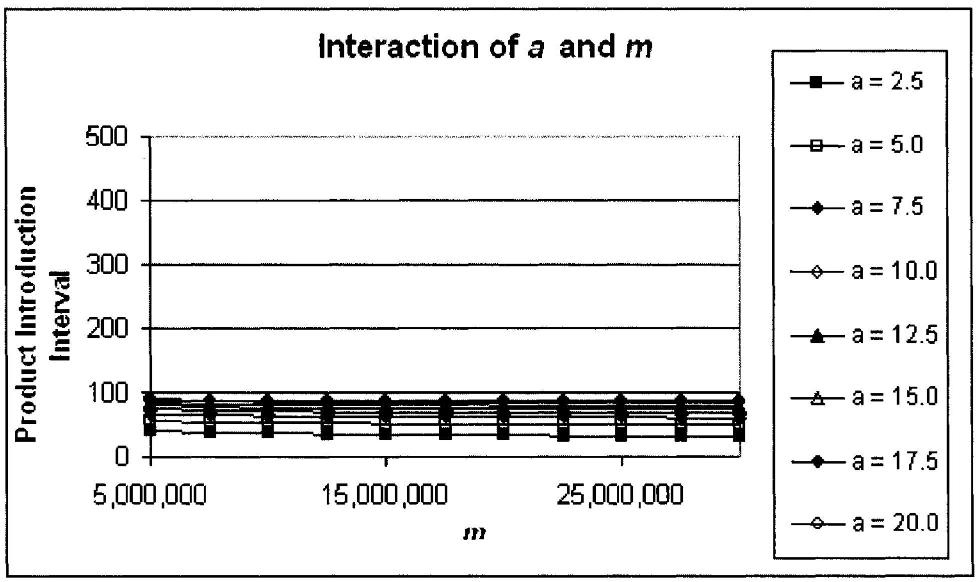

Figure 6 Interaction of *a*, *m*, and *T*.

Interaction Between a and η

Figures 7 and 8 show the impact of varying the values of the input parameters a and η from the values found in Table 3. The values of the remaining parameters are equal to the values found in Table 3. Figure 7 shows that when profit per period is the dependent variable, there is no noticeable interaction between a and η.

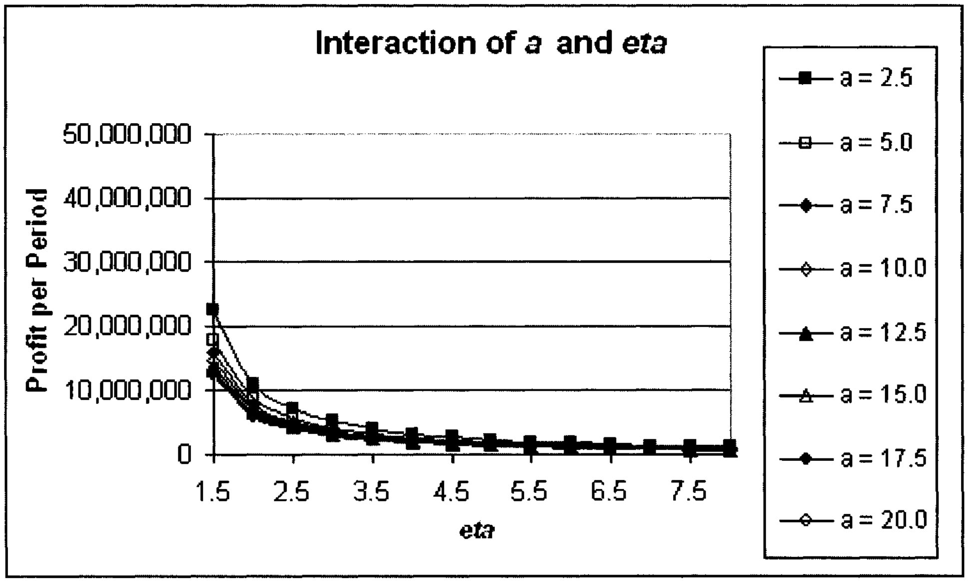

Figure 7 Interaction of a, η, and profit.

Figure 8 shows that when product introduction interval is the dependent variable, there is no noticeable interaction between a and η.

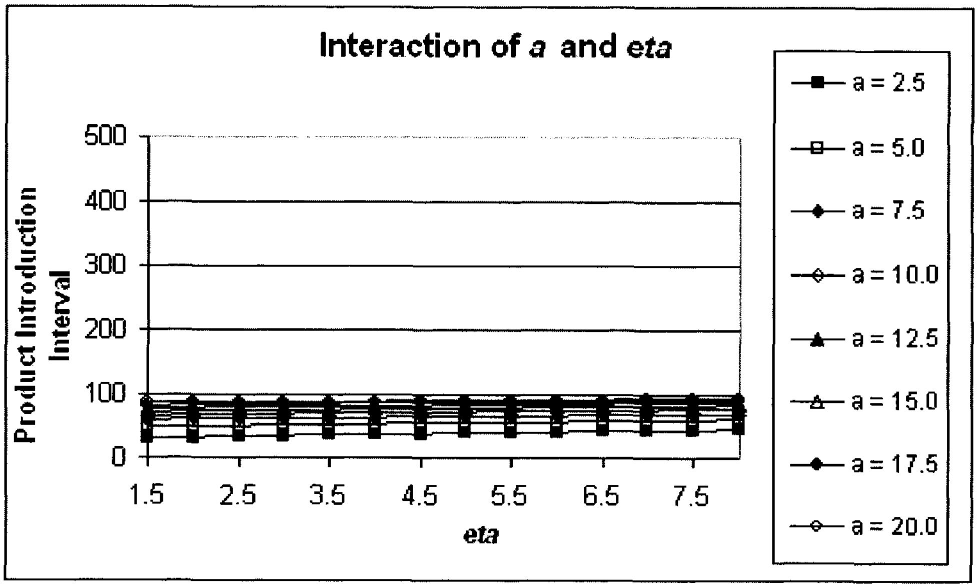

Figure 8 Interaction of a, η, and T.

Interaction Between b and m

Figures 9 and 10 show the impact of varying the values of the input parameters b and m from the values found in Table 3. The values of the remaining parameters are equal to the values found in Table 3. Figure 9 shows that when profit per period is the dependent variable, there is a strong interaction between b and m.

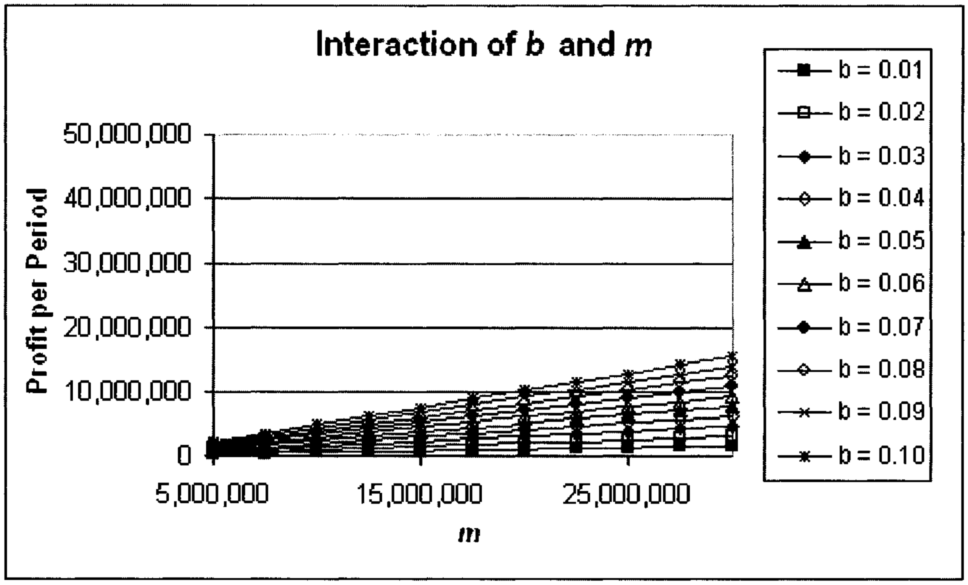

Figure 9 Interaction of *b*, *m*, and profit.

Figure 10 shows that when product introduction interval is the dependent variable, there is no noticeable interaction between *b* and *m*.

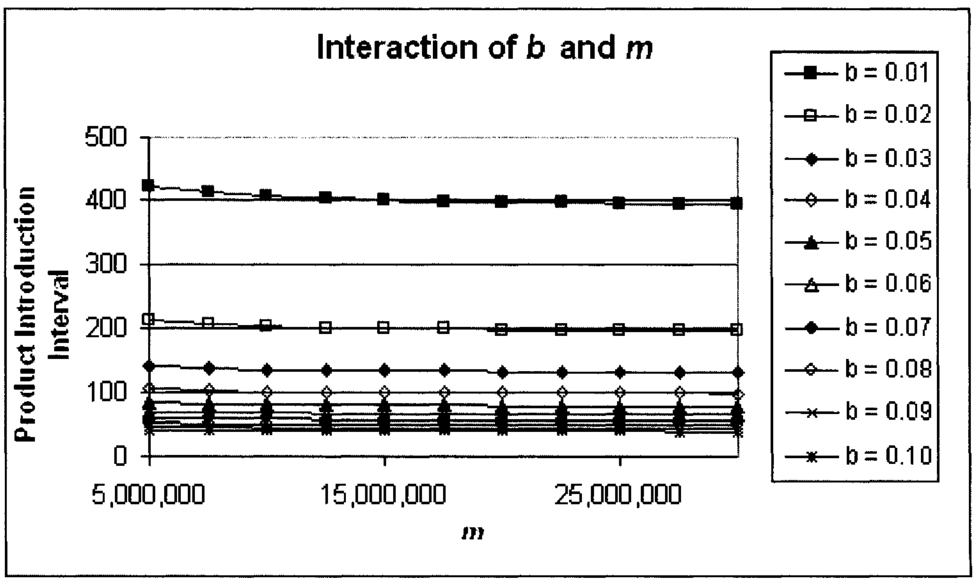

Figure 10 Interaction of *b*, *m*, and *T*.

Interaction Between b and η

Figures 11 and 12 show the impact of varying the values of the input parameters b and η from the values found in Table 3. The values of the remaining parameters are equal to the values found in Table 3. Figure 11 shows that when profit per period is the dependent variable, there is a strong interaction between b and η.

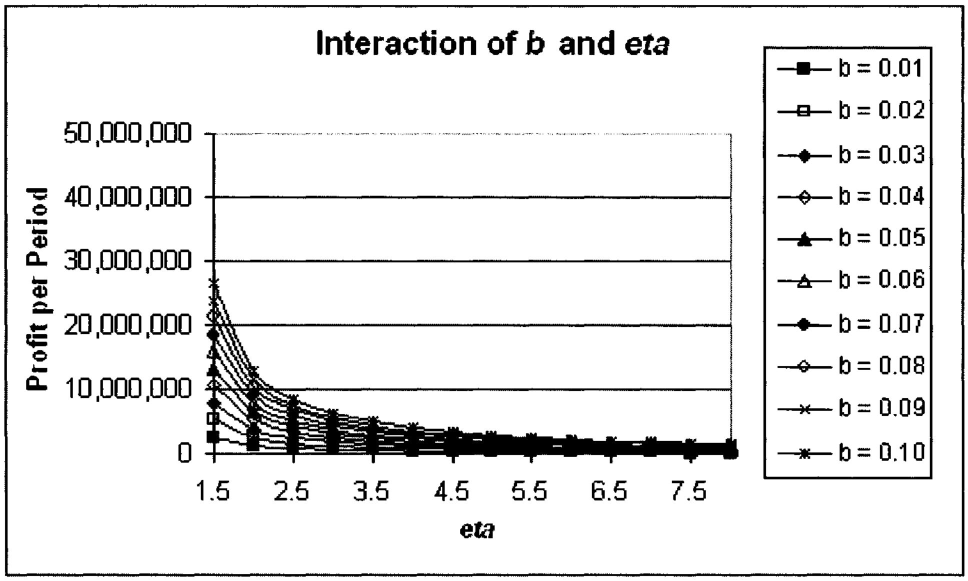

Figure 11 Interaction of b, η, and profit.

Figure 12 shows that when product introduction interval is the dependent variable, there is a slight interaction between b and η.

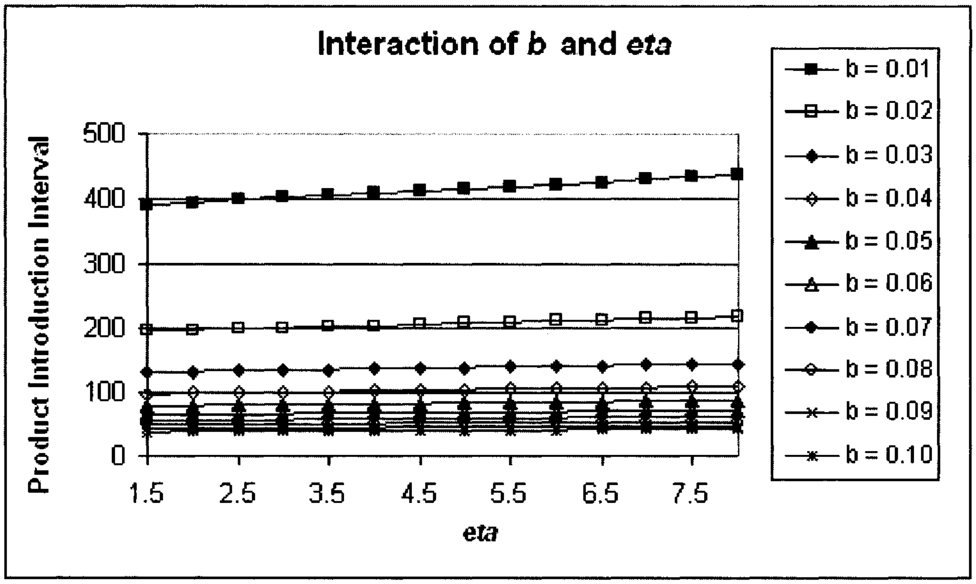

Figure 12 Interaction of b, η, and T.

<u>Interaction Between m and η</u>

Figures 13 and 14 show the impact of varying the values of the input parameters m and η from the values found in Table 2. The values of the remaining parameters are equal to the values found in Table 2. Figure 13 shows that when profit per period is the dependent variable, there is a strong interaction between m and η.

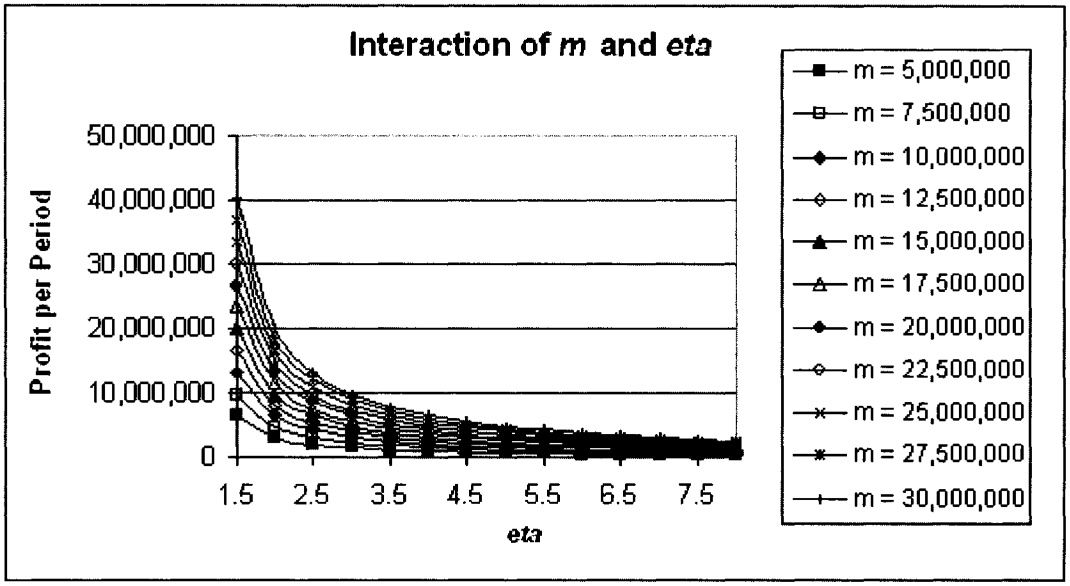

Figure 13 Interaction of *m*, η, and profit.

Figure 14 shows that when product introduction interval is the dependent variable, there is no noticeable interaction between *m* and η.

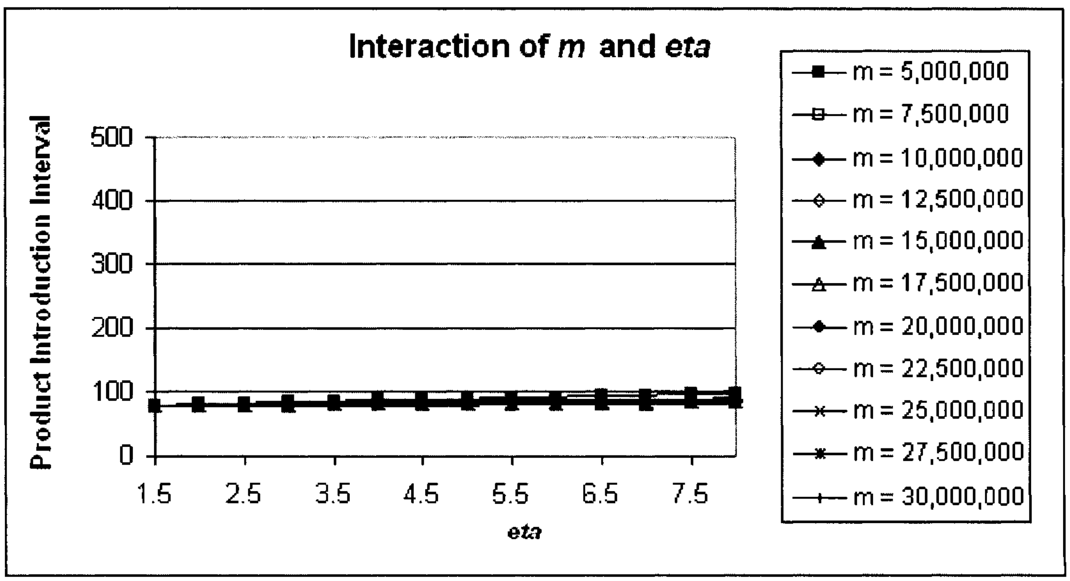

Figure 14 Interaction of *m*, η, and *T*.

Main Effects

In this section, the main effect of each input parameter on profit is investigated. First the interaction effects discovered in the previous section must be considered. Based on Figures 3 and 4, it appears that there is an interaction between parameters a and b so a is investigated at high and low values of b. Similarly, b is investigated at high and low values of a. The other interactions that are considered in the following analysis are between a and m, b and m, b and η, and m and η. The interaction between a and η is not considered because neither Figure 7 nor Figure 8 suggest that such an interaction exists.

Two curves are shown in each of the figures from Figures 15 to Figure 70. In each figure, one curve is marked with squares and is associated with perfect information. The other is marked with diamonds and is associated with imperfect information. In all cases, the horizontal axis represents the single parameter that is being manipulated and the vertical axis represents either profit per period, for odd numbered figures, or product introduction interval, for even numbered figures. Similarly to the even and odd numbered figures in the previous section, the even and odd numbered figures in this section all share the same vertical scale to ease comparison between figures.

In the odd numbered figures, where profit per period is the dependent variable, the two curves in each figure are sometimes collocated but never cross. At every point along the horizontal axis, the curve associated with perfect information is as high as or higher than the curve associated with imperfect information. In the even numbered figures, where product introduction interval is the dependent variable, the "curve" associated with imperfect information is a horizontal line. This is because the dependent variable is the decision variable T, and all decision variables are fixed constant in the imperfect information case. In the perfect information case, the decision variables are reoptimized at every point along the curve and thus the dependent variable varies as well.

One final point must be addressed before any numerical results are presented. It is assumed that the decision bases order size decisions on desired inventory position rather than strictly adhering to the planned values for X. For example, suppose for a set of input parameters that an optimal production plan exists such that beginning inventory in period t is zero and production in period t is 100 units. If due to a misspecification of some input parameter the beginning inventory in period t is 10 units then production in period t is set to 90 units instead of 100 units so that the inventory level after production is at the desired position. It is assumed that, aside from using inventory position to determine production volumes, the decision maker does nothing during the T weeks being considered to update or correct their estimation of the various inaccurate parameters. Thus, if in the above example there had been 99 units on hand at the beginning of period t, the decision maker would have no choice but to incur the costs associated with a setup in order to produce the single unit required to bring the inventory position up to 100.

The Main Effects of a

In this section, a ranges from 2.5 to 20 in increments of 2.5 while η, H, S, V, and I are held constant at 3.5, 0.3, 20000, 70, and 20000000 respectively. In this section b takes on values of 0.01 and 0.1 and m takes on values of 5,000,000 and 30,000,000. At every point along the perfect information curves of Figures 15 through 22 the decision maker knows the value of a and behaves optimally whereas at every point along the imperfect information curves the decision maker believes the value of a to be 15 and behaves in a manner that would be optimal if that were true.

Figures 15 and 16 are associated with *b* equal to 0.01 and *m* equal to 5,000,000. The perfect information curve of Figure 15 slopes downward from $341,264 per week when *a* is equal to 2.5 to $198,607 per week when *a* is equal to 20. This suggests that maximum profit per period is negatively related to *a*. The imperfect information curve slopes upward from $132,617 per week when *a* is equal to 2.5 to $211,350 when *a* is equal to 15 and then slopes downward to $188,240 when *a* is equal to 20. The relative closeness of the two curves suggests that, when *b* is equal to 0.01 and *m* is equal to 5,000,000, profit per period is not greatly influenced by misspecification of *a*.

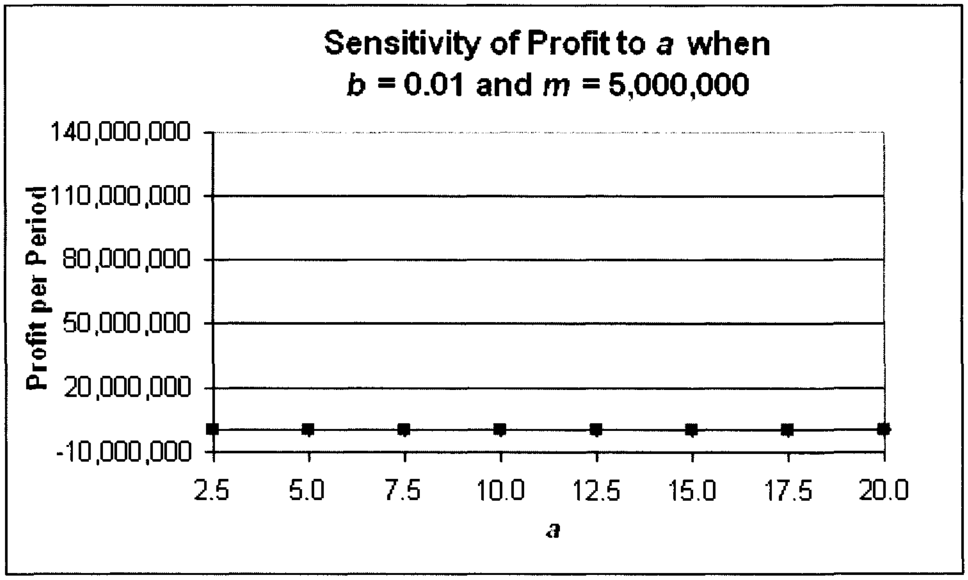

Figure 15 Sensitivity of profit to *a* when *b* = 0.01 and *m* = 5,000,000.

The perfect information curve of Figure 16 slopes upward from 210 weeks when *a* is equal to 2.5 to 459 weeks when *a* is equal to 20. This suggests that the product introduction interval is positively related to *a*. The imperfect information curve is constant at 423 weeks because this is the optimal product introduction interval when *a* is equal to 15. The slope of the perfect information curve suggests that, when *b* is equal to

0.01 and m is equal to 5,000,000, the product introduction interval is greatly influenced by the value of a.

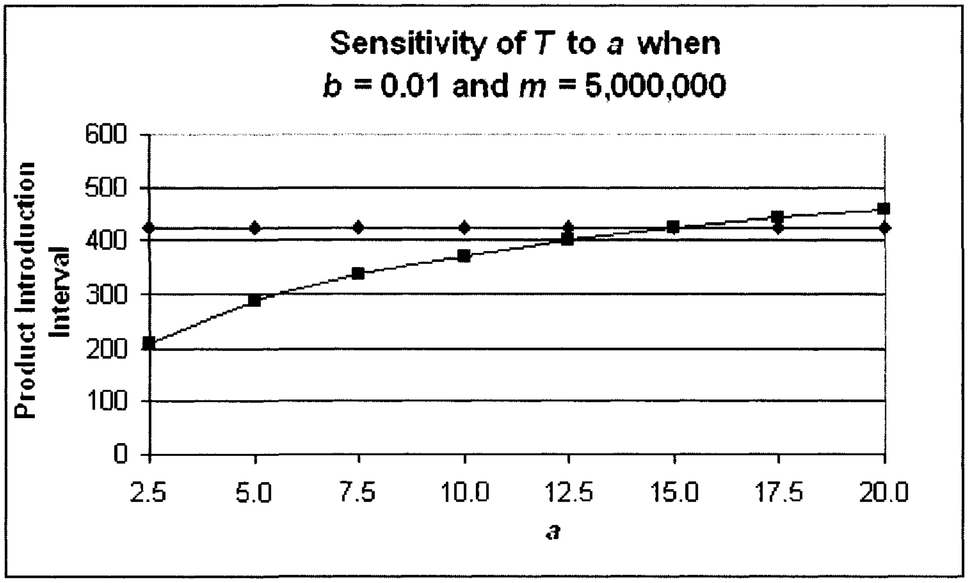

Figure 16 Sensitivity of T to a when $b = 0.01$ and $m = 5{,}000{,}000$.

Figures 17 and 18 are associated with b equal to 0.01 and m equal to 30,000,000. The perfect information curve of Figure 17 slopes downward from \$2,640,067 per week when a is equal to 2.5 to \$1,451,611 per week when a is equal to 20. This suggests that maximum profit per period is negatively related to a. The imperfect information curve slopes upward from \$1,132,499 per week when a is equal to 2.5 to \$1,549,570 when a is equal to 15 and then slopes downward to \$1,402,697 when a is equal to 20. The relative closeness of the two curves suggests that, when b is equal to 0.01 and m is equal to 30,000,000, profit per period is not greatly influenced by misspecification of a.

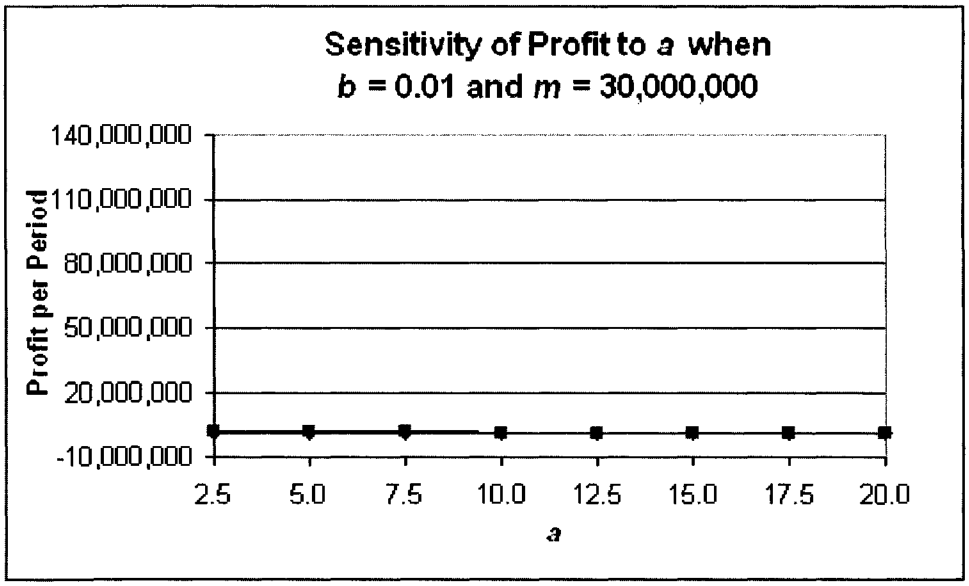

Figure 17 Sensitivity of profit to a when $b = 0.01$ and $m = 30{,}000{,}000$.

The perfect information curve of Figure 18 slopes upward from 155 weeks when a is equal to 2.5 to 431 weeks when a is equal to 20. This suggests that the product introduction interval is positively related to a. The imperfect information curve is constant at 394 weeks because this is the optimal product introduction interval when a is equal to 15. The slope of the perfect information curve suggests that, when b is equal to 0.01 and m is equal to 30,000,000, the product introduction interval is greatly influenced by the value of a.

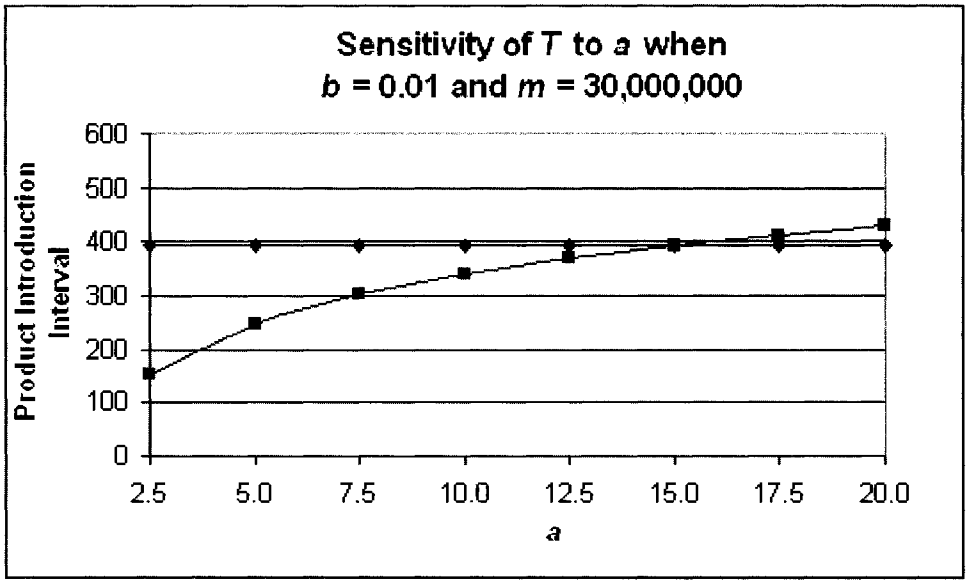

Figure 18 Sensitivity of T to a when $b = 0.01$ and $m = 30{,}000{,}000$.

Figures 19 and 20 are associated with b equal to 0.1 and m equal to 5,000,000. The perfect information curve of Figure 19 slopes downward from $3,505,486 per week when a is equal to 2.5 to $2,054,722 per week when a is equal to 20. This suggests that maximum profit per period is negatively related to a. The imperfect information curve slopes upward from $1,357,321 per week when a is equal to 2.5 to $2,184,834 when a is equal to 15 and then slopes downward to $1,955,878 when a is equal to 20. The relative closeness of the two curves suggests that, when b is equal to 0.1 and m is equal to 5,000,000, profit per period is not greatly influenced by misspecification of a.

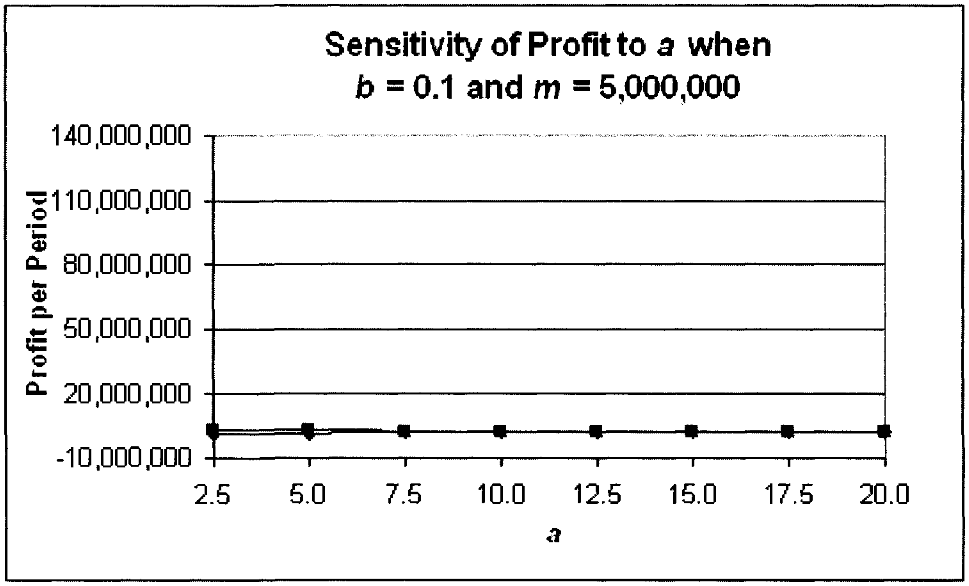

Figure 19 Sensitivity of profit to *a* when *b* = 0.1 and *m* = 5,000,000.

The perfect information curve of Figure 20 slopes upward from 21 weeks when *a* is equal to 2.5 to 46 weeks when *a* is equal to 20. This suggests that the product introduction interval is positively related to *a*. The imperfect information curve is constant at 42 weeks because this is the optimal product introduction interval when *a* is equal to 15. The near horizontal nature of the perfect information curve suggests that, when *b* is equal to 0.1 and *m* is equal to 5,000,000, the product introduction interval is not greatly influenced by the value of *a*.

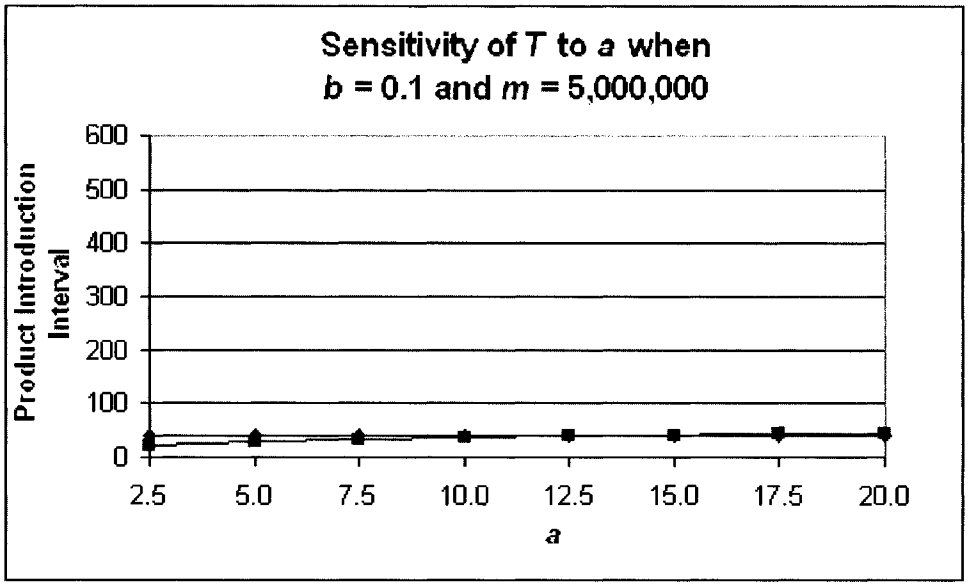

Figure 20 Sensitivity of *T* to *a* when *b* = 0.1 and *m* = 5,000,000.

Figures 21 and 22 are associated with *b* equal to 0.1 and *m* equal to 30,000,000. The perfect information curve of Figure 21 slopes downward from $26,559,927 per week when *a* is equal to 2.5 to $14,670,942 per week when *a* is equal to 20. This suggests that maximum profit per period is negatively related to *a*. The imperfect information curve slopes upward from $11,009,623 per week when *a* is equal to 2.5 to $15,653,556 when *a* is equal to 15 and then slopes downward to $14,176,136 when *a* is equal to 20. The relative closeness of the two curves suggests that, when *b* is equal to 0.1 and *m* is equal to 30,000,000, profit per period is not greatly influenced by misspecification of *a*. However, the distance between the two curves to the left of Figure 21 suggests that grossly overestimating *a* can lead to a noticeable reduction in profit.

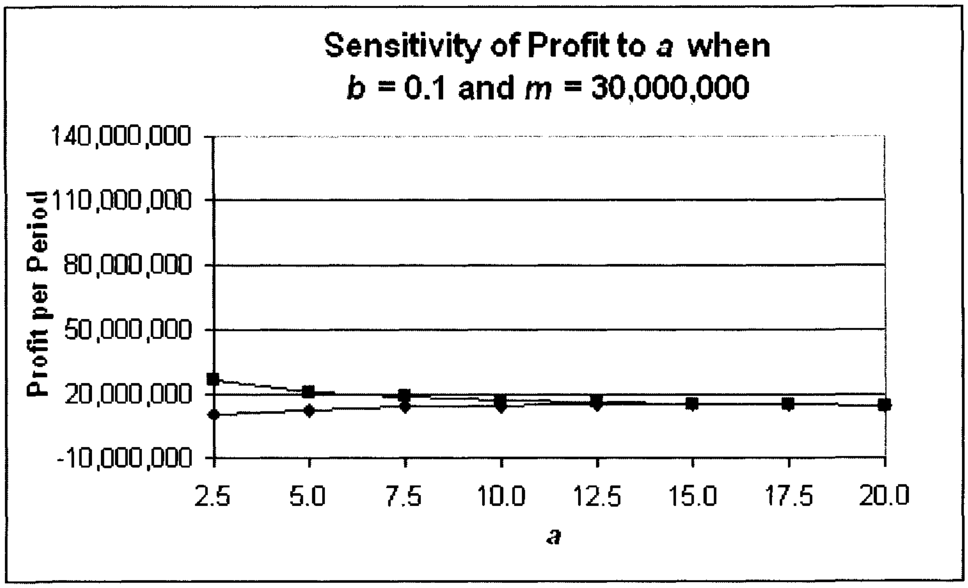

Figure 21 Sensitivity of profit to *a* when *b* = 0.1 and *m* = 30,000,000.

The perfect information curve of Figure 22 slopes upward from 16 weeks when *a* is equal to 2.5 to 43 weeks when *a* is equal to 20. This suggests that the product introduction interval is positively related to *a*. The imperfect information curve is constant at 39 weeks because this is the optimal product introduction interval when *a* is equal to 15. The near horizontal nature of the perfect information curve suggests that, when *b* is equal to 0.1 and *m* is equal to 30,000,000, the product introduction interval is not greatly influenced by the value of *a*.

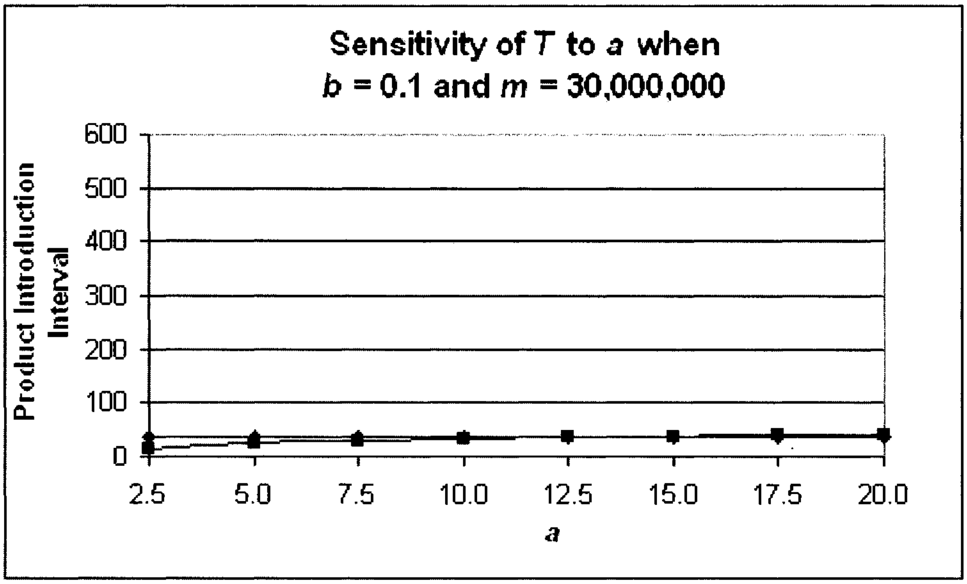

Figure 22 Sensitivity of T to a when $b = 0.1$ and $m = 30{,}000{,}000$.

Comparison of Figures 15, 17, 19, and 21 suggests that profit is negatively related to a regardless of the values of b and m. Comparison of Figures 16, 18, 20, and 22 suggests that although the product introduction interval is always positively related to a, this relationship is only strong for small values of b.

The Main Effects of b

In this section, b ranges from 0.01 to 0.1 in increments of 0.01 while H, S, V, and I are held constant at 0.3, 20000, 70, and 20000000 respectively. In this section, a takes on values of 2.5 and 20, m takes on values of 5,000,000 and 30,000,000, and η takes on values of 1.5 and 8. At every point along the perfect information curves of Figures 23 through 38, the decision maker knows the value of b and behaves optimally whereas at every point along the imperfect information curves the decision maker believes the value of b to be 0.05 and behaves in a manner that would be optimal if that were true.

Figures 23 and 24 are associated with *a* equal to 2.5, *m* equal to 5,000,000, and η equal to 1.5. The perfect information curve of Figure 23 slopes upward from $2,183,897 per week when *b* is equal to 0.01 to $21,933,088 per week when *b* is equal to 0.1. This suggests that maximum profit per period is positively related to *b*. The imperfect information curve slopes upward from $1,324,116 per week when *b* is equal to 0.01 to $10,957,272 when *b* is equal to 0.05 and then slopes downward to $10,719,432 when *b* is equal to 0.1. The relative closeness of the two curves suggests that, when *a* is equal to 2.5, *m* is equal to 5,000,000, and η is equal to 1.5, profit per period is not greatly influenced by misspecification of *b*. However, the distance between the two curves to the right of Figure 23 suggests that grossly underestimating *b* can lead to a noticeable reduction in profit.

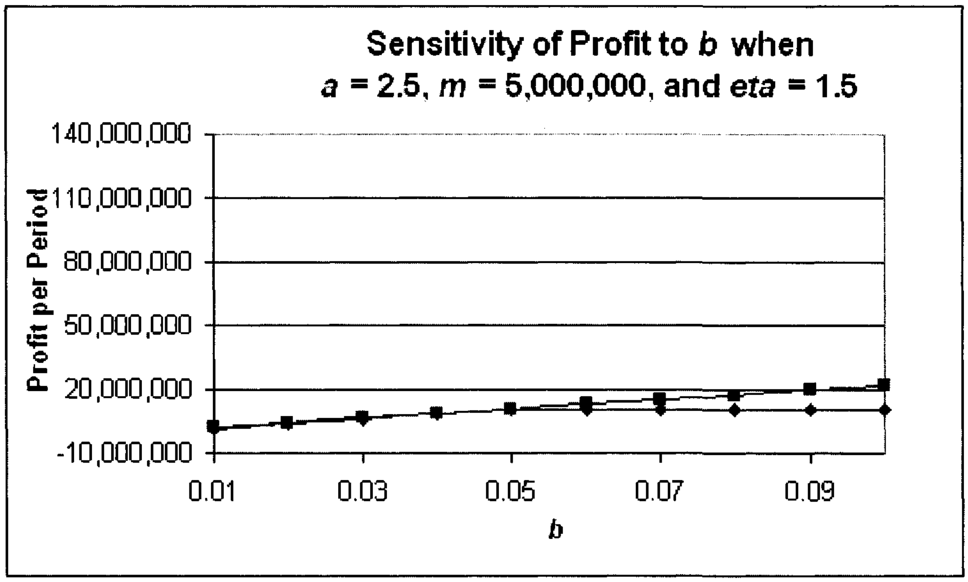

Figure 23 Sensitivity of profit to *b* when *a* = 2.5, *m* = 5,000,000, and η = 1.5.

The perfect information curve of Figure 24 slopes downward from 159 weeks when *b* is equal to 0.01 to 16 weeks when *b* is equal to 0.1. This suggests that the product introduction interval is negatively related to *b*. The imperfect information curve is constant at 32 weeks because this is the optimal product introduction interval when *b* is equal to 0.05. The slope of the perfect information curve suggests that, when *a* is equal to 2.5, *m* is equal to 5,000,000, and η is equal to 1.5, the product introduction interval is somewhat influenced by the value of *b*.

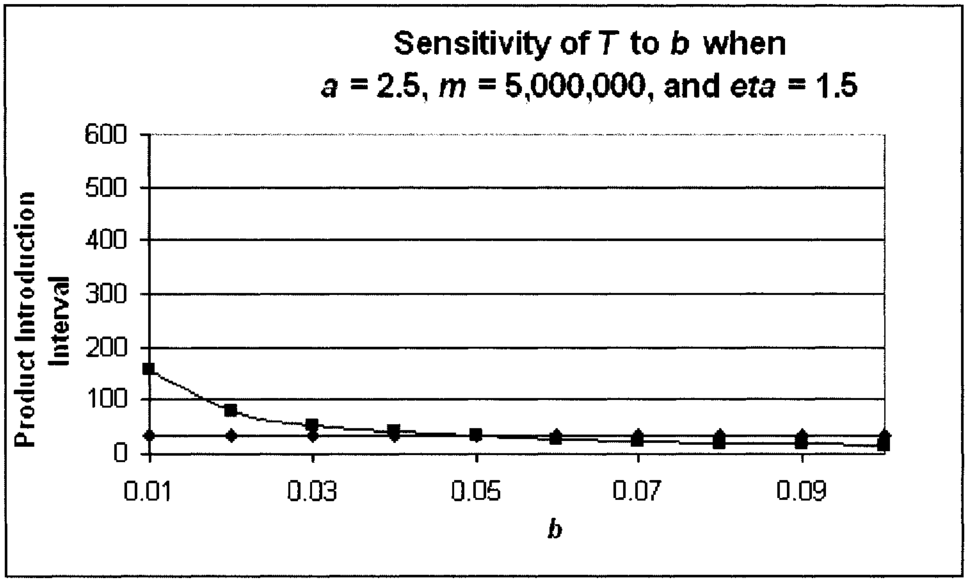

Figure 24 Sensitivity of *T* to *b* when $a = 2.5$, $m = 5{,}000{,}000$, and $\eta = 1.5$.

Figures 25 and 26 are associated with *a* equal to 2.5, *m* equal to 5,000,000, and η equal to 8. The perfect information curve of Figure 25 slopes upward from \$63,522 per week when *b* is equal to 0.01 to \$721,745 per week when *b* is equal to 0.1. This suggests that maximum profit per period is positively related to *b*. The imperfect information curve slopes upward from negative \$276,602 per week when *b* is equal to 0.01 to \$352,066 when *b* is equal to 0.05 and then slopes downward to \$103,103 when *b* is equal to 0.1. The relative closeness of the two curves suggests that, when *a* is equal to 2.5, *m* is equal to 5,000,000, and η is equal to 8, profit per period is not greatly influenced by misspecification of b.

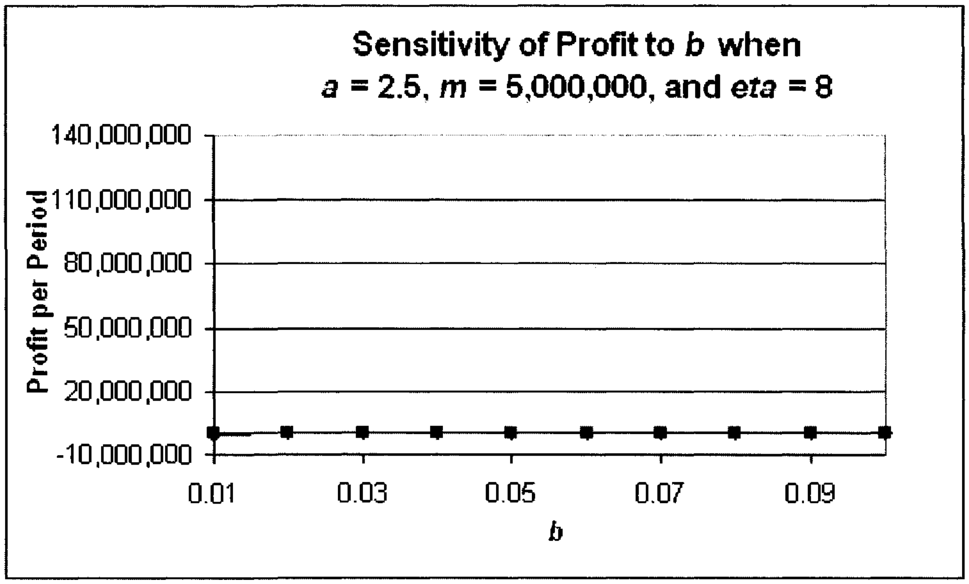

Figure 25 Sensitivity of profit to b when a = 2.5, m = 5,000,000, and η = 8.

The perfect information curve of Figure 26 slopes downward from 295 weeks when b is equal to 0.01 to 29 weeks when b is equal to 0.1. This suggests that the product introduction interval is negatively related to b. The imperfect information curve is constant at 58 weeks because this is the optimal product introduction interval when b is equal to 0.05. The slope of the perfect information curve suggests that, when a is equal to 2.5, m is equal to 5,000,000, and η is equal to 8, the product introduction interval is greatly influenced by the value of b.

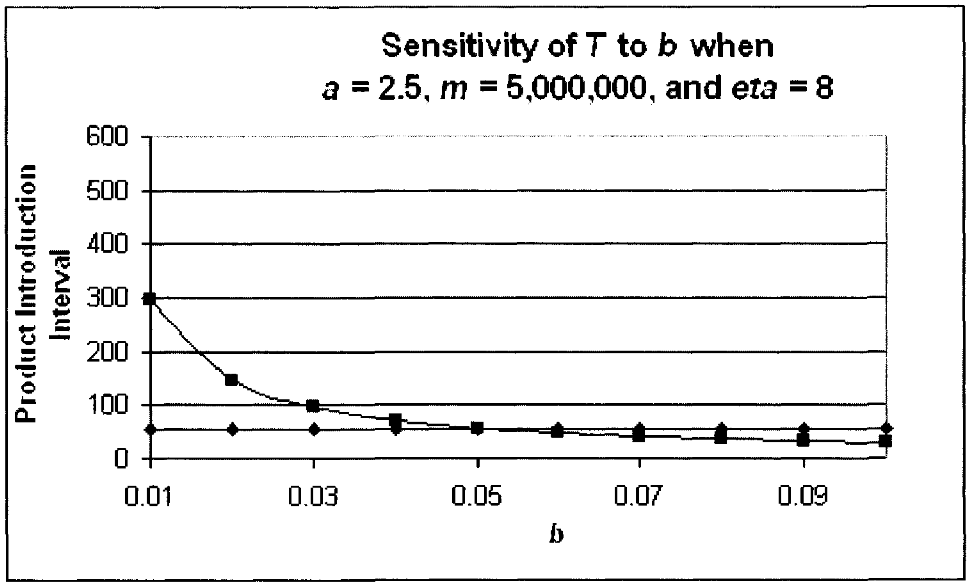

Figure 26 Sensitivity of T to b when $a = 2.5$, $m = 5{,}000{,}000$, and $\eta = 8$.

Figures 27 and 28 are associated with a equal to 2.5, m equal to 30,000,000, and η equal to 1.5. The perfect information curve of Figure 27 slopes upward from $13,819,402 per week when b is equal to 0.01 to $138,355,247 per week when b is equal to 0.1. This suggests that maximum profit per period is positively related to b. The imperfect information curve slopes upward from $10,799,857 per week when b is equal to 0.01 to $69,172,618 when b is equal to 0.05 and then slopes downward to $69,163,206 when b is equal to 0.1. The relative closeness of the two curves to the left of Figure 27 suggests that, when a is equal to 2.5, m is equal to 30,000,000, and η is equal to 1.5, profit per period is not greatly influenced by overestimation of b. The distance between the two curves to the right of Figure 27 suggests that, in the same situation, profit per period is greatly influenced by underestimation of b.

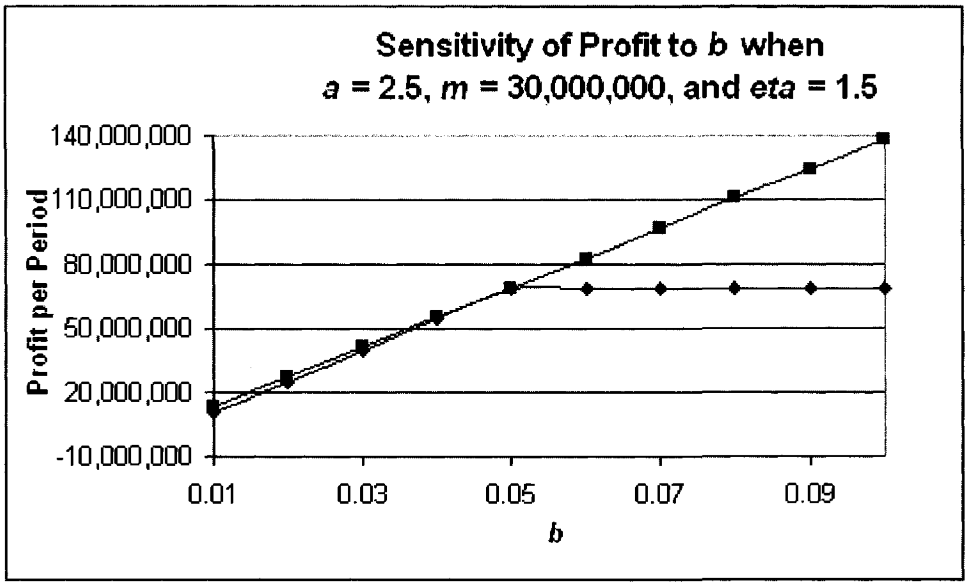

Figure 27 Sensitivity of profit to *b* when *a* = 2.5, *m* = 30,000,000, and η = 1.5.

The perfect information curve of Figure 28 slopes downward from 141 weeks when *b* is equal to 0.01 to 14 weeks when *b* is equal to 0.1. This suggests that the product introduction interval is negatively related to *b*. The imperfect information curve is constant at 28 weeks because this is the optimal product introduction interval when *b* is equal to 0.05. The slope of the perfect information curve suggests that, when *a* is equal to 2.5, *m* is equal to 30,000,000, and η is equal to 1.5, the product introduction interval is somewhat influenced by the value of *b*.

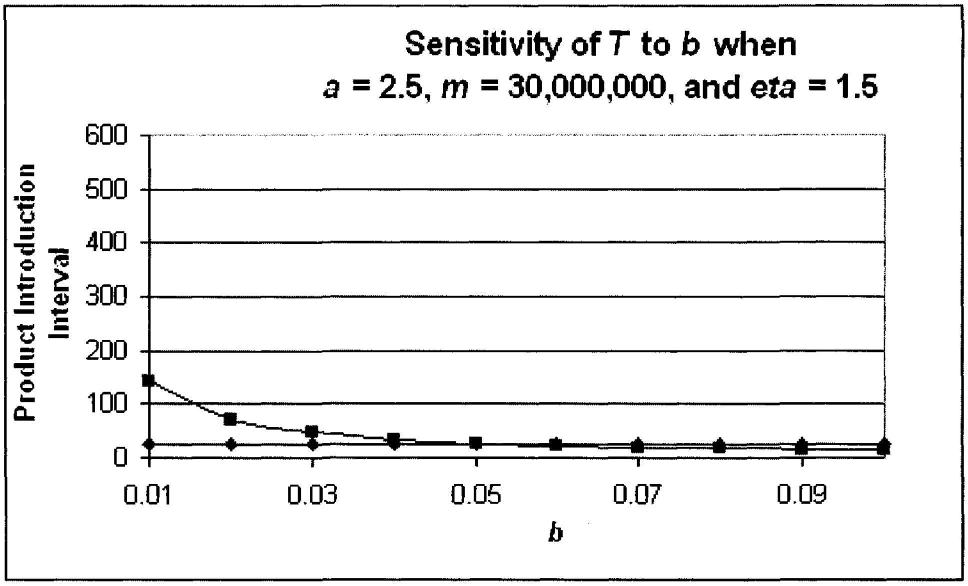

Figure 28 Sensitivity of *T* to *b* when *a* = 2.5, *m* = 30,000,000, and η = 1.5.

Figures 29 and 30 are associated with *a* equal to 2.5, *m* equal to 30,000,000, and η equal to 8. The perfect information curve of Figure 29 slopes upward from $853,250 per week when *b* is equal to 0.01 to $8,698,869 per week when *b* is equal to 0.1. This suggests that maximum profit per period is positively related to *b*. The imperfect information curve slopes upward from negative $451,699 per week when *b* is equal to 0.01 to $4,343,265 when *b* is equal to 0.05 and then slopes downward to $3,748,160 when *b* is equal to 0.1. The relative closeness of the two curves suggests that, when *a* is equal to 2.5, *m* is equal to 30,000,000, and η is equal to 8, profit per period is not greatly influenced by misspecification of *b*. However, the distance between the two curves to the right of Figure 29 suggests that grossly underestimating *b* can lead to a noticeable reduction in profit.

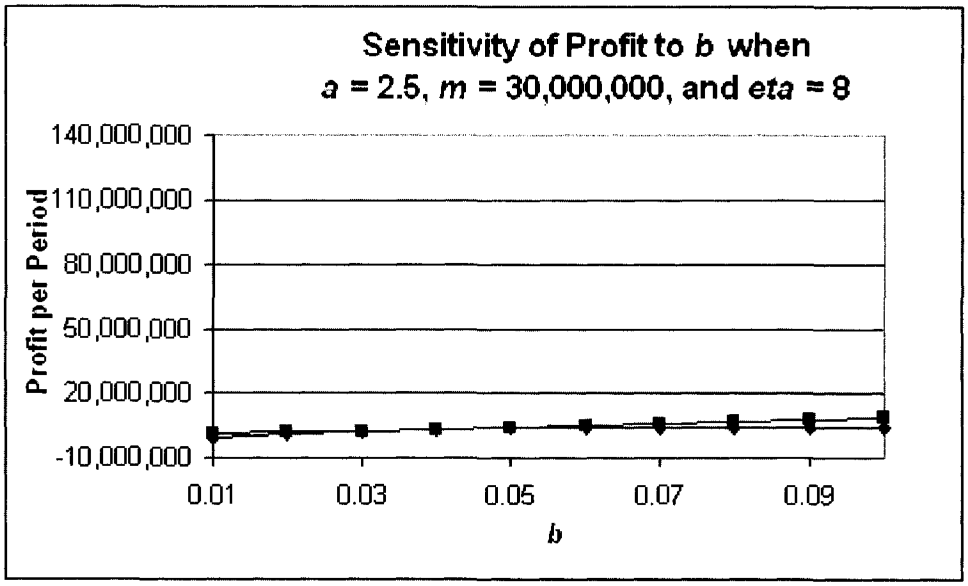

Figure 29 Sensitivity of profit to *b* when *a* = 2.5, *m* = 30,000,000, and η = 8.

The perfect information curve of Figure 30 slopes downward from 179 weeks when *b* is equal to 0.01 to 18 weeks when *b* is equal to 0.1. This suggests that the product introduction interval is negatively related to *b*. The imperfect information curve is constant at 36 weeks because this is the optimal product introduction interval when *b* is equal to 0.05. The slope of the perfect information curve suggests that, when *a* is equal to 2.5, *m* is equal to 30,000,000, and η is equal to 8, the product introduction interval is greatly influenced by the value of *b*.

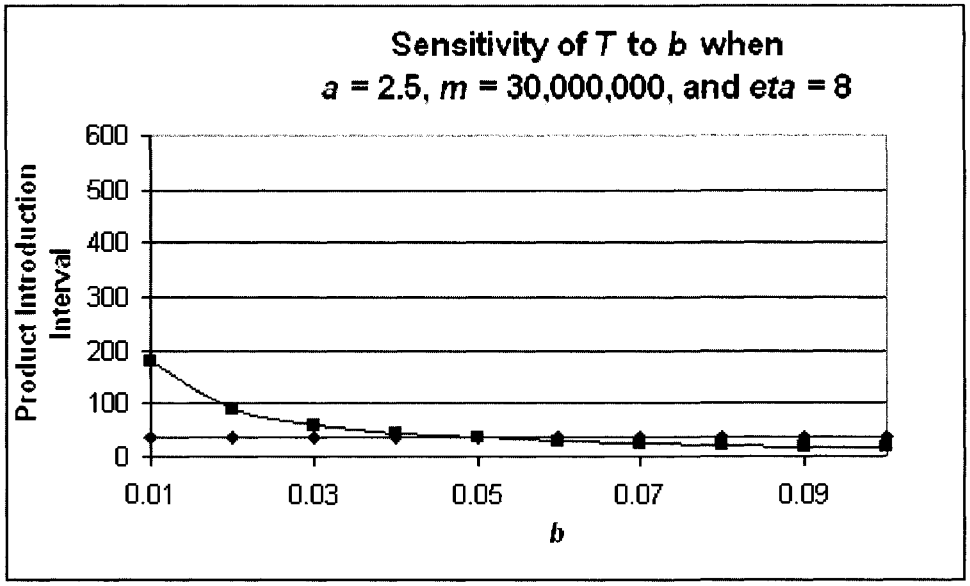

Figure 30 Sensitivity of T to b when $a = 2.5$, $m = 30{,}000{,}000$, and $\eta = 8$.

Figures 31 and 32 are associated with a equal to 20, m equal to 5,000,000, and η equal to 1.5. The perfect information curve of Figure 31 slopes upward from $1,207,768 per week when b is equal to 0.01 to $12,146,479 per week when b is equal to 0.1. This suggests that maximum profit per period is positively related to b. The imperfect information curve slopes upward from $162,447 per week when b is equal to 0.01 to $6,066,099 when b is equal to 0.05 and then slopes downward to $3,473,621 when b is equal to 0.1. The relative closeness of the two curves suggests that, when a is equal to 20, m is equal to 5,000,000, and η is equal to 1.5, profit per period is not greatly influenced by misspecification of b. However, the distance between the two curves to the right of Figure 31 suggests that grossly underestimating b can lead to a noticeable reduction in profit.

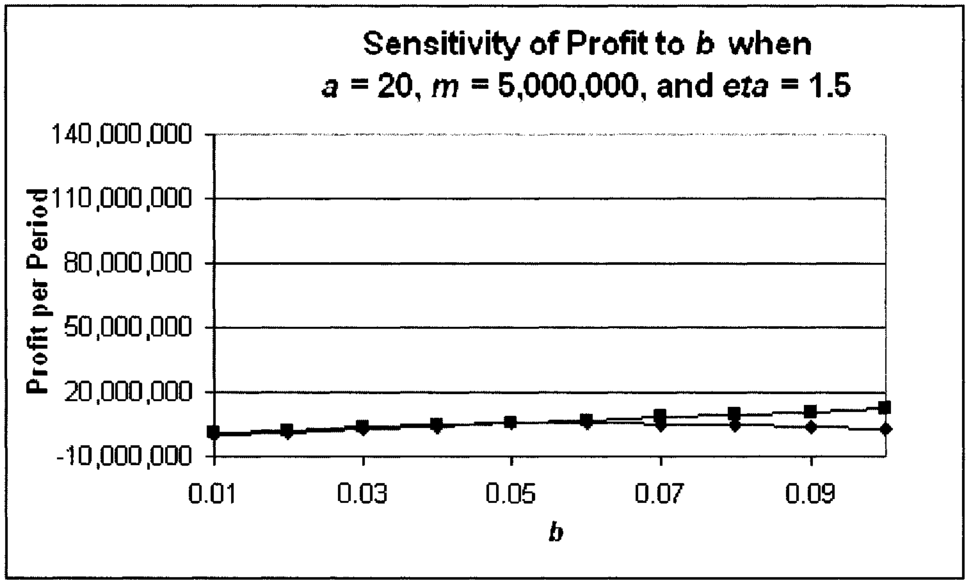

Figure 31 Sensitivity of profit to b when $a = 20$, $m = 5{,}000{,}000$, and $\eta = 1.5$.

The perfect information curve of Figure 32 slopes downward from 432 weeks when b is equal to 0.01 to 43 weeks when b is equal to 0.1. This suggests that the product introduction interval is negatively related to b. The imperfect information curve is constant at 86 weeks because this is the optimal product introduction interval when b is equal to 0.05. The slope of the perfect information curve suggests that, when a is equal to 20, m is equal to 5,000,000, and η is equal to 1.5, the product introduction interval is greatly influenced by the value of b.

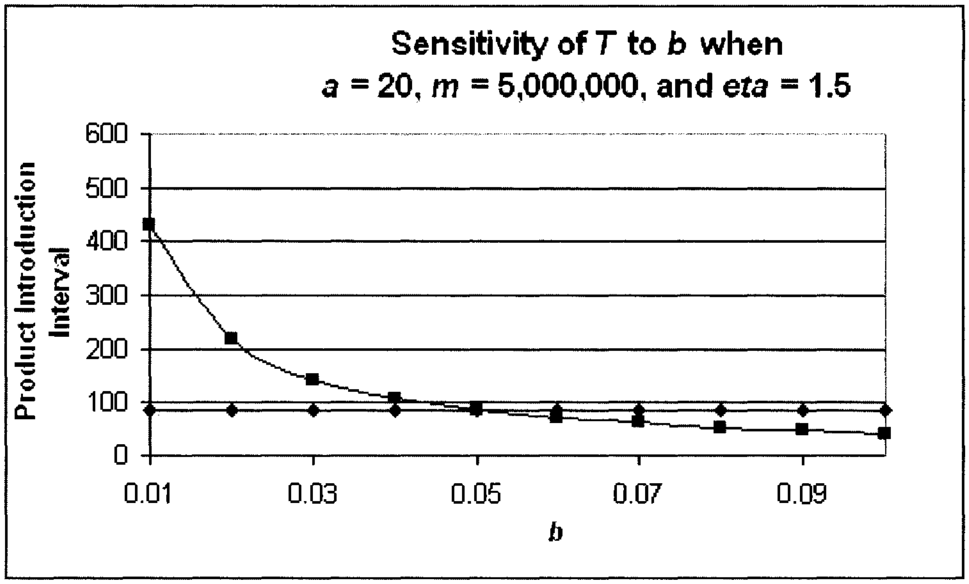

Figure 32 Sensitivity of *T* to *b* when *a* = 20, *m* = 5,000,000, and η = 1.5.

Figures 33 and 34 are associated with *a* equal to 20, *m* equal to 5,000,000, and η equal to 8. The perfect information curve of Figure 33 slopes upward from $39,126 per week when *b* is equal to 0.01 to $457,618 per week when *b* is equal to 0.1. This suggests that maximum profit per period is positively related to *b*. The imperfect information curve slopes upward from negative $212,643 per week when *b* is equal to 0.01 to $222,057 when *b* is equal to 0.05 and then slopes downward to negative $38,699 when *b* is equal to 0.1. The relative closeness of the two curves suggests that, when *a* is equal to 20, *m* is equal to 5,000,000, and η is equal to 8, profit per period is not greatly influenced by misspecification of *b*.

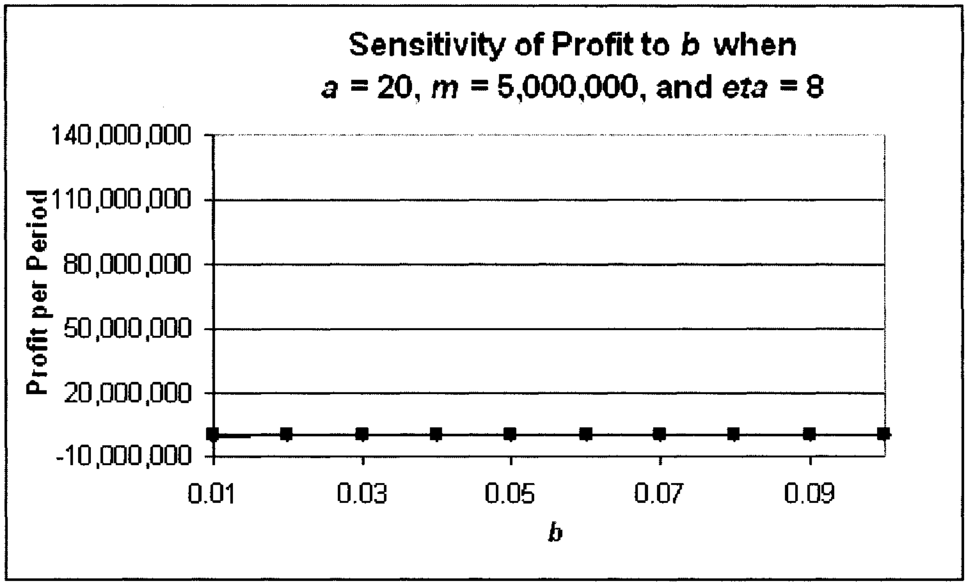

Figure 33 Sensitivity of profit to *b* when $a = 20$, $m = 5{,}000{,}000$, and $\eta = 8$.

The perfect information curve of Figure 34 slopes downward from 523 weeks when *b* is equal to 0.01 to 52 weeks when *b* is equal to 0.1. This suggests that the product introduction interval is negatively related to *b*. The imperfect information curve is constant at 104 weeks because this is the optimal product introduction interval when *b* is equal to 0.05. The slope of the perfect information curve suggests that, when *a* is equal to 20, *m* is equal to 5,000,000, and η is equal to 8, the product introduction interval is greatly influenced by the value of *b*.

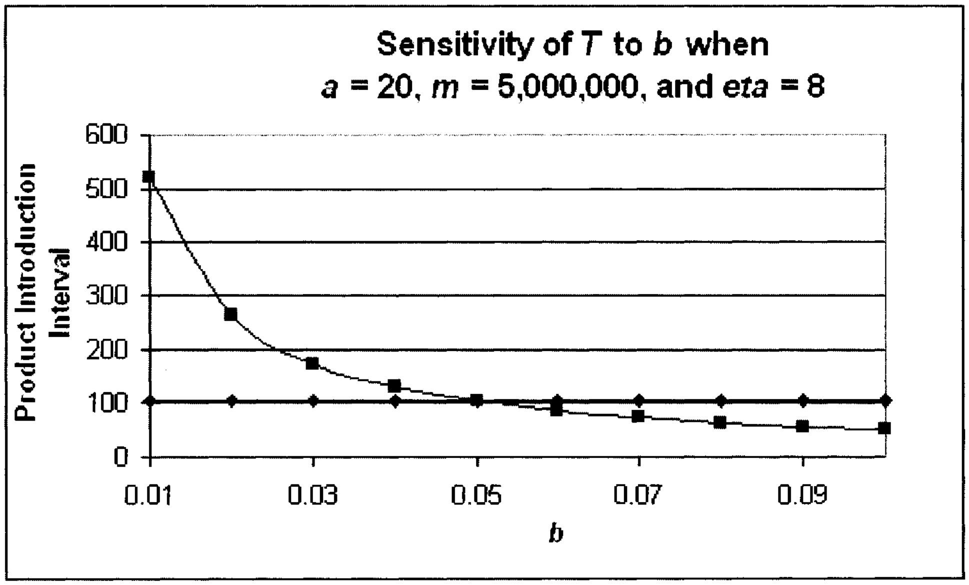

Figure 34 Sensitivity of T to b when a = 20, m = 5,000,000, and η = 8.

Figures 35 and 36 are associated with a equal to 20, m equal to 30,000,000, and η equal to 1.5. The perfect information curve of Figure 35 slopes upward from $7,514,720 per week when b is equal to 0.01 to $75,297,837 per week when b is equal to 0.1. This suggests that maximum profit per period is positively related to b. The imperfect information curve slopes upward from $2,429,587 per week when b is equal to 0.01 to $37,640,513 when b is equal to 0.05 and then slopes downward to $22,574,044 when b is equal to 0.1. The relative closeness of the two curves to the left of Figure 35 suggests that, when a is equal to 20, m is equal to 30,000,000, and η is equal to 1.5, profit per period is not greatly influenced by overestimation of b. The distance between the two curves to the right of Figure 35 suggests that, in the same situation, profit per period is greatly influenced by underestimation of b.

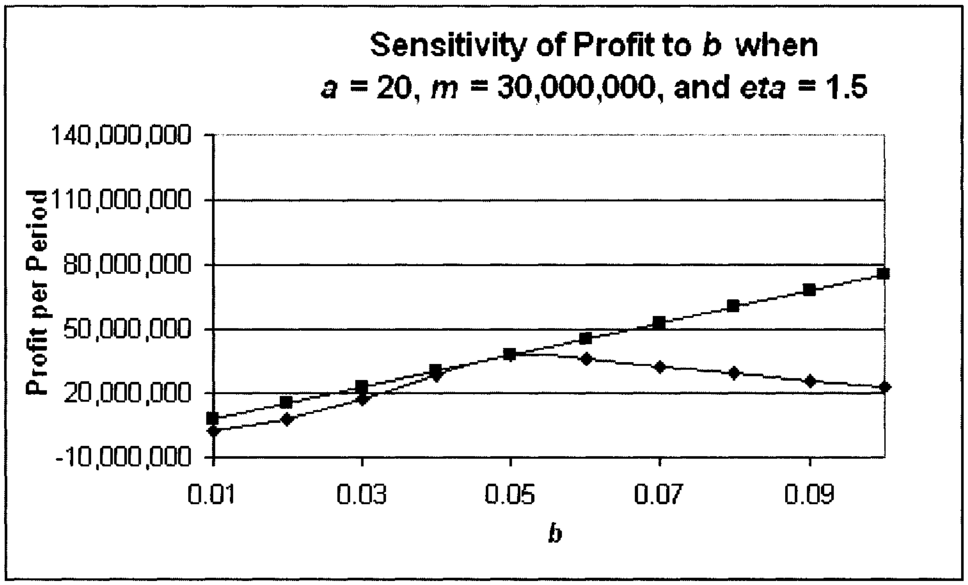

Figure 35 Sensitivity of profit to b when a = 20, m = 30,000,000, and η = 1.5.

The perfect information curve of Figure 36 slopes downward from 427 weeks when b is equal to 0.01 to 43 weeks when b is equal to 0.1. This suggests that the product introduction interval is negatively related to b. The imperfect information curve is constant at 85 weeks because this is the optimal product introduction interval when b is equal to 0.05. The slope of the perfect information curve suggests that, when a is equal to 20, m is equal to 30,000,000, and η is equal to 1.5, the product introduction interval is greatly influenced by the value of b.

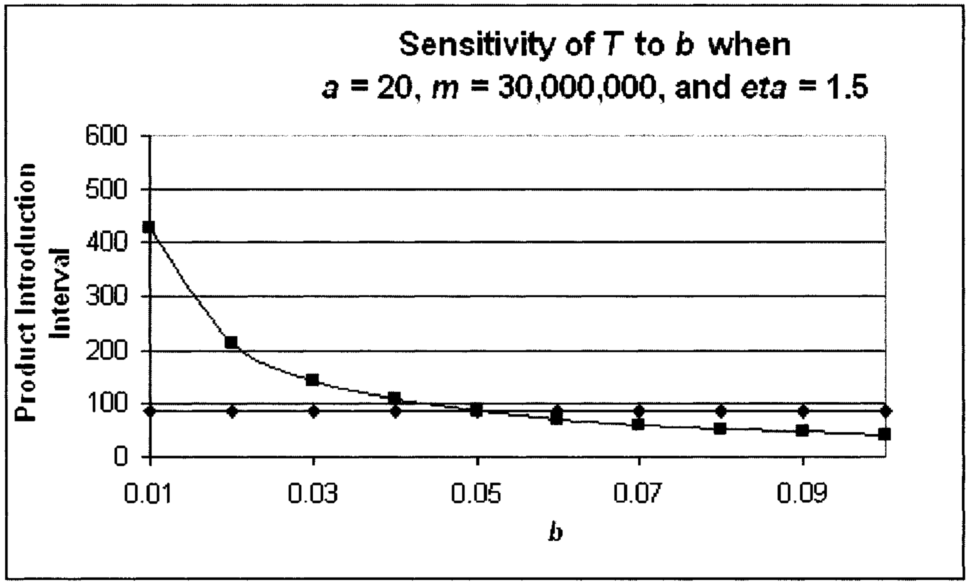

Figure 36 Sensitivity of *T* to *b* when *a* = 20, *m* = 30,000,000, and η = 1.5.

Figures 37 and 38 are associated with *a* equal to 20, *m* equal to 30,000,000, and η equal to 8. The perfect information curve of Figure 37 slopes upward from $477,712 per week when *b* is equal to 0.01 to $4,931,654 per week when *b* is equal to 0.1. This suggests that maximum profit per period is positively related to *b*. The imperfect information curve slopes upward from negative $285,079 per week when *b* is equal to 0.01 to $2,456,119 when *b* is equal to 0.05 and then slopes downward to $1,121,997 when *b* is equal to 0.1. The relative closeness of the two curves suggests that, when *a* is equal to 20, *m* is equal to 30,000,000, and η is equal to 8, profit per period is not greatly influenced by misspecification of *b*.

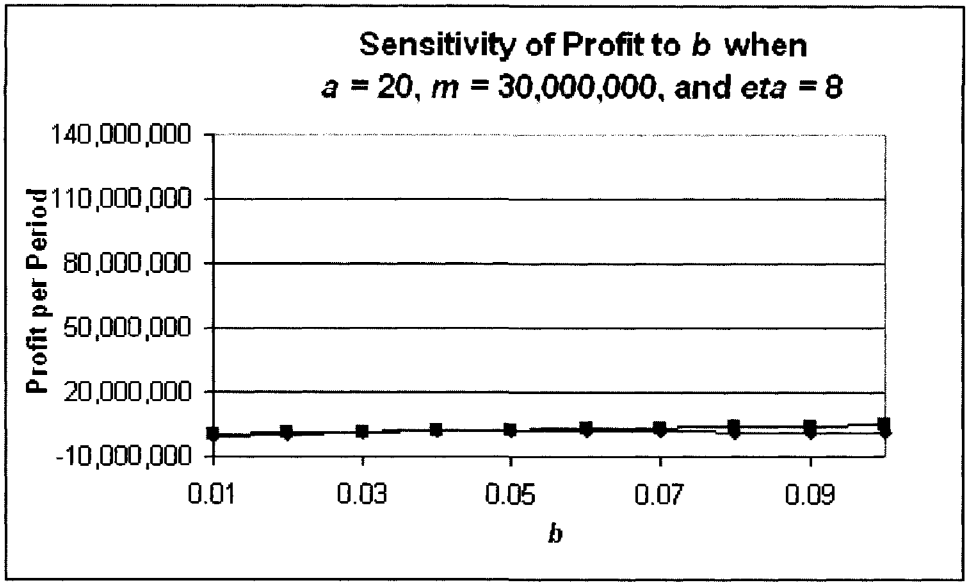

Figure 37 Sensitivity of profit to b when a = 20, m = 30,000,000, and η = 8.

The perfect information curve of Figure 37 slopes downward from 441 weeks when b is equal to 0.01 to 44 weeks when b is equal to 0.1. This suggests that the product introduction interval is negatively related to b. The imperfect information curve is constant at 88 weeks because this is the optimal product introduction interval when b is equal to 0.05. The slope of the perfect information curve suggests that, when a is equal to 20, m is equal to 30,000,000, and η is equal to 8, the product introduction interval is greatly influenced by the value of b.

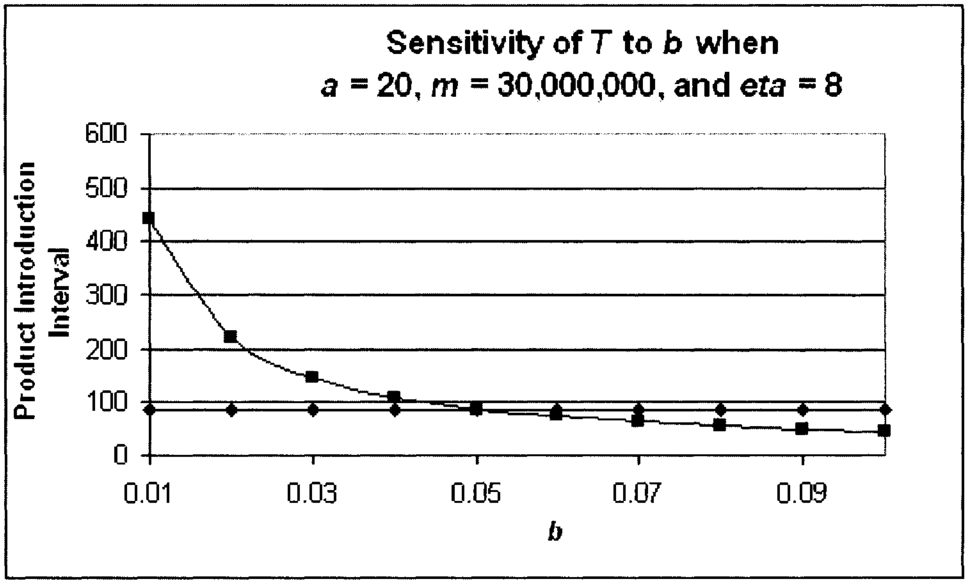

Figure 38 Sensitivity of T to b when $a = 20$, $m = 30{,}000{,}000$, and $\eta = 8$.

Comparison of Figures 23, 25, 27, 29, 31, 33, 35 and 37 suggests that, although profit is positively related to b, the relationship is not generally strong. A notable exception to this is in the case where m is large and η is small. In this circumstance, profit appears to be strongly positively related to b. The sensitivity of profit to underestimation of b is situational, while profit does not appear to ever be particularly sensitive to overestimation of b. Comparison of Figures 24, 26, 28, 30, 32, 34, 36 and 38 suggests that the relationship between b and the product introduction interval is always negative but not always strong. It appears that the product introduction interval is much more sensitive to b when a is large than when a is small. Further, it appears that the product introduction interval is much more sensitive to overestimation of b than underestimation of b.

<u>The Main Effects of *m*</u>

In this section, *m* ranges from 5,000,000 to 30,000,000 in increments of 2,500,000 while *H*, *S*, *V*, and *I* are held constant at 0.3, 20000, 70, and 20000000 respectively. In this section, *a* takes on values of 2.5 and 20, *b* takes on values of 0.01 and 0.1, and η takes on values of 1.5 and 8. At every point along the perfect information curves of Figures 39 through 54 the decision maker knows the value *m* and behaves optimally whereas at every point along the imperfect information curves the decision maker believes the value of *m* to be 10,000,000 and behaves in a manner that would be optimal if that were true.

Figures 39 and 40 are associated with *a* equal to 2.5, *b* equal to 0.01, and η equal to 1.5. The perfect information curve of Figure 39 slopes upward from $2,183,897 per week when *m* is equal to 5,000,000 to $13,819,402 per week when *m* is equal to 30,000,000. This suggests that maximum profit per period is positively related to *m*. The imperfect information curve slopes upward from $2,155,818 per week when *m* is equal to 5,000,000 to $4,506,165 when *m* is equal to 10,000,000 and then slopes downward to $4,496,077 when *m* is equal to 30,000,000. The relative closeness of the two curves suggests that, when *a* is equal to 2.5, *b* is equal to 0.01, and η is equal to 1.5, profit per period is not greatly influenced by misspecification of *m*. However, the distance between the two curves to the right of Figure 39 suggests that grossly underestimating *m* can lead to a noticeable reduction in profit.

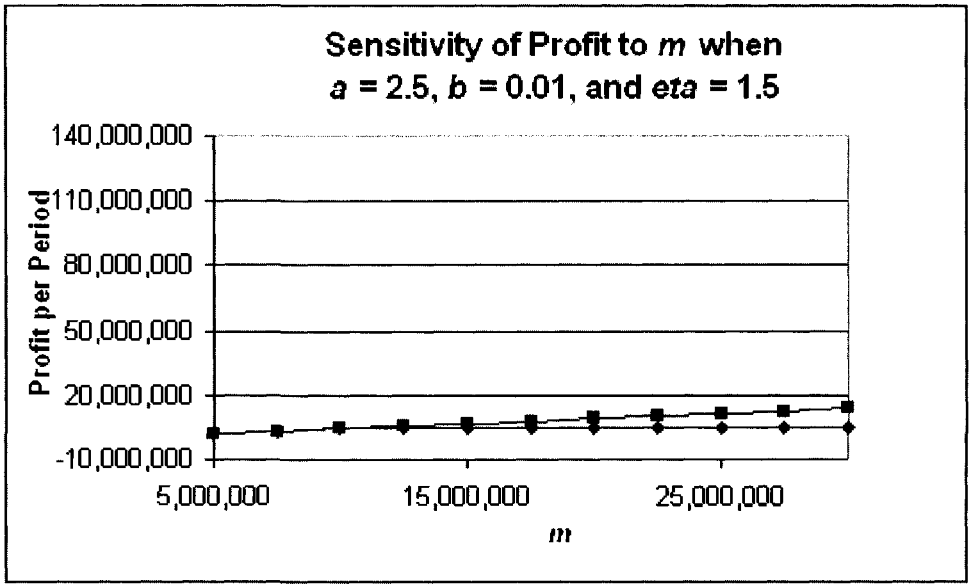

Figure 39 Sensitivity of profit to m when $a = 2.5$, $b = 0.01$, and $\eta = 1.5$.

The perfect information curve of Figure 40 slopes downward from 159 weeks when m is equal to 5,000,000 to 141 weeks when m is equal to 30,000,000. This suggests that the product introduction interval is negatively related to m. The imperfect information curve is constant at 149 weeks because this is the optimal product introduction interval when m is equal to 10,000,000. The near horizontal nature of the perfect information curve suggests that, when a is equal to 2.5, b is equal to 0.01, and η is equal to 1.5, the product introduction interval is not greatly influenced by the value of m.

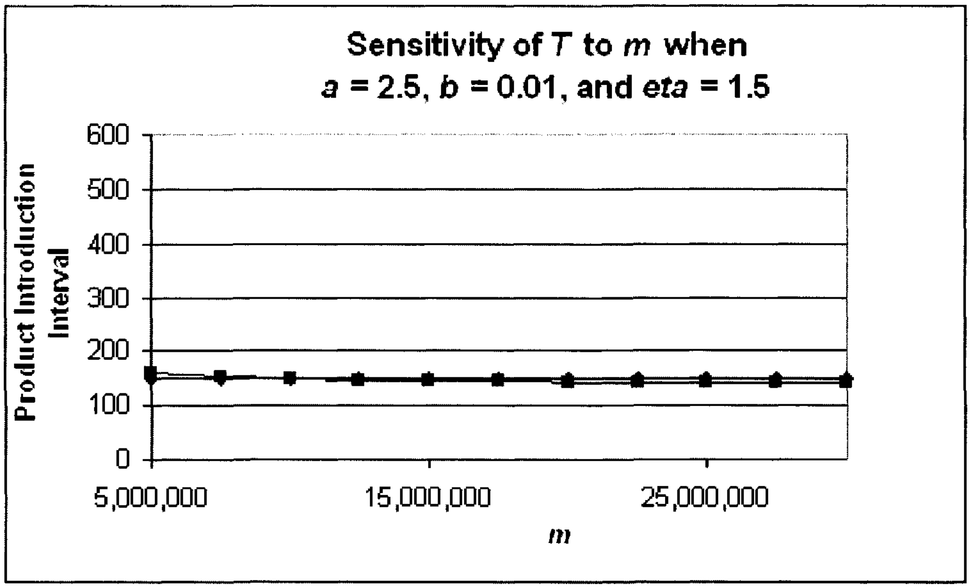

Figure 40 Sensitivity of *T* to *m* when *a* = 2.5, *b* = 0.01, and η = 1.5.

Figures 41 and 42 are associated with *a* equal to 2.5, *b* equal to 0.01, and η equal to 8. The perfect information curve of Figure 41 slopes upward from $63,522 per week when *m* is equal to 5,000,000 to $853,250 per week when *m* is equal to 30,000,000. This suggests that maximum profit per period is positively related to *m*. The imperfect information curve slopes upward from $37,877 per week when *m* is equal to 5,000,000 to $211,226 when *m* is equal to 10,000,000 and then slopes downward to $210,567 when *m* is equal to 30,000,000. The relative closeness of the two curves suggests that, when *a* is equal to 2.5, *b* is equal to 0.01, and η is equal to 8, profit per period is not greatly influenced by misspecification of *m*.

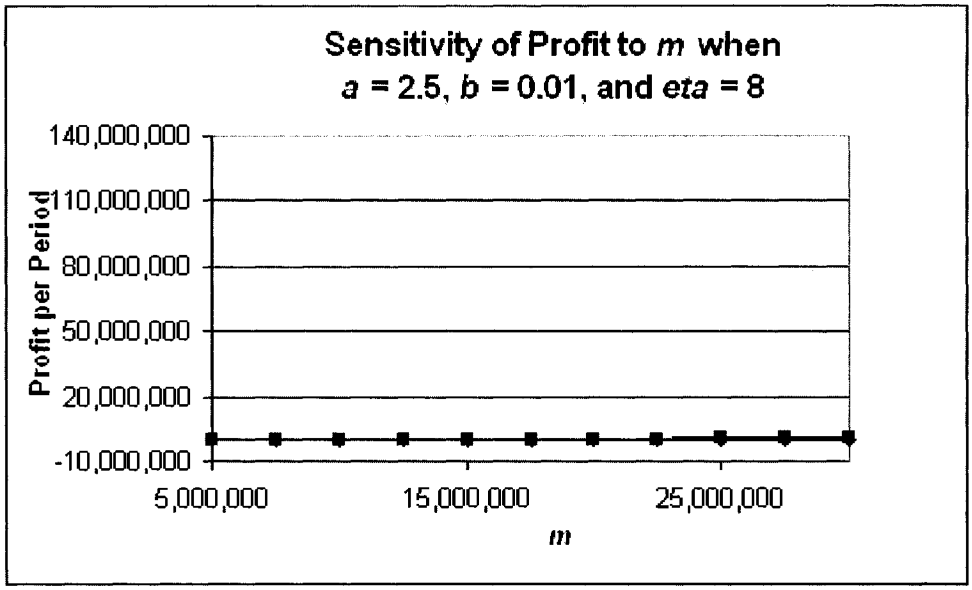

Figure 41 Sensitivity of profit to m when $a = 2.5$, $b = 0.01$, and $\eta = 8$.

The perfect information curve of Figure 42 slopes downward from 295 weeks when m is equal to 5,000,000 to 179 weeks when m is equal to 30,000,000. This suggests that the product introduction interval is negatively related to m. The imperfect information curve is constant at 230 weeks because this is the optimal product introduction interval when m is equal to 10,000,000. The slope of the perfect information curve suggests that, when a is equal to 2.5, b is equal to 0.01, and η is equal to 8, the product introduction interval is greatly influenced by the value of m.

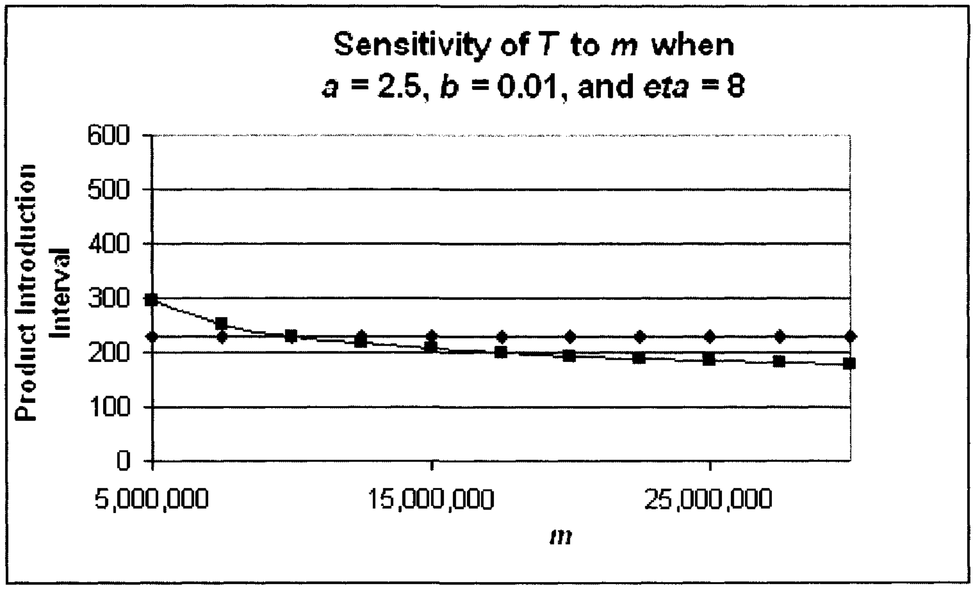

Figure 42 Sensitivity of T to m when $a = 2.5$, $b = 0.01$, and $\eta = 8$.

Figures 43 and 44 are associated with a equal to 2.5, b equal to 0.1, and η equal to 1.5. The perfect information curve of Figure 43 slopes upward from \$21,933,088 per week when m is equal to 5,000,000 to \$138,355,247 per week when m is equal to 30,000,000. This suggests that maximum profit per period is positively related to m. The imperfect information curve slopes upward from \$21,100,297 per week when m is equal to 5,000,000 to \$45,186,229 when m is equal to 10,000,000 and then slopes downward to \$45,174,177 when m is equal to 30,000,000. The relative closeness of the two curves to the left of Figure 43 suggests that, when a is equal to 2.5, b is equal to 0.1, and η is equal to 1.5, profit per period is not greatly influenced by overestimation of m. The distance between the two curves to the right of Figure 43 suggests that, in the same situation, profit per period is greatly influenced by underestimation of m.

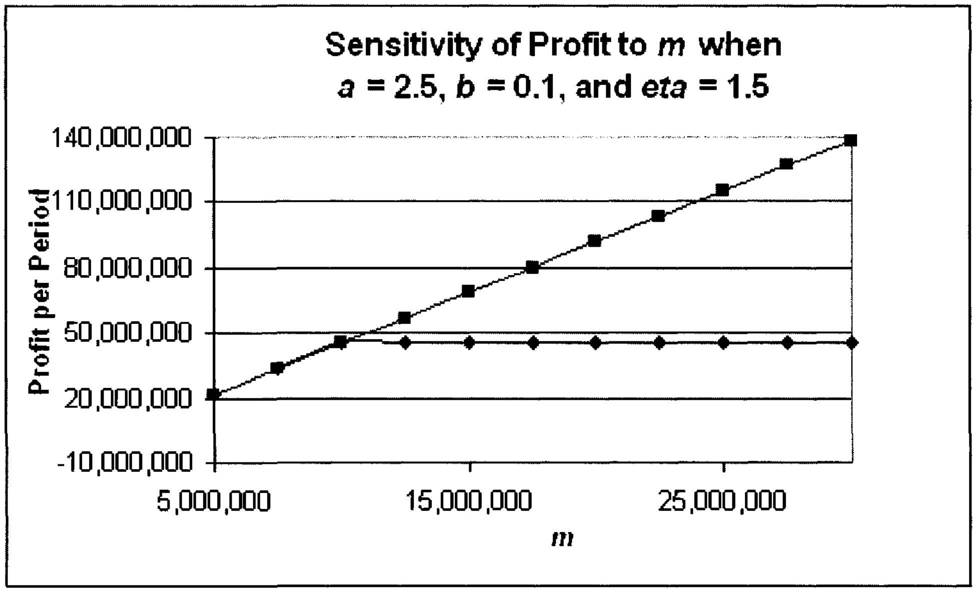

Figure 43 Sensitivity of profit to m when $a = 2.5$, $b = 0.1$, and $\eta = 1.5$.

The perfect information curve of Figure 44 slopes downward from 16 weeks when m is equal to 5,000,000 to 14 weeks when m is equal to 30,000,000. This suggests that the product introduction interval is negatively related to m. The imperfect information curve is constant at 15 weeks because this is the optimal product introduction interval when m is equal to 10,000,000. The relatively horizontal nature of the perfect information curve suggests that, when a is equal to 2.5, b is equal to 0.1, and η is equal to 1.5, the product introduction interval is not greatly influenced by the value of m.

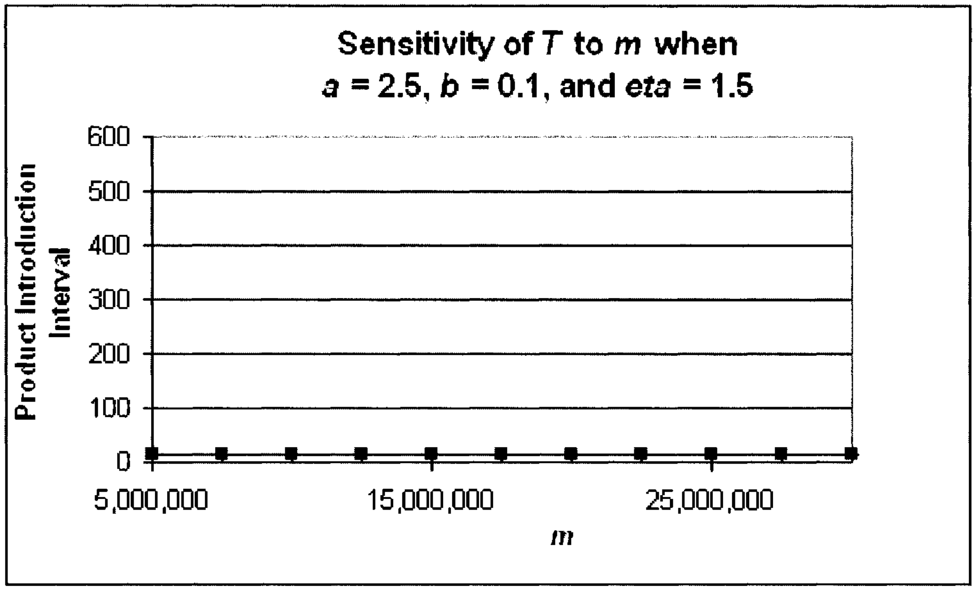

Figure 44 Sensitivity of T to m when $a = 2.5$, $b = 0.1$, and $\eta = 1.5$.

Figures 45 and 46 are associated with a equal to 2.5, b equal to 0.1, and η equal to 8. The perfect information curve of Figure 45 slopes upward from \$721,745 per week when m is equal to 5,000,000 to \$8,698,869 per week when m is equal to 30,000,000. This suggests that maximum profit per period is positively related to m. The imperfect information curve slopes upward from \$271,012 per week when m is equal to 5,000,000 to \$2,235,331 when m is equal to 10,000,000 and then slopes downward to \$2,234,785 when m is equal to 30,000,000. The relative closeness of the two curves suggests that, when a is equal to 2.5, b is equal to 0.1, and η is equal to 8, profit per period is not greatly influenced by misspecification of b. However, the distance between the two curves to the right of Figure 45 suggests that grossly underestimating m can lead to a noticeable reduction in profit.

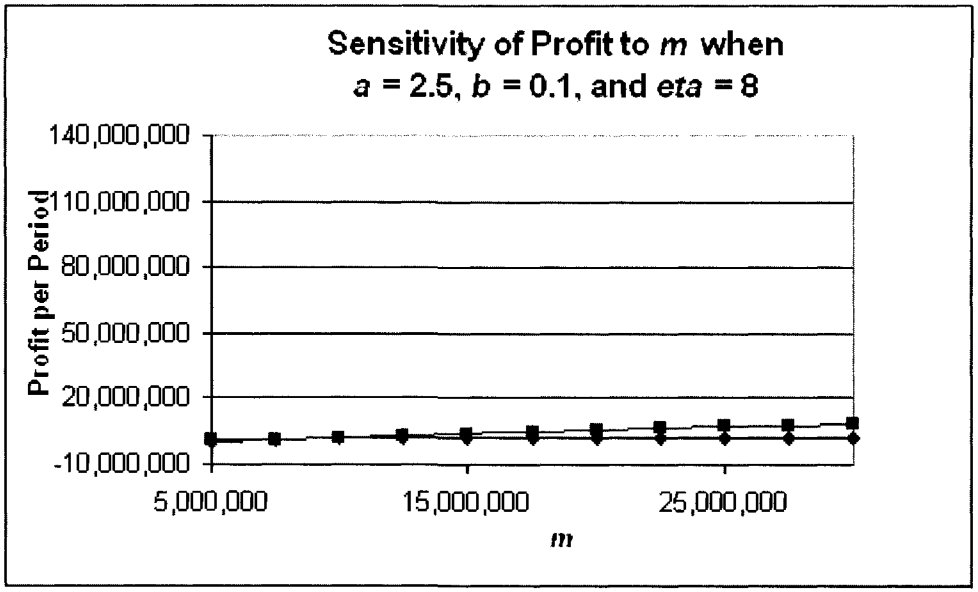

Figure 45 Sensitivity of profit to m when a = 2.5, b = 0.1, and η = 8.

The perfect information curve of Figure 46 slopes downward from 29 weeks when m is equal to 5,000,000 to 18 weeks when m is equal to 30,000,000. This suggests that the product introduction interval is negatively related to m. The imperfect information curve is constant at 23 weeks because this is the optimal product introduction interval when m is equal to 10,000,000. The near horizontal nature of the perfect information curve suggests that, when a is equal to 2.5, b is equal to 0.1, and η is equal to 8, the product introduction interval is not greatly influenced by the value of m.

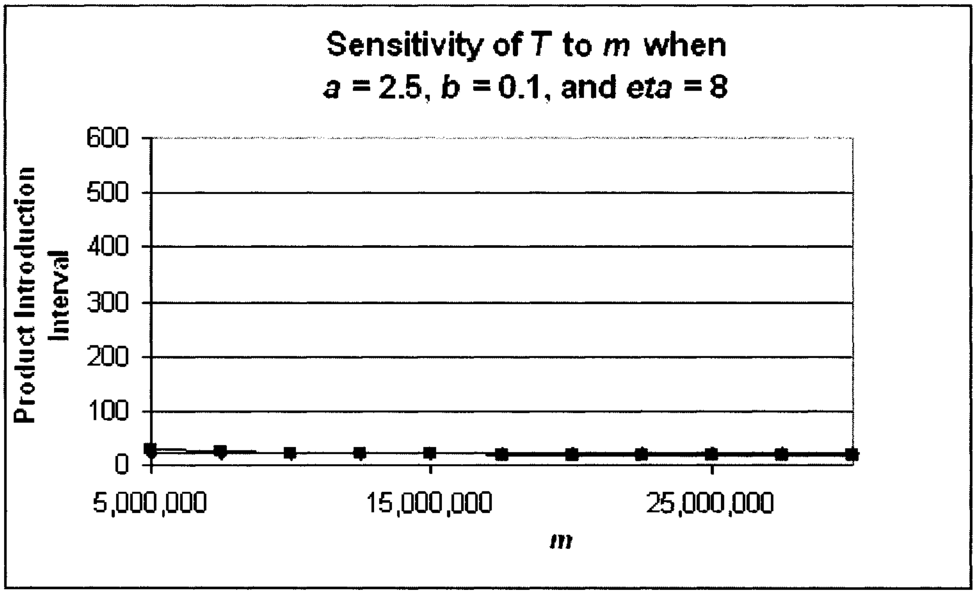

Figure 46 Sensitivity of *T* to *m* when $a = 2.5$, $b = 0.1$, and $\eta = 8$.

Figures 47 and 48 are associated with *a* equal to 20, *b* equal to 0.01, and η equal to 1.5. The perfect information curve of Figure 47 slopes upward from $1,207,768 per week when *m* is equal to 5,000,000 to $7,514,720 per week when *m* is equal to 30,000,000. This suggests that maximum profit per period is positively related to *m*. The imperfect information curve slopes upward from $1,195,512 per week when *m* is equal to 5,000,000 to $2,467,842 when *m* is equal to 10,000,000 and then slopes downward to $2,459,469 when *m* is equal to 30,000,000. The relative closeness of the two curves suggests that, when *a* is equal to 20, *b* is equal to 0.01, and η is equal to 1.5, profit per period is not greatly influenced by misspecification of *m*. However, the distance between the two curves to the right of Figure 47 suggests that grossly underestimating *m* can lead to a noticeable reduction in profit.

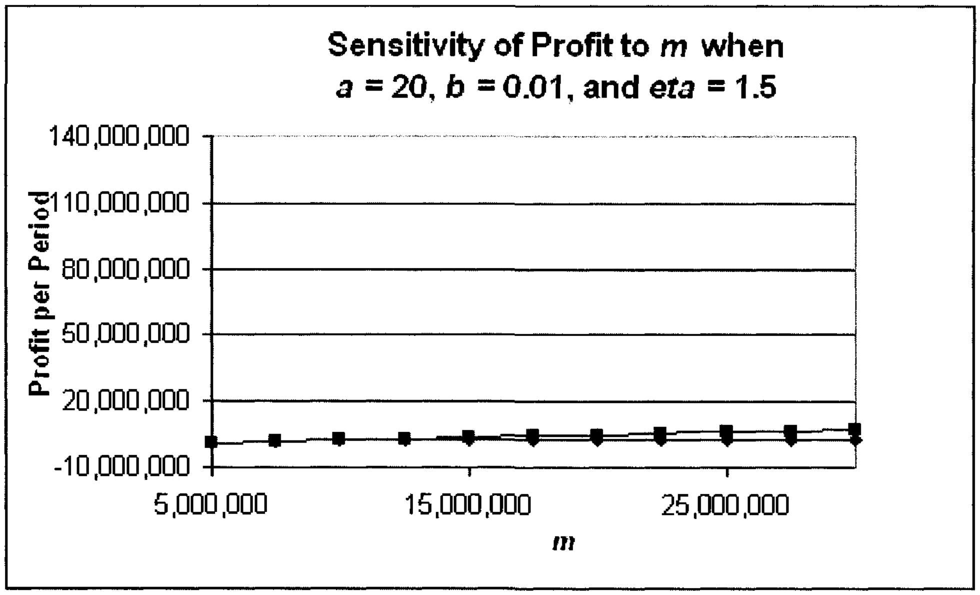

Figure 47 Sensitivity of profit to m when $a = 20$, $b = 0.01$, and $\eta = 1.5$.

The perfect information curve of Figure 48 slopes downward from 432 weeks when m is equal to 5,000,000 to 427 weeks when m is equal to 30,000,000. This suggests that the product introduction interval is negatively related to m. The imperfect information curve is constant at 429 weeks because this is the optimal product introduction interval when m is equal to 10,000,000. The near horizontal nature of the perfect information curve suggests that, when a is equal to 20, b is equal to 0.01, and η is equal to 1.5, the product introduction interval is not greatly influenced by the value of m.

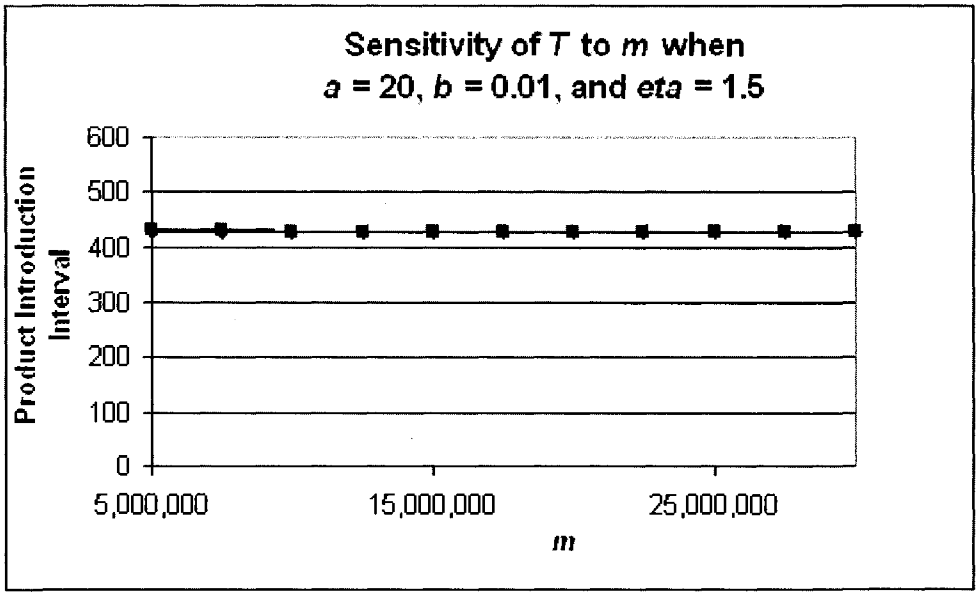

Figure 48 Sensitivity of *T* to *m* when *a* = 20, *b* = 0.01, and η = 1.5.

Figures 49 and 50 are associated with *a* equal to 20, *b* equal to 0.01, and η equal to 8. The perfect information curve of Figure 49 slopes upward from $39,126 per week when *m* is equal to 5,000,000 to $477,712 per week when *m* is equal to 30,000,000. This suggests that maximum profit per period is positively related to *m*. The imperfect information curve slopes upward from $27,382 per week when *m* is equal to 5,000,000 to $124,195 when *m* is equal to 10,000,000 and then slopes downward to $123,599 when *m* is equal to 30,000,000. The relative closeness of the two curves suggests that, when *a* is equal to 20, *b* is equal to 0.01, and η is equal to 8, profit per period is not greatly influenced by misspecification of *m*.

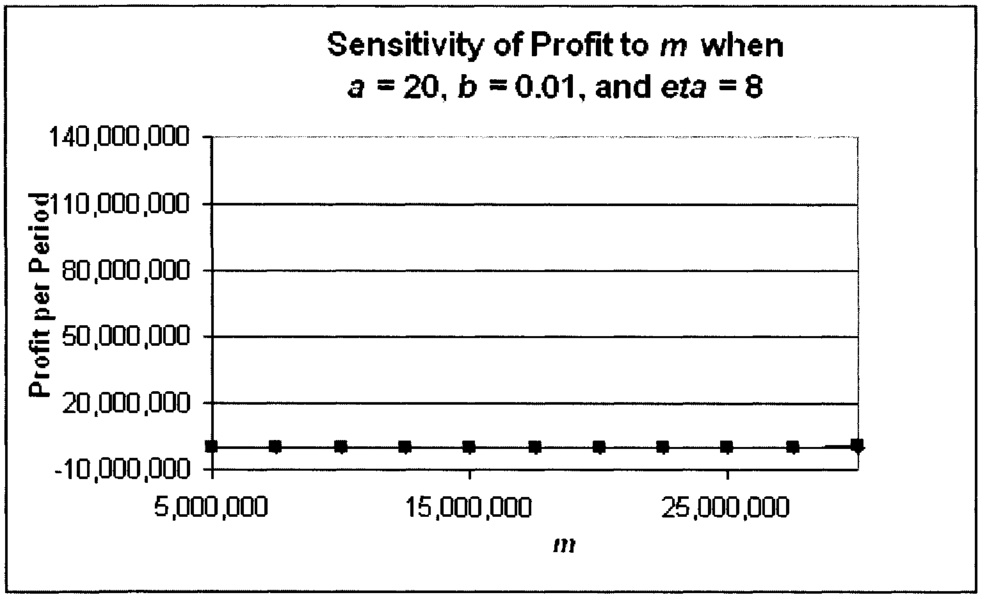

Figure 49 Sensitivity of profit to m when $a = 20$, $b = 0.01$, and $\eta = 8$.

The perfect information curve of Figure 50 slopes downward from 523 weeks when m is equal to 5,000,000 to 441 weeks when m is equal to 3,000,000. This suggests that the product introduction interval is negatively related to m. The imperfect information curve is constant at 472 weeks because this is the optimal product introduction interval when m is equal to 10,000,000. The slope of the perfect information curve suggests that, when a is equal to 20, b is equal to 0.01, and η is equal to 8, the product introduction interval is somewhat influenced by the value of m.

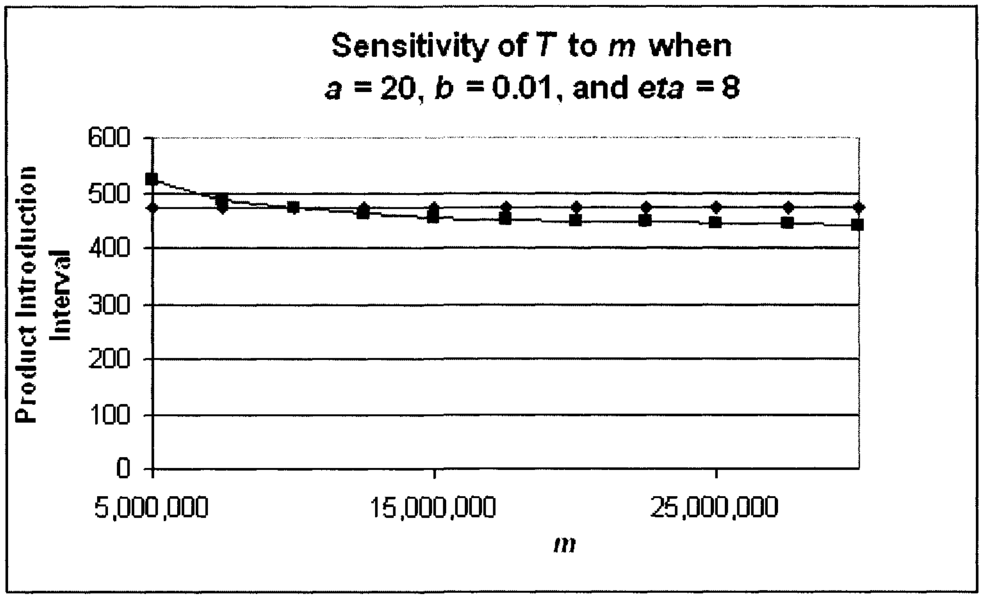

Figure 50 Sensitivity of T to m when a = 20, b = 0.01, and η = 8.

Figures 51 and 52 are associated with a equal to 20, b equal to 0.1, and η equal to 1.5. The perfect information curve of Figure 51 slopes upward from $12,146,479 per week when m is equal to 5,000,000 to $75,297,837 per week when m is equal to 30,000,000. This suggests that maximum profit per period is positively related to m. The imperfect information curve slopes upward from $11,970,552 per week when m is equal to 5,000,000 to $24,776,192 when m is equal to 10,000,000 and then slopes downward to $24,773,989 when m is equal to 30,000,000. The relative closeness of the two curves to the left of Figure 51 suggests that, when a is equal to 20, b is equal to 0.1, and η is equal to 1.5, profit per period is not greatly influenced by overestimation of m. The distance between the two curves to the right of Figure 51 suggests that, in the same situation, profit per period is greatly influenced by underestimation of m.

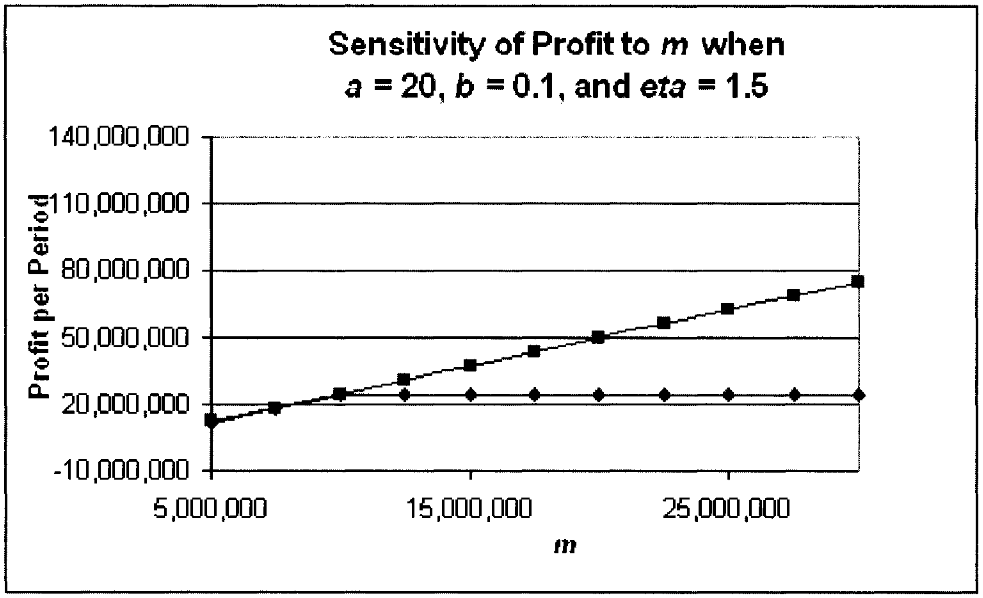

Figure 51 Sensitivity of profit to m when $a = 20$, $b = 0.1$, and $\eta = 1.5$.

Both the perfect information and imperfect information curves of Figure 52 are constant at 43 weeks. The horizontal nature of the perfect information curve suggests that, when a is equal to 20, b is equal to 0.1, and η is equal to 1.5, the product introduction interval is not influenced by the value of m.

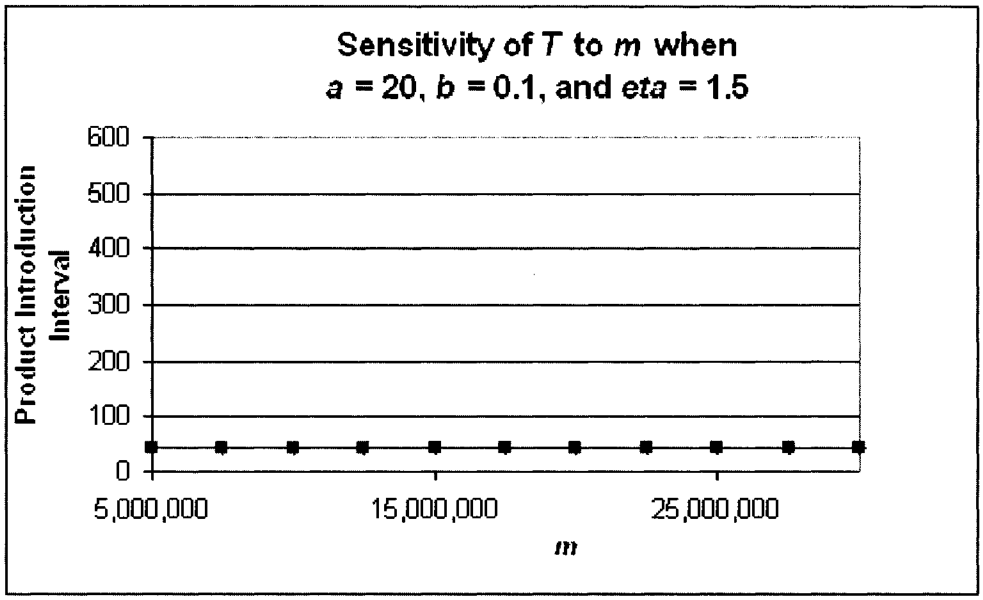

Figure 52 Sensitivity of T to m when $a = 20$, $b = 0.1$, and $\eta = 1.5$.

Figures 53 and 54 are associated with a equal to 20, b equal to 0.1, and η equal to 8. The perfect information curve of Figure 53 slopes upward from $457,618 per week when m is equal to 5,000,000 to $4,931,654 per week when m is equal to 30,000,000. This suggests that maximum profit per period is positively related to m. The imperfect information curve slopes upward from $314,012 per week when m is equal to 5,000,000 to $1,338,390 when m is equal to 10,000,000 and then slopes downward to $1,338,285 when m is equal to 30,000,000. The relative closeness of the two curves suggests that, when a is equal to 20, b is equal to 0.1, and η is equal to 8, profit per period is not greatly influenced by misspecification of b. However, the distance between the two curves to the right of Figure 53 suggests that grossly underestimating m can lead to a noticeable reduction in profit.

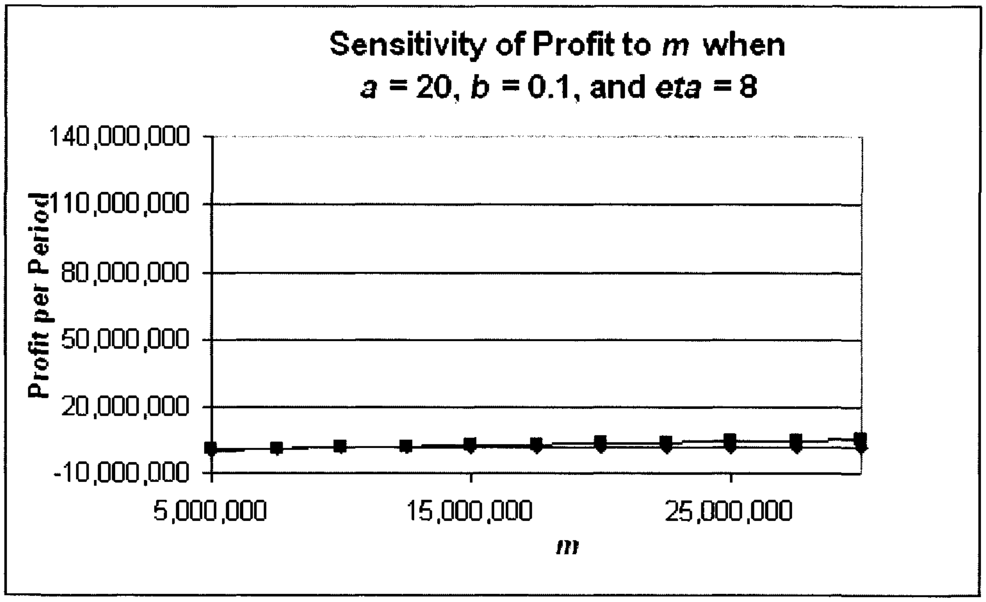

Figure 53 Sensitivity of profit to m when a = 20, b = 0.1, and η = 8.

The perfect information curve of Figure 54 slopes downward from 52 weeks when m is equal to 5,000,000 to 44 weeks when m is equal to 30,000,000. This suggests that the product introduction interval is negatively related to m. The imperfect information curve is constant at 47 weeks because this is the optimal product introduction interval when m is equal to 10,000,000. The near horizontal nature of the perfect information curve suggests that, when a is equal to 20, b is equal to 0.1, and η is equal to 8, the product introduction interval is not greatly influenced by the value of m.

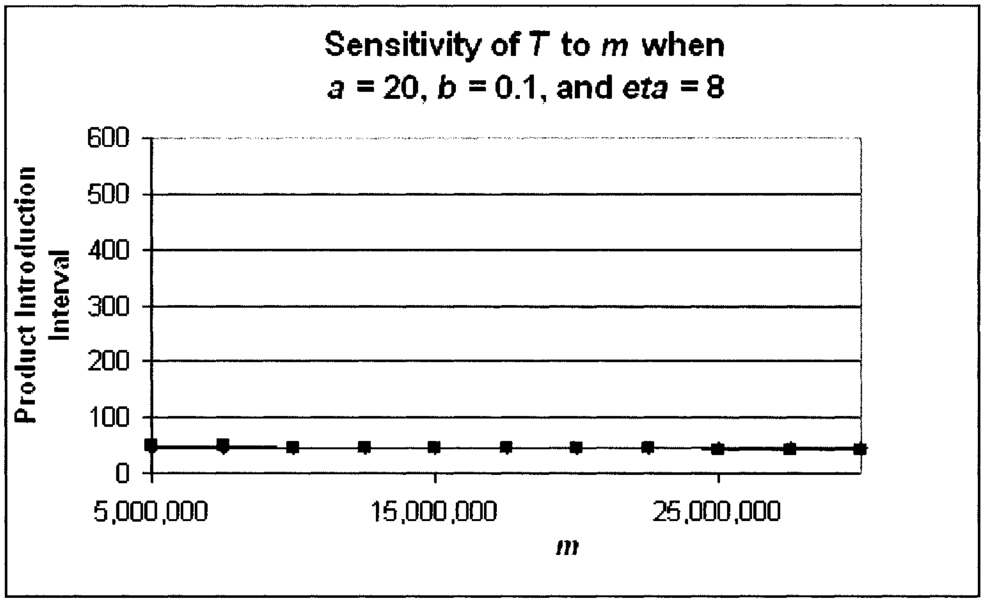

Figure 54 Sensitivity of T to m when a = 20, b = 0.1, and η = 8.

Comparison of Figures 39, 41, 43, 45, 47, 49, 51 and 53 suggests that profit is positively related to m but that the relationship is not generally strong. A notable exception to this is in the case where b is large and η is small. In this circumstance, profit appears to be strongly positively related to m. The sensitivity of profit to underestimation of m is situational while profit does not appear to ever be particularly sensitive to overestimation of m. Comparison of Figures 40, 42, 44, 46, 48, 50, 52 and 54 suggests that the product introduction interval is never positively related to m. It is only strongly positively related to m when b is small and η is large.

The Main Effects of η

In this section, η ranges from 1.5 to 8 in increments of 0.5 while a, H, S, V, and I are held constant at 15, 0.3, 20000, 70, and 20000000 respectively. In this section, b takes on values of 0.01 and 0.1 and m takes on values of 5,000,000 and 30,000,000. At every point along the perfect information curves of Figures 55 through 62 the decision maker knows the value of η and behaves optimally whereas at every point along the imperfect information curves the decision maker believes the value of η to be 3.5 and behaves in a manner that would be optimal if that were true.

Figures 55 and 56 are associated with b equal to 0.01 and m equal to 5,000,000. The perfect information curve of Figure 55 slopes downward from \$1,288,768 per week when η is equal to 1.5 to \$41,487 per week when η is equal to 8. This suggests that maximum profit per period is negatively related to η. The imperfect information curve slopes upward from \$210,014 per week when η is equal to 1.5 to \$211,350 when η is equal to 3.5 and then slopes downward to negative \$14,296 when η is equal to 8. The relative closeness of the two curves suggests that, when b is equal to 0.01 and m is equal to 5,000,000, profit per period is not greatly influenced by misspecification of η.

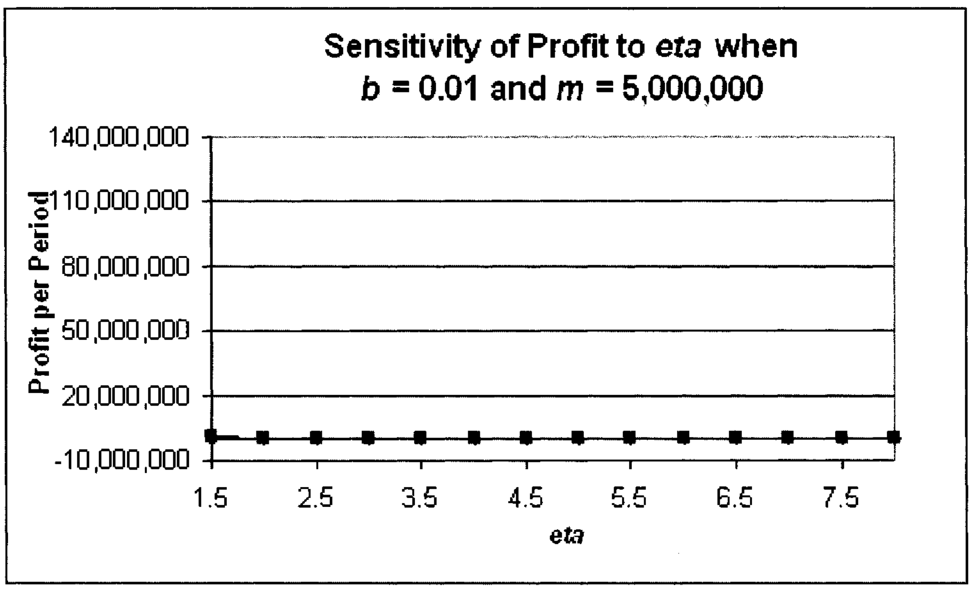

Figure 55 Sensitivity of profit to η when $b = 0.01$, and $m = 5,000,000$.

The perfect information curve of Figure 56 slopes upward from 395 weeks when η is equal to 1.5 to 489 weeks when η is equal to 8. This suggests that the product introduction interval is positively related to η. The imperfect information curve is constant at 423 weeks because this is the optimal product introduction interval when η is equal to 3.5. The slope of the perfect information curve suggests that, when b is equal to 0.01 and m is equal to 5,000,000, the product introduction interval is somewhat influenced by the value of η.

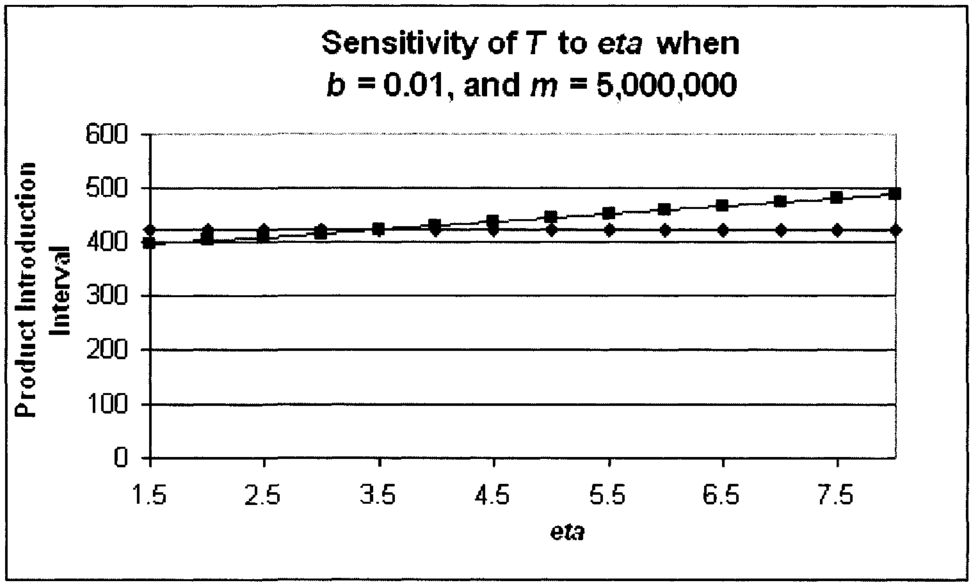

Figure 56 Sensitivity of T to η when $b = 0.01$, and $m = 5{,}000{,}000$.

Figures 57 and 58 are associated with b equal to 0.01, and m equal to 30,000,000. The perfect information curve of Figure 57 slopes downward from $8,024,434 per week when η is equal to 1.5 to $509,633 per week when η is equal to 8. This suggests that maximum profit per period is negatively related to η. The imperfect information curve slopes upward from $1,548,023 per week when η is equal to 1.5 to $1,549,570 when η is equal to 3.5 and then slopes downward to $215,909 when η is equal to 8. The relative closeness of the two curves suggests that, when b is equal to 0.01 and m is equal to 30,000,000, profit per period is not greatly influenced by misspecification of η. However, the distance between the two curves to the left of Figure 57 suggests that grossly overestimating η can lead to a noticeable reduction in profit.

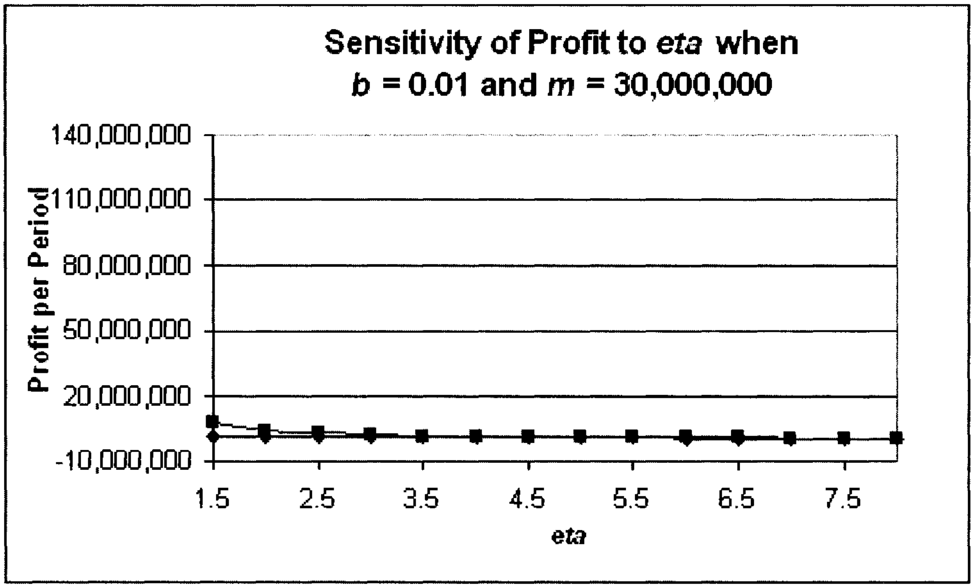

Figure 57 Sensitivity of profit to η when $b = 0.01$, and $m = 30{,}000{,}000$.

The perfect information curve of Figure 58 slopes upward from 389 weeks when η is equal to 1.5 to 404 weeks when η is equal to 8. This suggests that the product introduction interval is positively related to η. The imperfect information curve is constant at 394 weeks because this is the optimal product introduction interval when η is equal to 3.5. The near horizontal nature of the perfect information curve suggests that, when b is equal to 0.01 and m is equal to 30,000,000, the product introduction interval is not greatly influenced by the value of η.

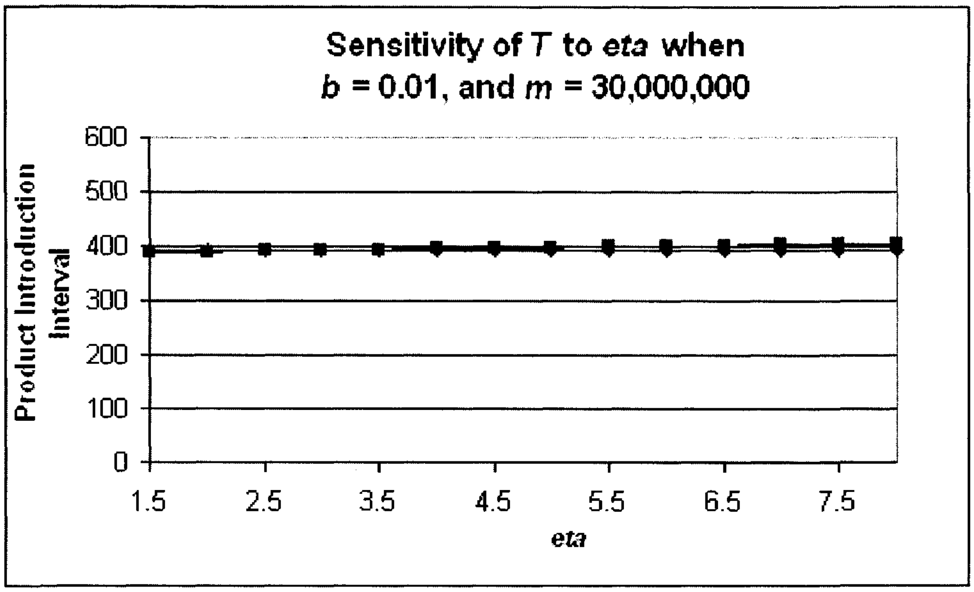

Figure 58 Sensitivity of T to η when b = 0.01, and m = 30,000,000.

Figures 59 and 60 are associated with b equal to 0.1, and m equal to 5,000,000. The perfect information curve of Figure 59 slopes downward from $12,957,521 per week when η is equal to 1.5 to $483,571 per week when η is equal to 8. This suggests that maximum profit per period is negatively related to η. The imperfect information curve slopes upward from $2,184,124 per week when η is equal to 1.5 to $2,184,834 when η is equal to 3.5 and then slopes downward to negative $102,695 when η is equal to 8. The relative closeness of the two curves suggests that, when b is equal to 0.1 and m is equal to 5,000,000, profit per period is not greatly influenced by η. However, the distance between the two curves to the left of Figure 59 suggests that grossly overestimating η can lead to a noticeable reduction in profit.

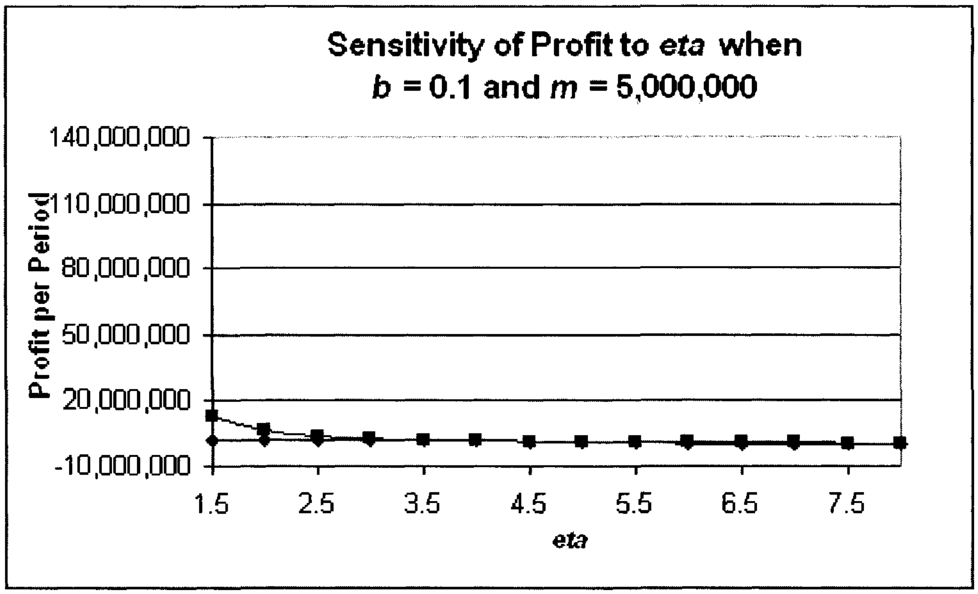

Figure 59 Sensitivity of profit to η when $b = 0.1$, and $m = 5{,}000{,}000$.

The perfect information curve of Figure 60 slopes upward from 40 weeks when η is equal to 1.5 to 49 weeks when η is equal to 8. This suggests that the product introduction interval is positively related to η. The imperfect information curve is constant at 42 weeks because this is the optimal product introduction interval when η is equal to 3.5. The relatively horizontal nature of the perfect information curve suggests that, when b is equal to 0.1, and m is equal to 5,000,000, the product introduction interval is not greatly influenced by the value of η.

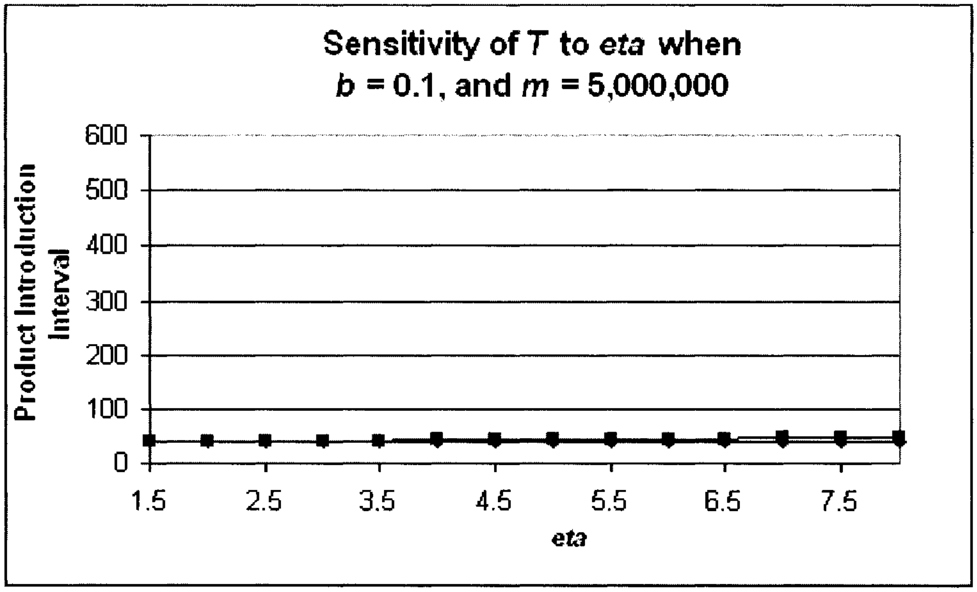

Figure 60 Sensitivity of T to η when $b = 0.1$, and $m = 5{,}000{,}000$.

Figures 61 and 62 are associated with b equal to 0.1, and m equal to 30,000,000. The perfect information curve of Figure 61 slopes downward from \$80,403,527 per week when η is equal to 1.5 to \$5,254,742 per week when η is equal to 8. This suggests that maximum profit per period is negatively related to η. The imperfect information curve slopes upward from \$15,652,909 per week when η is equal to 1.5 to \$15,653,556 when η is equal to 3.5 and then slopes downward to \$1,670,636 when η is equal to 8. The relative closeness of the two curves to the right of Figure 61 suggests that, when b is equal to 0.1 and m is equal to 30,000,000, profit per period is not greatly influenced by underestimation of η. The distance between the two curves to the left of Figure 61 suggests that, in the same situation, profit per period is greatly influenced by overestimation of η.

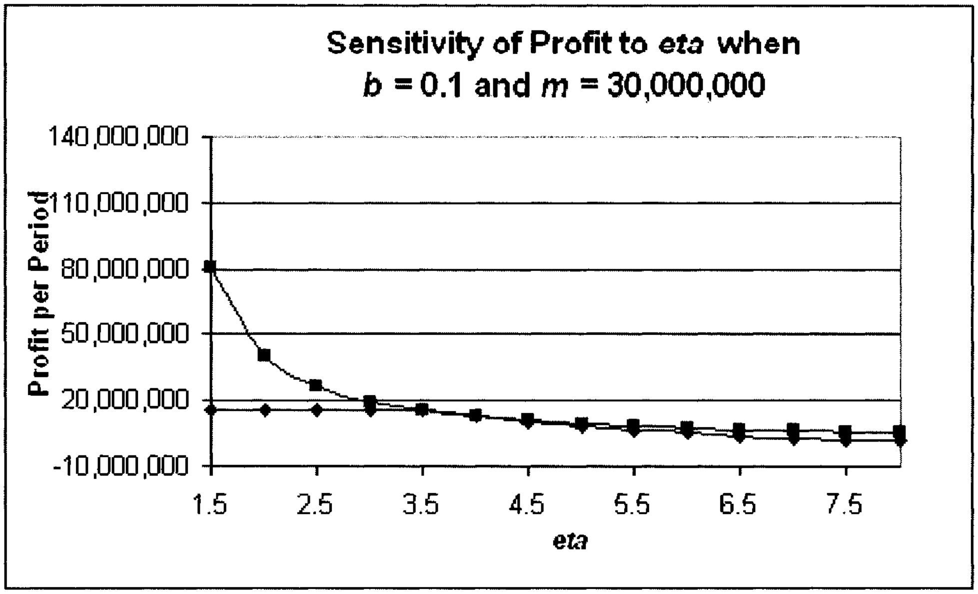

Figure 61 Sensitivity of profit to η when $b = 0.1$, and $m = 30{,}000{,}000$.

The perfect information curve of Figure 62 slopes upward from 39 weeks when η is equal to 1.5 to 40 weeks when η is equal to 8. This suggests that the product introduction interval is positively related to η. The imperfect information curve is constant at 39 weeks because this is the optimal product introduction interval when η is equal to 3.5. The near horizontal nature of the perfect information curve suggests that, when b is equal to 0.1, and m is equal to 30,000,000, the product introduction interval is not greatly influenced by the value of η.

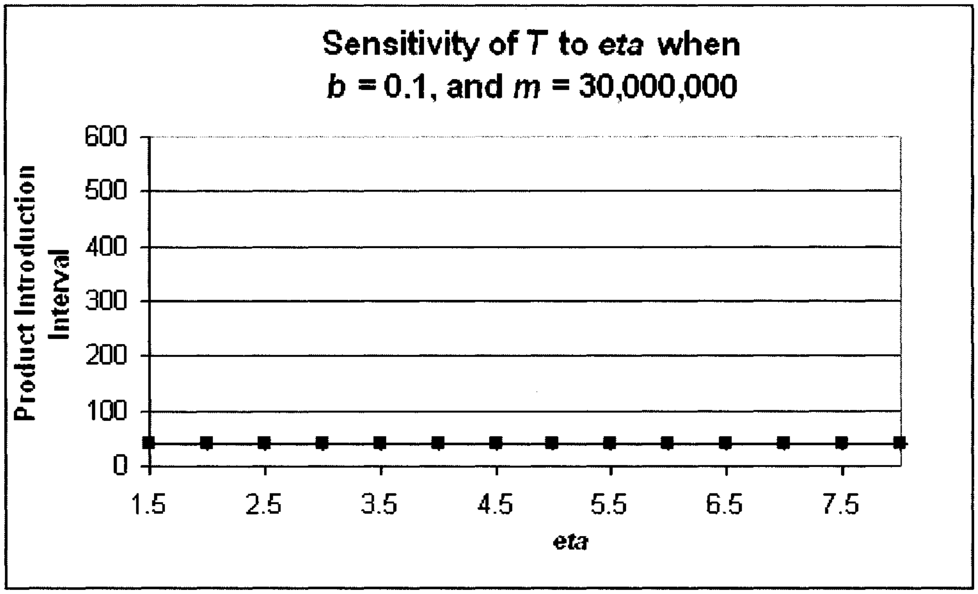

Figure 62 Sensitivity of T to η when $b = 0.1$, and $m = 30{,}000{,}000$.

Comparison of Figures 55, 57, 59, and 61 suggests that profit is negatively related to η but that the relationship is not generally strong unless b or m is large. Profit does not appear to be sensitive to underestimation of η, and appears to be sensitive to overestimation only when b and m are both large. Comparison of Figures 56, 58, 60, and 62 suggests that the optimal product introduction interval is not particularly sensitive to η. However, it appears that T is somewhat positively related to η when b and m are both small.

<u>The Main Effects of *H*</u>

In this section, H ranges from 0.2 to one in increments of 0.1 while a, b, m, η, S, V, and I are held constant at 15, 0.05, 10000000, 3.5, 20000, 70, and 20000000 respectively. At every point along the perfect information curves of Figures 63 and 64 the decision maker knows the value of H and behaves optimally whereas at every point along the imperfect information curves the decision maker believes the value of *H* to be 0.3 and behaves in a manner that would be optimal if that were true.

The perfect information curve of Figure 63 slopes downward from $2,427,441 per week when *H* is equal to 0.2 to $2,425,763 per week when *H* is equal to one. This suggests that maximum profit per period is negatively related to *H*. The imperfect information curve slopes downward from $2,427,092 per week when *H* is equal to 0.2 to $2,422,944 when *H* is equal to one. The near horizontal nature of the perfect information curve suggests that profit is not greatly influenced by *H*. The relative closeness of the two curves suggests that profit per period is not greatly influenced by misspecification of *H*.

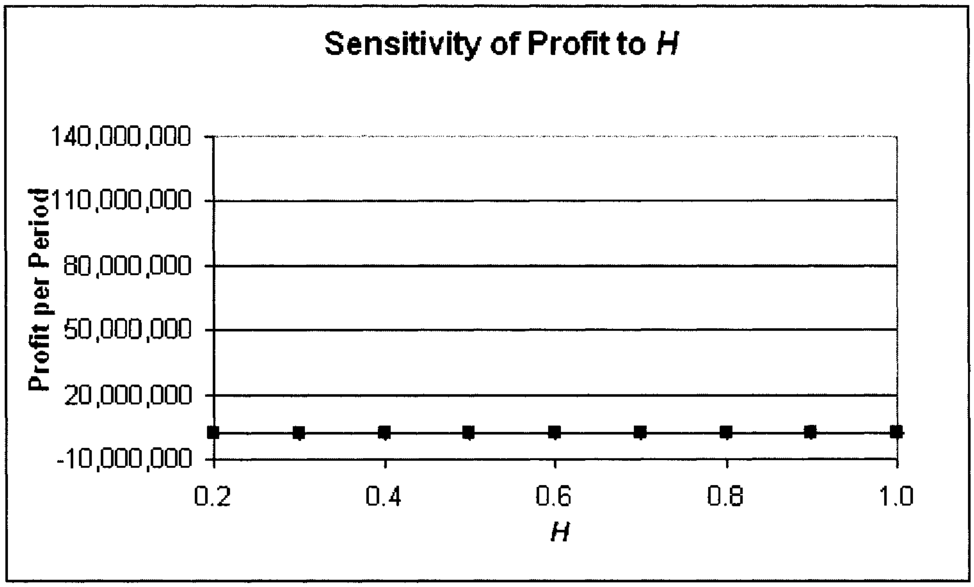

Figure 63 Sensitivity of profit to *H*.

Both the perfect information and imperfect information curves of Figure 64 are constant at 81 weeks. The horizontal nature of the perfect information curve suggests that the product introduction interval is not influenced by the value of *H*.

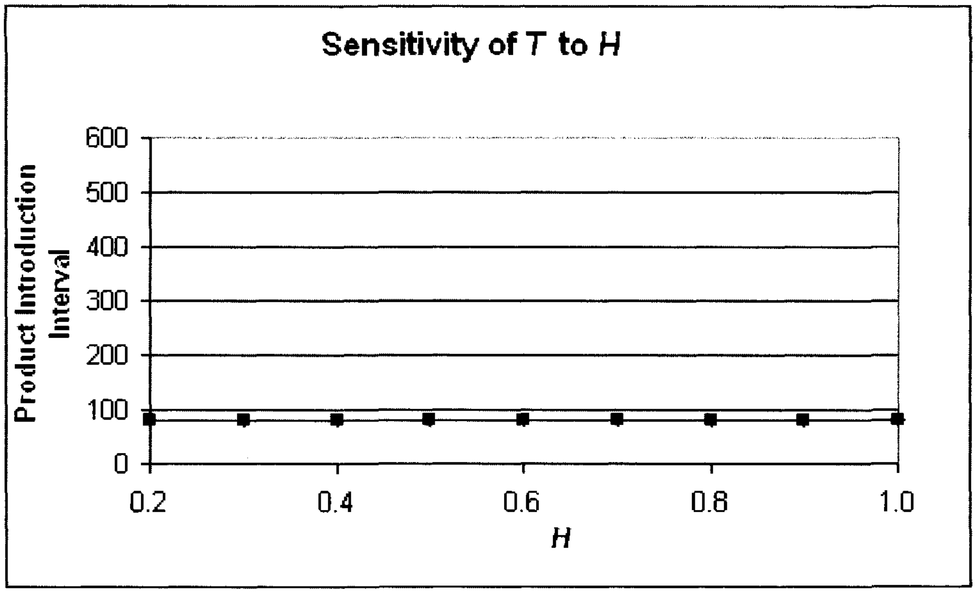

Figure 64 Sensitivity of *T* to *H*.

The Main Effects of *S*

In this section, *S* ranges from 2,500 to 30,000 in increments of 2,500 while *a*, *b*, *m*, η, *H*, *V*, and *I* are held constant at 15, 0.05, 10000000, 3.5, 0.2, 70, and 20000000 respectively. At every point along the perfect information curves of Figures 65 and 66 the decision maker knows the value of *S* and behaves optimally whereas at every point along the imperfect information curves the decision maker believes the value of *S* to be 20,000 and behaves in a manner that would be optimal if that were true.

The perfect information curve of Figure 65 slopes downward from $2,443,326 per week when *S* is equal to 2,500 to $2,418,214 per week when *S* is equal to 30,000. This suggests that maximum profit per period is negatively related to *S*. The imperfect information curve slopes downward from $2,442,129 per week when *S* is equal to 2,500 to $2,417,684 when *S* is equal to 30,000. The near horizontal nature of the perfect information curve suggests that profit is not greatly influenced by *S*. The relative closeness of the two curves suggests that profit per period is not greatly influenced by misspecification of *S*.

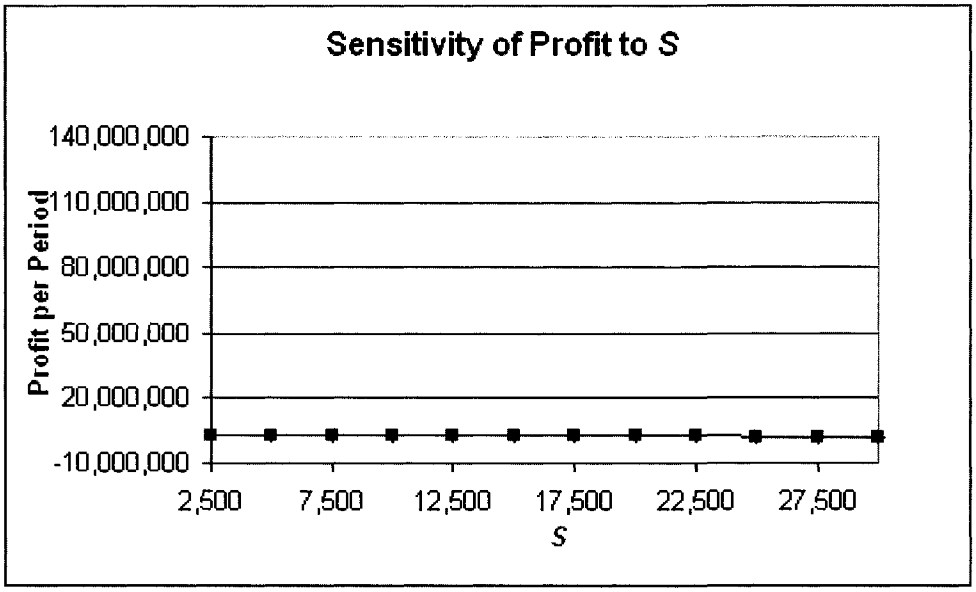

Figure 65 Sensitivity of profit to *S*.

Both the perfect information and imperfect information curves of Figure 66 are constant at 81 weeks. The horizontal nature of the perfect information curve suggests that the product introduction interval is not influenced by the value of *S*.

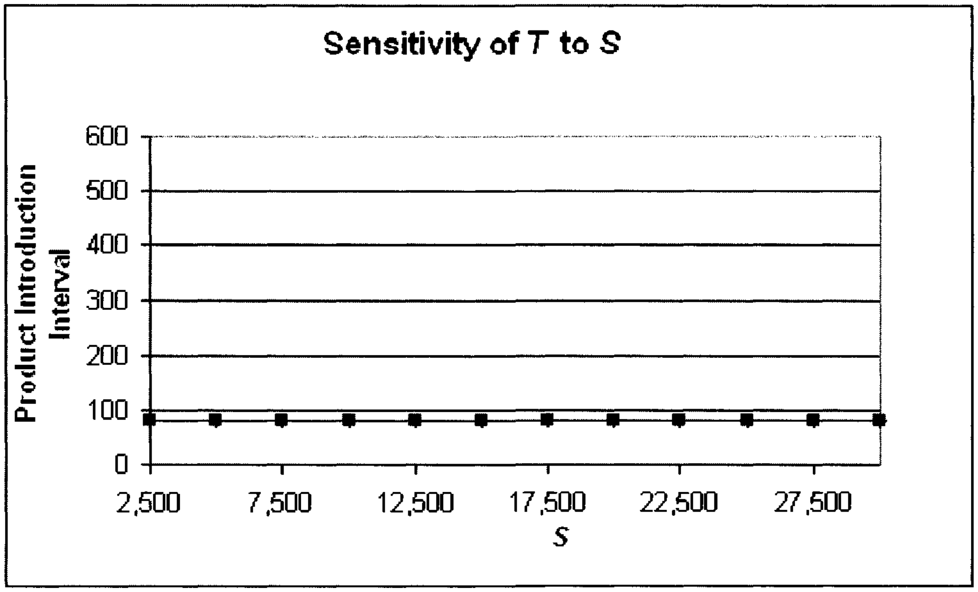

Figure 66 Sensitivity of *T* to *S*.

<u>The Main Effects of *V*</u>

In this section, *V* ranges from 50 to 100 in increments of 5 while *a*, *b*, *m*, η, *H*, *S*, and *I* are held constant at 15, 0.05, 10000000, 3.5, 0.2, 20000, and 20000000 respectively. At every point along the perfect information curves of Figures 67 and 68 the decision maker knows the value of *V* and behaves optimally whereas at every point along the imperfect information curves the decision maker believes the value of *V* to be 70 and behaves in a manner that would be optimal if that were true.

The perfect information curve of Figure 67 slopes upward from $1,657,799 per week when *V* is equal to 50 to $3,581,204 per week when *V* is equal to 100. This suggests that maximum profit per period is positively related to *V*. Note that this counterintuitive result is due to the fact that prices, and thus revenue, are positively related to *V*. The imperfect information curve slopes upward from $1,096,044 per week when *V* is equal to 50 to $2,426,573 when *V* is equal to 70 then downward to negative $456,880 when *V* is equal to 100. The near horizontal nature of the perfect information curve suggests that profit is not greatly influenced by *V*. The relative closeness of the two curves suggests that profit per period is not greatly influenced by misspecification of *V*. However, the distance between the two curves to the right of Figure 57 suggests that grossly underestimating *V* can lead to a noticeable reduction in profit.

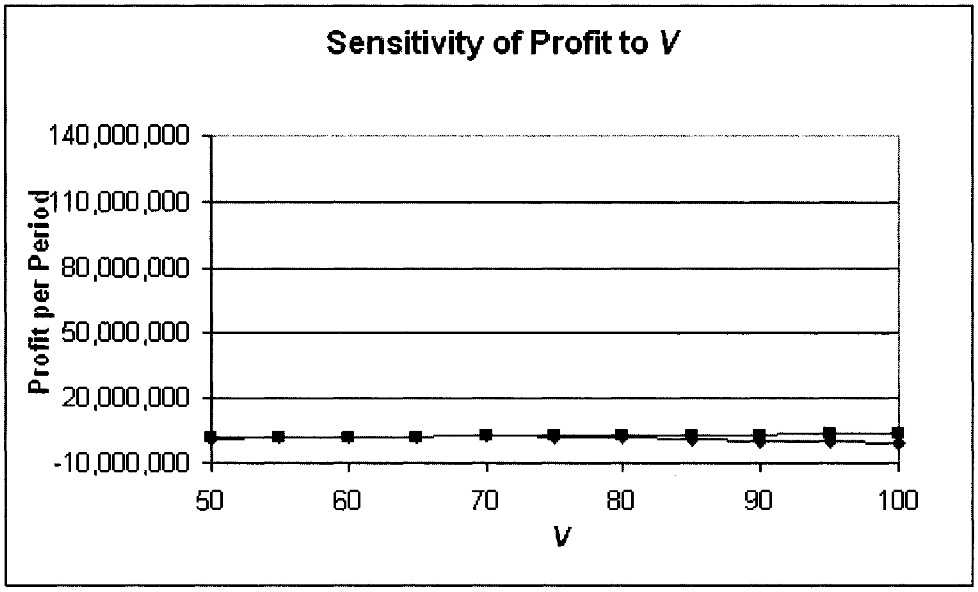

Figure 67 Sensitivity of profit to *V*.

The perfect information curve of Figure 68 slopes downward from 82 weeks when *V* is equal to 50 to 80 weeks when *V* is equal to 100. This suggests that the product introduction interval is negatively related to *V*. The imperfect information curve is constant at 81 weeks because this is the optimal product introduction interval when *V* is equal to 70. The near horizontal nature of the perfect information curve suggests that the product introduction interval is not greatly influenced by the value of *V*.

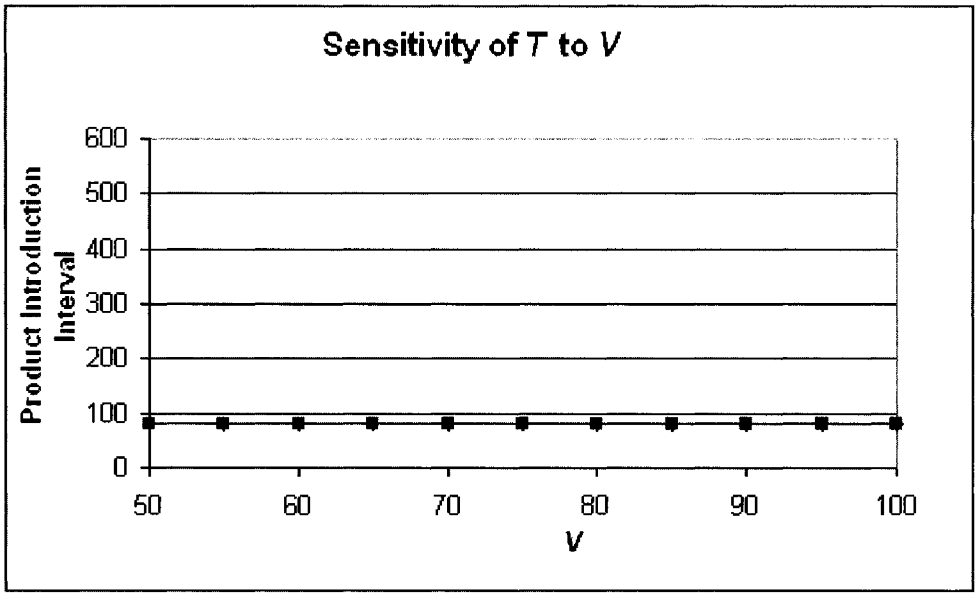

Figure 68 Sensitivity of *T* to *V*.

The Main Effects of *I*

In this section, *I* ranges from 5,000,000 to 25,000,000 in increments of 2,500,000 while *a*, *b*, *m*, η, *H*, *S*, and *V* are held constant at 15, 0.05, 10000000, 3.5, 0.2, 20000, and 70 respectively. At every point along the perfect information curves of Figures 69 and 70 the decision maker knows the value of *I* and behaves optimally whereas at every point along the imperfect information curves the decision maker believes the value of *I* to be 20,000,000 and behaves in a manner that would be optimal if that were true.

The perfect information curve of Figure 69 slopes downward from $2,614,493 per week when *I* is equal to 5,000,000 to $2,365,238 per week when *I* is equal to 25,000,000. This suggests that maximum profit per period is negatively related to *I*. The imperfect information curve slopes downward from $2,611,758 per week when *I* is equal to 5,000,000 to $2,364,845 when *I* is equal to 25,000,000. The near horizontal nature of the perfect information curve suggests that profit is not greatly influenced by *I*. The relative closeness of the two curves suggests that profit per period is not greatly influenced by misspecification of *I*.

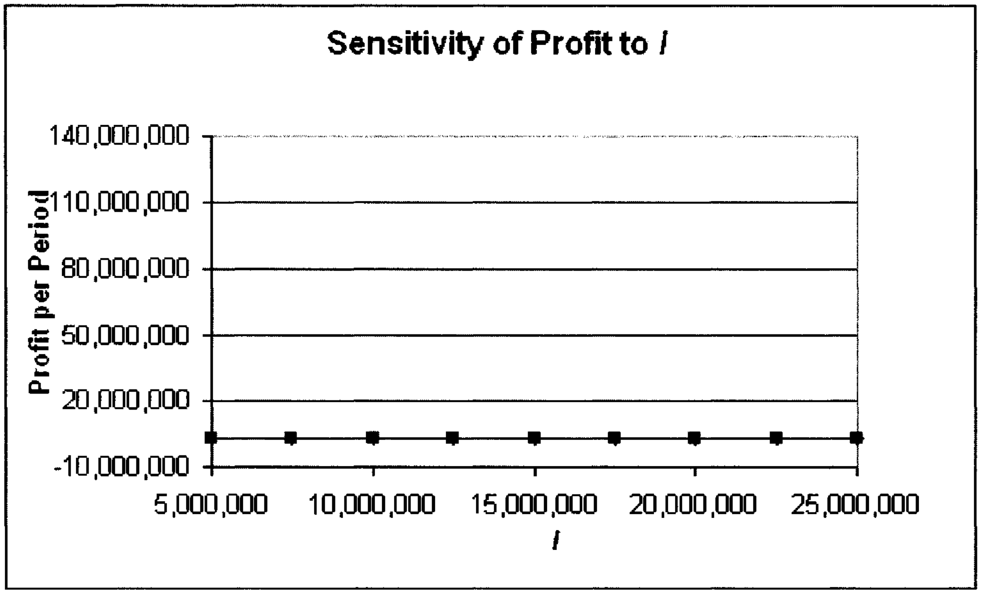

Figure 69 Sensitivity of profit to *I*.

The perfect information curve of Figure 70 slopes upward from 78 weeks when *I* is equal to 5,000,000 to 82 weeks when *I* is equal to 25,000,000. This suggests that the product introduction interval is positively related to *I*. The imperfect information curve is constant at 81 weeks because this is the optimal product introduction interval when *I* is equal to 20,000,000. The near horizontal nature of the perfect information curve suggests that the product introduction interval is not greatly influenced by the value of *I*.

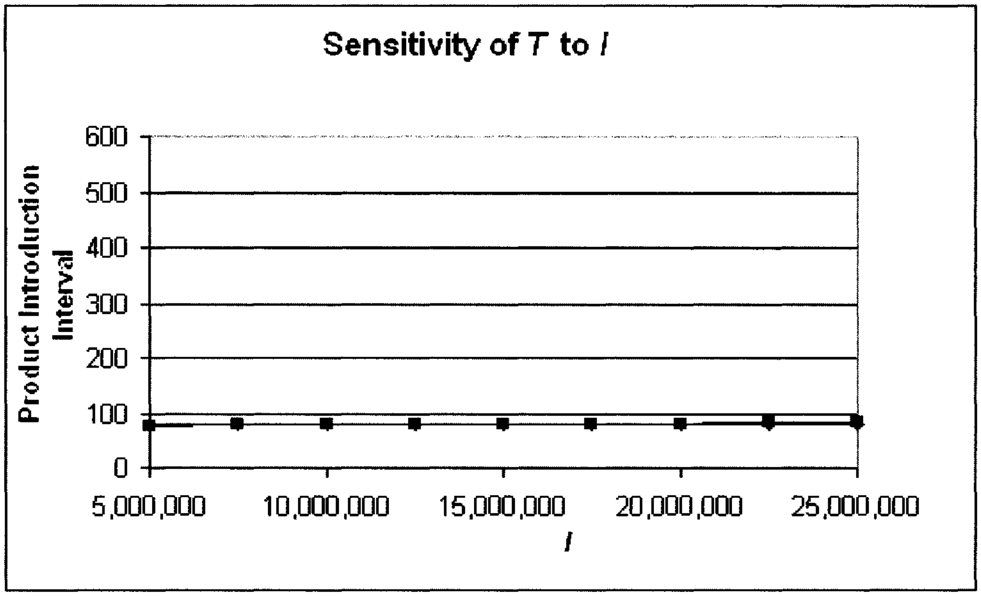

Figure 70 Sensitivity of *T* to *I*.

CHAPTER 6

SUMMARY AND FUTURE RESEARCH

Dissertation Overview

This dissertation presents an exact solution methodology that simultaneously optimizes the pricing, lot-sizing, and new product introduction timing decisions. Chapter 1 grounds the study by defining the types of products being considered. This study is concerned with products in industries characterized by innovations in fashion and technology where technological innovations occur often but are iterative in nature and where fashion innovations are disruptive. Cellular telephones and other boutique electronics items, such as personal organizers or iPods, are examples of this type of product.

Chapter 2 is concerned with summarizing and presenting the pertinent literature. The pertinent literature can be divided into three broad categories. The first category concerns the various lot-sizing methodologies that have been presented over the years. The second category concerns the diffusion of innovations. The third category is concerned with new product introduction timing.

Chapter 3 is concerned with presenting and describing the mathematical model used to represent the problem. The objective is to maximize the long-term average profit per period by manipulating product prices, production quantities, production timing, and timing of new product introductions. The mathematical model presented in Chapter 3 is similar to that of Wagner and Whitin (1958b) but has been modified to reflect the inclusion of a relationship between price and demand in a manner similar to Wagner and Whitin (1958a) and Thomas (1970).

Chapter 4 is concerned with the solution methodology used to solve the problem presented by the model developed in Chapter 3. The solution methodology resembles the method presented by Thomas (1970) and is similar to, but more elaborate than, the method presented by Wagner and Whitin (1958b). All three methods are based conceptually on the methodology outlined, but not explicitly defined, by Wagner and Whitin (1958a).

Chapter 5 is concerned with the sensitivity of both profit per period and the new product introduction interval to the various model input parameters. Interactions are investigated both at optimality and in the presence of imperfect information concerning input parameter values.

Chapter 6 serves to remind the reader of the contribution of each chapter to the conclusion of the dissertation. This chapter also includes further discussion and analysis of material from the other chapters. Finally, likely areas of future research suggested by this dissertation are discussed.

Summary of Dissertation Results

The major contribution of this dissertation is the scope of the problem addressed. It spans three separate bodies of literature in the management and marketing disciplines: lot sizing, diffusion of innovations, and new product introduction timing. In this section, each of these three bodies of literature is discussed briefly and the dissertation's place within each is addressed.

Results Relative to the
Lot-Sizing Literature

The lot-sizing literature is a very mature field with seminal works nearly a century old (e.g., Harris, 1913). Despite the tenure of this body of literature, it has remained relatively isolated from other bodies of literature that address issues that may impact lot-sizing decisions. As shown in Chapter 2, few articles incorporate product pricing decisions and lot-sizing decisions. Thomas (1970) presents a methodology that simultaneously optimizes pricing and lot-sizing decisions. This dissertation represents an extension of the work of Thomas along two main directions. First, Thomas examines the problem from a strictly operations research point of view. As such, no consideration is given to the appropriateness of the shape of the demand function or to the values of the various model parameters. In contrast, this dissertation includes a thorough grounding in the form of justification for the basic demand curve, for the price response function that relates demand to price, and for the values of the various input parameter values. These business considerations are ignored by Thomas. In addition, this dissertation differs from the work of Thomas in that it presents a model that includes new product introduction timing and thus bridges the gap between the lot-sizing and new product introduction timing literature.

Results Relative to the Diffusion of
Innovation Literature

Although not as old as the lot-sizing literature, the diffusion of innovation literature is also well established and mature. Chapter 2 divides the diffusion of innovation literature into two sections: that which considers price, and that which considers multiple product iterations. This dissertation exists at the intersection of these two sub-categories of the literature. Although the diffusion of innovation literature is quite mature, it has also remained somewhat insulated from other bodies of literature. Of the diffusion of innovation articles addressed in Chapter 2, very few consider operational issues such as production costs and lot-sizing. A few (e.g., Bass, 1980) do consider production costs, particularly the impact of learning on production costs; however, after a thorough review of the literature it appears that this dissertation, to the best knowledge of its author, is the only study that incorporates production lot-sizing decisions into a diffusion of innovation model that incorporates price.

Results Relative to the New Product Introduction Timing Literature

The new product introduction timing literature is the youngest and least developed of the three bodies of literature that relate to this dissertation. That being said, it is admirable that connections have already been made between this body of literature and the diffusion of innovation literature (e.g., Kurawarwala and Matsuo, 1996). However, after a thorough review of the literature it appears that this dissertation, to the best knowledge of its author, is the only study that incorporates new product introduction timing, diffusion of innovation, production lot-sizing, and product pricing all into a single model.

Managerial Implications

Based on the findings in Chapter 5, it appears that profit is much more sensitive to misspecification of the a, b, m, and η parameters than it is to misspecification of the H, S, *V*, and *I* parameters. In this study, *a* is a measure that captures the relative market influence of imitators and innovators, *b* is a measure that captures the market influence of both innovators and imitators, *m* is a measure that captures potential market size, η is a measure that captures price elasticity of demand. These are market-oriented parameters. On the other hand, *H* is a measure that captures the cost of holding one unit of inventory for one period, *S* is a measure that captures the cost of setting up a production run, *V* is a measure that captures the variable cost per unit of the product, and *I* is a measure that captures the fixed costs associated with introducing a new product iteration. These are operations-oriented parameters. The relative sensitivity of profit to misspecification of these two groups of input parameters has major managerial implications. It means that it is more important to precisely estimate the market-oriented parameters than it is to precisely estimate the operations-oriented parameters.

One important caveat is required for the previous statement. That is, the results have not yet been shown to be generalizable beyond the model presented in Chapter 3. Chapter 4 presents an optimal solution methodology for a particular dynamic lot-sizing problem. Equation 1 to Equation 3 and Table 2 and Table 3 are based on a survey of the literature rather than data from a particular manufacturer. With minor modification, the method described in Chapter 4 can be used to solve a variety of dynamic lot-sizing problems of similar difficulty. A major direction for future research is to develop an alternate model that is tuned empirically to match a particular manufacturing organization. Not only would such a model be a valuable decision support tool for making pricing, lot-sizing, and new product introduction timing decisions, it would also provide very valuable information in the form of a sensitivity analysis similar to that found in Chapter 5. In essence, it would let the organization know which parameters must be accurately assessed and which parameters are keys to higher profits.

The ultimate goal of this research stream is the generation of a decision support tool that is of practical value to actual decision makers in industry. The tool will help decision makers make decisions concerning pricing, production timing, production quantities, and new product introduction timing. The software described in Chapter 4 represents an important first step in this direction. Determination of the model's relative insensitivity to most parameter values is another important step. However, before the model can be used in a practical setting, its sensitivity to various assumptions must also be determined. This could progress in a systematic way once appropriate empirical data becomes available.

Model Limitations and Extensions

With slight modifications, the solution methodology presented in Chapter 4 can be used to solve many other models of similar difficulty to the one presented in Chapter 3. The notion of similar difficulty is rather complex. It may be possible to add additional complicating factors to the problem without changing the model's difficulty. For example, it may be possible to include the impact of learning on production costs in the model (e.g., Bass, 1980) without requiring more than slight modification to the solution process. The most important characteristic of the set of models that can be solved by slight, as opposed to major, modification to the existing procedure is that they must be deterministic. After some initial investigation, it appears that changes to the first two equations have very little impact on the difficulty of solving the problem so long as they do not introduce uncertainty into the model (e.g. Krankel et al, 2006). Changes to the price response function require moderate changes to the solution processes. This suggests that relatively substantial model changes can be incorporated with little in the way of required changes to the solution process. This opens up the possibility to include elements that were previously omitted from the problem so as to keep the complexity to an acceptabe level. For example, it now seems very likely that the solution logic presented in the Chapter 4 can be used on models that include the effect of learning on production cost.

Despite the flexibility of the solution logic, it is still constrained greatly by certain assumptions. A major extension of this research concerns relaxing some of the assumptions that currently limit the model. It is very likely that some manufacturing organizations face situations too complex to accurately be captured by any model of the same class of difficulty as the model presented in Chapter 3. The inclusion of capacity restrictions (e.g., Haugen, Olstad, & Petterson, 2007), stock outs (e.g. Graves, 1996), backorders (e.g., Chu & Chung, 2004), or model parameters that change over time (Federgruen & Tzur, 1994) may be required for the model to be of sufficient fidelity to be of practical relevance to these organizations. The creation of a less stylized model is a major avenue for future research.

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