Spring 2009

The relationship between pedagogical content knowledge and mathematics teacher questioning strategies

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THE RELATIONSHIP BETWEEN PEDAGOGICAL CONTENT KNOWLEDGE AND MATHEMATICS TEACHER QUESTIONING STRATEGIES

by

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A Dissertation Presented in Partial Fulfillment
Of the Requirements for the Degree
Doctor of Education

COLLEGE OF EDUCATION
LOUISIANA TECH UNIVERSITY

May 2009

LOUISIANA TECH UNIVERSITY

THE GRADUATE SCHOOL

April 15, 2009

Date

We hereby recommend that the dissertation prepared under our supervision

By Lynne Smith Nielsen

entitled The Relationship Between Pedagogical Content Knowledge and Mathematics Teacher Questioning Strategies

be accepted in partial fulfillment of the requirements for the Degree of

Doctor of Education

Supervisor of Dissertation Research

Head of Department

Department

Recommendation concurred in:

Advisory Committee

Approved:

Director of Graduate Studies

Dean of College

Approved:

Dean of the Graduate School

GS form 13
(5/03)

ABSTRACT

The purpose of this study was to determine whether an intense two-week professional development program for middle school mathematics teachers, along with follow-up classroom visits, video review of lessons (with feedback), and two six-hour follow-up sessions each semester would improve teacher questioning strategies and promote higher-level questioning based on Bloom's (1956) Taxonomy of cognitive categories. A second purpose of the study was to determine whether, through reflection and instructor feedback, teachers would gain the ability to involve students in *high-press* questioning situations. The third purpose of the study was to investigate whether relevant professional development would result in an increase in teacher content knowledge. The fourth purpose was to determine if there was a relationship between teachers' pedagogical content knowledge, as measured by the CKT-M, and their ability to ask better questions, as determined by Bloom's (1956) Taxonomy of cogitative categories.

The researcher used four statistical tests including chi-square tes*t*, *z*-test, *t*-test, and Spearman correlation. The population for this study was a group of 18 middle school mathematics teachers from southwest Arkansas. The instruments used for the study included several forms created by the instructor team as well as the Content Knowledge for Teaching-Mathematics pre- test and post-test. Prior to gathering data, human use forms and participant consent forms were completed from both Southern Arkansas University and Louisiana Tech University. In addition each teacher was required to have parental consent for each student involved in the video recording. Results showed

significance in the first three of four hypotheses. Significance was found in the first hypothesis using a chi-square test that compared the number of high-level questions asked in the first video compared to the number of high-level questions asked in the last video. Significance was found in the second hypothesis using a two-proportion *z*-test that compared the number of high-press exchanges in the first video to the number in the last video. Significance was found in the third hypothesis using a paired *t*-test that compared the pre-test score to the post-test score on the CKT-M. There was no significant relationship found between teachers' pedagogical content knowledge on the post-test and teachers' use of high-level questions as defined by Bloom's (1956) Taxonomy.

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DEDICATION

This dissertation is the culminating project prepared during a journey into learning that began many long years ago. Without a doubt this endeavor would never have become a reality without the love and support of my wonderful, encouraging, soul mate and husband Dr. Bill Nielsen, my beautiful and inspiring daughter, Maggi, my independent and fun-loving son, Tanner, and my caring and compassionate son, Jake. They sacrificed their mom and wife in order for this to happen.

The urge to begin this life-long goal to become a doctor would not have been without the sincere love for learning that was instilled in me as a small child by my selfless, endearing mother, Betty Smith and her lovely mother, Elizabeth Wyrick (fondly referred to as Grandmother). My mom encouraged me, she laughed with me and cried with me, she believed in me, and she loved me unconditionally and immensely, as I loved her. She gave me the desire to make a great life for myself, the courage to set lofty goals, and the hard-headedness to set about meeting those goals at all cost. Even though she will not celebrate this accomplishment with us in person, she believed it would someday happen and pride would have filled her heart.

My Grandmother is a wonderful, 93 year old bundle of joy and is a very well-respected and loved woman in our community. She made sure that I had every opportunity to go and do as other kids, even though my mother couldn't fill this role because of her handicapping condition. Grandmother carried me to church, she taught me about God, she taught me honesty and integrity, she taught me practicality and appreciation, like hanging clothes out to dry-to save energy and smell fresh, and she taught me to stop and smell the flowers on an early spring morning.

It is to these family members mentioned and so many other family members and friends that I express my sincere love and appreciation and dedicate this dissertation. You have been and always will be my inspiration and I love you all!

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ACKNOWLEDGMENTS

There are several special people to whom I would like to express my sincere appreciation. First and foremost, to my committee chair, Dr. Lawrence Leonard, "thank you" does not seem adequate as indicative of my gratitude for your unfailing support and gentle reminders. You have read revision after revision and returned it to me in record time. You became the chair of my committee because the other faculty members in my area were "full." I know your plate was full as well, however, you made time for me, and I can't thank you enough for your time, professionalism, support, encouragement, and expertise along this road that we have traveled together. I will never forget your kindness, your integrity, and your dedication, not only to me, but also to your work. The students of Louisiana Tech University and the Louisiana Education Consortium are blessed to have you as a professor and you are an outstanding faculty member of this university and friend to many. You have impacted many lives here and you have definitely made a difference in mine, particularly.

To Dr. Bill White, Dr. Berand Schroder, and Dr. Beverly Flowers-Gibson, I'm sure there were times you were doubtful that I would ever bring this project to fruition. My experience seemed to be a vicious cycle of "hurry up and wait." However, you were all wonderful in your support, professional expertise, and willingness to provide comments and insight with each juncture.

To my friends at Southern Arkansas University, Dr. Debe Kincaid, Dr. Ed Kardas, and Debbie Wilson, you have each provided me with an assortment of irreplaceable advice and assistance that could not be matched anywhere. Each of you helped in your own special way, always with a smile.

George (Dr. George Bratton)...thanks man!

Dr. Cage, even though you didn't serve on my committee, you were an integral part of the completion process. You are my most respected educational statistician and I'm so blessed to have learned from the best.

To the faculties of the three universities comprising the LEC program, I extend my appreciation and congratulations to you for successfully managing the doctoral programs. This would not be possible without the continued collaboration and wonderful people at Grambling State University, University of Louisiana Monroe, and Louisiana Tech University.

CHAPTER 1

INTRODUCTION

The first significant publication in the current generation of American educational reform appeared in April of 1983: *A Nation at Risk* (Denning, 1983). The national study was ordered by U.S. Secretary of Education Terrel H. Bell in response to "a widespread public perception that something is seriously remiss in our educational system" (Education Week, 2000, p. 130). He created the National Commission on Excellence in Education to study the problems of public schooling. *A Nation at Risk* has been called the most influential and controversial report in history. This report described a society that no longer dominated the international economy and an education system confused about its purpose. The report concluded that

> The educational foundations of our society are presently being eroded by a rising tide of mediocrity that threatens our very future as a nation and a people. We have, in effect, been committing an act of unthinking, unilateral educational disarmament. (p. 130)

Some of the greatest changes that came about during this first wave of reform as a result of the report were higher graduation requirements, standardized curriculum, increased teacher and student testing, and higher certification requirements.

The second wave of reform came about in the mid-1980s as researchers found that despite the actions resulting from *A Nation at Risk*, the discussion around curricular matters was disappointing. Boyer stated, "It [*A Nation at Risk*] has not led to a serious and creative look at the curriculum, instead, states have simply been adding units along traditional lines, almost mindlessly, without asking what it is we ought to be teaching in them" (Education Week, 2000, p.130). In 1989 a national summit on these issues took place in Charlottesville, Virginia. During that gathering, governors and educational leaders committed themselves to working toward a set of goals for the nation's schools. This group displayed high aspirations for all students to master challenging subject matter in core disciplines and for U.S. students to become "first in the world" in science and mathematics achievement (p. 131). This second reform wave of the late 1980s concentrated on teacher empowerment, site-based management, parental involvement, decentralization, and increased use of technology.

By the 1990s, policy makers and educators decided the country needed to translate those goals into academic standards that spelled out exactly what students should know and be able to do. Under the Clinton administration, on March 26, 1994, the Goals 2000: Educate America Act was passed by the U.S. Congress. The intention of this bill was to

> improve learning and teaching by providing a national framework for education reform; to promote research, consensus building, and systemic changes needed to ensure equitable educational opportunities and high levels of educational achievement for all American students; and to promote the development and adoption of a voluntary national system of skill standards and certifications. (Schugurensky, 2002, ¶ 1)

The legislation mandated distinct goals in eight categories that were to be met by the year 2000: school readiness; school completion; student achievement and citizenship; teacher education and professional development; mathematics and science; adult literacy and lifelong learning; safe, disciplined and drug-free schools; and school and home partnership.

The beginning of the 21st century brought with it the No Child Left Behind Act (NCLB) (2001) under the administration of President George W. Bush. Under NCLB guidelines, all students were mandated to be tested in reading and mathematics by criterion-referenced tests in grades 3 through 8. The tests were aligned with state standards and all students were to be *proficient* by the year 2014, although the definition of *proficient* varies from state to state. NCLB also required that all states make adequate yearly progress (AYP) by improving the scores of their students each year and that all teachers must be considered *highly qualified* by the school year 2005-2006. The emphasis of NCLB, as re-issued by congress in 2007, is on teacher quality, student performance, school choice, and instructional strategies based on proven scientific research (NCLB, 2007). Each of these benchmark initiatives described in educational history has had specific effects on mathematics education.

The movement to reform mathematics education began in the mid-1980s in response to the documented failure of traditional methods of teaching mathematics (Battista, 1999). Public school students of the United States ranked dismally low and well below some of their foreign counterparts as reflected by decades of documented performance on the National Assessment of Educational Progress (NAEP) and by three international studies, including the Third International Mathematics and Science Study.

According to Stigler and Hiebert (1999), assuming that the traditional mathematics programs have shown themselves to be successful is ignoring the largest database available. In response to the work by Stigler and Hiebert, Reys (2001) stated, "The evidence indicates that the traditional curriculum and instructional methods in the United States are not serving our students well" (p. 256).

Reacting to this body of negative research results, major mathematics education professional organizations, such as the National Council of Teachers of Mathematics (NCTM) and the Mathematical Sciences Education Board recommended fundamental changes in the way Americans teach and assess their children (Huntley, Rasmussen, Villarubi, Sangton, & Fey, 2000). The recommendations sought drastic restructuring of traditional mathematics curricula (Manoucherhri & Goodman, 1998). Recommendations for reform in mathematics education uniformly call for an increased emphasis on meaningful experiences in mathematics and a decreased emphasis on the repeated practice of computational algorithms. The NCTM standards recommend that the curriculum emphasize problem solving, reasoning, making connections between mathematical topics, communicating mathematical ideas, and providing opportunities for all students to learn (NCTM, 1989, 1991, 1995, 2000, 2006; Riordan & Noyce, 2001). In essence, reform recommendations found in these NCTM documents dealt with how mathematics is taught, what mathematics is taught, and at a more fundamental level, the very nature of school mathematics (Battista, 1999).

With the mathematics reform movement in full swing since the mid-1990s and the publication of the national mathematics standards (NCTM, 1989, 2000), teachers have been encouraged to shift their classroom practices away from an exclusive focus on computational accuracy and to strive toward a focus on deeper understanding of the mathematical ideas, relations, and concepts that span all five strands of the NCTM standards (Hiebert & Carpenter, 1992; Lampert, 1991). The standards set forth by the NCTM were created with the most current mathematical research in mind. They charge teachers with the task of creating learning conditions for students that most teachers have not only never experienced themselves as students but also that they were never trained to do in their pre-service programs (Cohen & Ball, 1990; Fullan, 1991; Lloyd & Frykholm, 2000).

Implementing recommendations from research in the classroom is a complex and sometimes daunting task for teachers (Kazemi & Stipek, 2001; Spielman & Lloyd, 2004). Studies of teacher learning have found that teachers are able to incorporate some strategies from the reform-minded curricula into their teaching. They can do things like assign students multi-level, real world problems and provide manipulatives and technology for their use. They may even allow children to work collaboratively and then share their strategies and solutions. The NCTM (1989, 2000) endorses all of these teaching strategies; however, many teachers only superficially implemented the standards and teachers failed to stimulate students' conceptual understanding of the mathematics involved (Ball, 1993; Cobb, Wood, & Yackel, 1993; Cobb, Wood, Yackel, & McNeal, 1993; Cohen, 1990; Fennema, Carpenter, Franke, & Carey, 1993; Jacobs & Morita, 2002; Ma, 1999; Prawat, 1992). These new teaching strategies are necessary, but they are not enough to ensure that students are building a sophisticated understanding of mathematics (Kazemi & Stipek). Boaler (2002) found that the manner in which teachers present material to students had a direct impact not only on student understanding but also on equity. The recommended teaching practices included introducing activities through discussion, teaching students to explain and justify, and making real world contexts accessible to all students.

Statement of the Problem

Careful, intentional, and mindful questioning is one of the most powerful tools a skillful teacher possesses. Good questions help students make sense of mathematics. Questions should be open-ended in answer and approach. Such questions empower students to unravel their misconceptions and not only require the application of facts and procedures but also encourage students to make connections and generalizations. Good questions are accessible to all students in their language and offer an entry point for all students. Most importantly, their answers lead students to wonder more about a topic and to construct new questions on their own as they investigate newly found interests (Costa & Kallick, 2000).

Many teachers find it easy to pose questions and to ask students to describe their strategies; however, it is more challenging, pedagogically, to engage students in genuine mathematical inquiry and push them beyond what might come easily for them. Doing so is often more difficult because some teachers are unsure of their own understanding of mathematics. They may have a shallow conceptual understanding of the connectedness of mathematical ideas and procedures, and without depth of understanding it can be difficult to ascertain when explanations made by students are appropriate in all mathematical cases (Ball & Bass, 2000; Chazan & Ball, 1995; Fennema et al., 1996; Franke, Carpenter, Fennema, Ansell, & Behrend, 1998; Junk, 2005; Kazemi & Stipek, 2001).

According to Jacobs and Ambrose (2003), teacher questioning has become a popular topic of interest and research. Educators have addressed this issue by the development of multiple lists of potential questions that teachers can use with children during problem solving situations. However, although Jacobs and Ambrose agree that such lists can provide teachers with a starting point to get their students involved in the problem, they contend that the most effective questions cannot be preplanned but must occur in response to a child's specific action or idea. Thus, the effectiveness of a teacher's question can be determined only by considering how it is situated in the context of the teacher-student interaction.

Teachers need the opportunity to practice and analyze their own teaching. Grant, Kline, and VanZoest (2001) found positive results using video-taped lessons to enhance teachers' reflection on the launch of a lesson, the support students need during their exploration, and the closure/summary of the lesson. One study (Chval, 2004) showed through self-reflection that teachers were able to understand how the same lesson taught multiple times might go in a different direction depending on the students in the group, their language and discourse, and their responses to the questions asked by the teacher. Therefore, there is a need to determine a way to improve teacher pedagogical content knowledge and questioning strategies and to determine whether there is a correlation between the two. Also, teachers need to be able to engage and monitor students in high-level mathematical conversations, requiring students to go beyond short descriptions of a series of steps used to solve a problem to being able to relate their problem solving to other mathematical strategies and ideas.

Purposes of the Study

There were four main purposes of this study. One purpose was to determine whether an intense two-week professional development program for middle school teachers, along with follow-up classroom visits, video review of lessons (with feedback), and two six-hour follow-up sessions each semester would improve their questioning strategies and promote higher level questioning based on Bloom's (1956) Taxonomy of cognitive categories. Over the course of time, many lists of cognitive skills have been delineated. There are a variety of taxonomies that educators can consult for help. These include Sternberg's (1985) psychological schema and Lipman, Sharp, and Oscanyan's (1977) philosophical schema among others. However, according to Kloss (1988), Bloom's (1956) Taxonomy is the most useful for considering the relationship of cognitive levels to question asking. He contended that Bloom's six levels provide not only a structural model for the creation of questions but also a means of assessment and evaluation that has become increasingly important with the increased emphasis on teacher accountability.

A second purpose of the study was to determine whether, through reflection and instructor feedback, teachers would gain the ability to involve students in *high-press* questioning situations. (The instructors for the professional development program were two Southern Arkansas University mathematics faculty members, one math specialist from the SAU math/science center, and one mathematics specialist from the Arkansas Department of Education.) Kazemi and Stipek (2001) described high-press questions as those that engage students in a mathematical explanation that consists of more than just a description of a procedure. These questions require students to understand relationships among multiple strategies as well as explore contradictions in solutions and the pursuit of alternative strategies.

The third purpose of the study was to investigate whether professional development would result in an increase in teacher content knowledge. The fourth purpose was to determine if there was a relationship between teachers' pedagogical mathematics content knowledge as measured by the CKT-M (Hill, Ball, Schilling, & Bass, 2005) and their ability to ask better questions. Schoen, Cebulla, Finn, and Fi (2003) showed that student achievement gains were higher when teachers used certain instructional techniques including questioning and student interviews. This study was conducted across a wide range of students in schools with varying socio-economic status levels, sizes and ethnic mixes of school populations, beginning achievement levels, length of classes, and class size.

Justification of the Study

This study used video reflection in an effort to improve questioning strategies in middle school teachers. Jacobs and Ambrose (2003) conducted a similar video study on questioning strategies in an interview setting. They discussed teachers' instructional strategies (especially questioning) in terms of how responsive teachers are to individual children's ideas and actions. However, the researchers noted the need to explore questioning strategies in a more complex setting, the classroom environment. Their comments inspired the present study.

Many reform efforts encourage teachers to reflect on their instructional techniques, either individually or collaboratively. Jacobs and Morita (2002) conducted a videotape study with teachers in which the participants watched videos of teachers teaching lessons. The participants were Japanese and American teachers who viewed an American lesson and a Japanese lesson on the same topic. After the teachers watched the lesson they were asked to define the parts of the lesson or the instructional strategies that they thought were most effective. One limitation Jacobs and Morita found in their study was that the teachers found it difficult to express their thoughts when asked very general or decontextualized questions. In response to that limitation, the present study narrows the focus from a general perspective on questions to a more specific one, the questioning techniques of the teachers.

Jacobs and Morita's (2002) study was based on the premise that watching a lesson on videotape should activate the implicit schemas, scripts, or instructional strategies that teachers hold in mind regarding instruction. Stigler and Hiebert (1999) defined *scripts* as mental pictures or descriptions of what teachers expect classroom instruction to look and feel like. By comparing their own criteria for good instruction against what they see in the lesson, teachers should be able to produce judgments about the lesson that reflect these scripts (Jacobs & Morita). Jacobs and Morita implied that the instructional scripts of American teachers could be easily changed in the direction that U.S. reformers would like to see. They also determined that this type of reflective exercise may produce more reflective teachers, more informed researchers, and more effective practice in the classroom.

In addition to previous research that justifies more study on instructional strategies, specifically concerning teacher questioning, there was another need for this study. The project that this study was based on was funded by NCLB (NCLB of 2001, Public Law 107-110) monies that were filtered down through the Arkansas Department of Education. These federal dollars required that project effectiveness be formally evaluated.

Theoretical Framework

The theoretical framework for this study was adapted from the work of Vygotsky (1930/1978) and Bloom (1956). Bloom is most well-known for his three learning dimensions or taxonomies: cognitive, affective, and psychomotor. One focus of this study was based on Bloom's cognitive domain taxonomy. Bloom's cognitive taxonomy was developed to define a method of classification for thinking behaviors that were believed to be important in the process of learning. According to Eisner (2002), Bloom developed this scheme in an effort to hierarchically order cognitive processes, but the classification scheme would also provide a framework for the formulation of learning objectives by teachers. It is a well known, widely applied scheme that has provided educators with a systematic classification of the process of thinking and learning. Bloom's Taxonomy has been applied to a variety of educational situations and is helpful almost any time an instructor desires to move a group through a learning process utilizing an organized framework. Thus, this study investigated the questions that teachers ask and how they rank according to Bloom's cognitive levels.

Vygotsky (1930/1978) refers to four mental functions in learners: attention, sensation, perception, and memory. He contends that through interaction within the socio-cultural environment these four functions are developed into more sophisticated and effective mental processes/strategies which he refers to as *higher mental functions.* These higher mental functions correspond to the higher levels of Bloom's Taxonomy of cognitive levels. In addition, according to Vygotsky, much important learning by the child occurs through social interaction with a *more knowledgeable other* (MKO). The highlighted MKO in this study was the teacher, and the interaction was facilitated by the teacher's ability to question students at a higher level, according to Bloom's Taxonomy (1956). Also, Vygotsky's research defines the *Zone of Proximal Development* (ZPD) as the area where the most sensitive instruction or guidance should be given. He believed that when students are at the ZPD for a particular task, providing the appropriate assistance, which he called scaffolding, will give the students the necessary boost they need to achieve the tasks. This boost or scaffolding described in this study was the embedding of high-level questions. Vygotsky contended that full cognitive development requires social interaction. In this study, the social interaction investigated was that interaction among the teacher and students.

Research Questions

The following research questions were investigated in this study:

1. Will video review and reflection of teachers' lessons, along with feedback from instructors viewing the videos, increase the number of high-level questions asked by middle school teachers, as measured by Bloom's Taxonomy?
2. Will video review and reflection of teachers' lessons, along with feedback from instructors, encourage teachers to elicit more *high-press* questioning exchanges as defined by Kazemi and Stipek (2001)?
3. Will teachers show a significant difference in mathematical pedagogical content knowledge between the pre- test and post-test as measured by the Content Knowledge for Teaching Mathematics test?

4. Is there a relationship between mathematical pedagogical content knowledge as measured by participants' CKT-M score and the ability of the teacher to ask higher-level questions as defined by Bloom (1956)?

Hypotheses

The following hypotheses were tested.

H_1 Video review of and reflection on teacher's lessons, along with feedback from instructors viewing the videos will increase the number of high-level questions asked by middle school teachers based on Bloom's Taxonomy .

H_2 Video review of and reflection on teachers' lessons, along with feedback from instructors will encourage teachers to elicit more high-press questioning exchanges as defined by Kazemi and Stipek (2001).

H_3 Teachers participating in the professional development sessions will show a significant difference in mathematical pedagogical content knowledge from before the professional development sessions to after the professional development sessions, as reflected by the Content Knowledge for Teaching Mathematics (CKTM) by Deborah Ball and Associates, University of Michigan.

H_4 There will be a relationship between the participants' mathematical pedagogical content knowledge as measured by the CKT-M and their ability to ask higher-level questions as defined by Bloom (1956).

Limitations

Specific limitations which could have influenced the results of this study are as follows:

1. There was limited time for professional development in content knowledge sessions. These were held as a two-week institute with four follow-up sessions during the following school year (two in the fall and two in the spring).
2. There were a small sample (*N*=18) of teachers included in the study.
3. Teachers participated in professional development opportunities in addition to this one, so it was difficult to attribute any or all gains to this treatment.
4. Questions could have been inaccurately coded: The teachers coded the questions they asked in each lesson according to Bloom's Taxonomy. Then the researcher coded each question, as well. Coding questions, even when strong descriptors are used, still has a subjective component, resulting in possible inconsistencies.
5. Teachers planned the lesson that they taught for the video review and reflection. The researcher visited each teacher's classroom twice each semester. However, one cannot be sure that the lesson that the teacher reflects upon is characteristic of what he or she does in the classroom on a daily basis.

Definitions of Terms

High-Press: In high-press exchanges, students must go beyond descriptions of summaries of steps used to solve a problem; they must link their problem-solving strategies to mathematical reasons. Further, high-press questioning must elicit (a) an explanation consisting of mathematical arguments, not simply a procedural description or summary; (b) mathematical thinking that involves understanding relations among multiple strategies; (c) the belief that errors provide opportunities to reconceptualize a problem, explore contradictions in solutions, or pursue alternative strategies; and (d) collaborative work in groups involving individual accountability and reaching consensus through mathematical argumentation (Kazemi & Stipek, 2001). Through high-press questioning teachers are, in most cases, asking students to respond to higher level questions, as defined by Bloom (1956).

Bloom's Taxonomy: The following cognitive levels were used in ranking questions: knowledge, comprehension, application, analysis, synthesis, and evaluation (Bloom, 1956; Kloss, 1988).

Knowledge is the ability to remember material previously learned.

Comprehension is the ability to grasp the meaning of material.

Application is the ability to use learned material in new, concrete situations.

Analysis is the ability to break down material into its components so that its organizational structure can be understood.

Synthesis is the ability to put parts together to form a new whole.

Evaluation is the ability to judge the value of material for a given purpose.

Low-level questions: Low-level questions were defined as questions from the knowledge or comprehension levels of Bloom's (1956) Taxonomy. These kinds of questions can be answered with factual knowledge and the ability to understand the factual knowledge. These questions can often be answered with one word or phrase and often require students to recall or recognize information, ideas, and principles in the approximate form in which they were learned. Low-level questions check for understanding of main ideas and sometimes ask students to interpret or summarize the ideas in his/her own words.

Middle-level questions: Middle-level questions were defined as questions from the application or analysis levels of Bloom's (1956) Taxonomy. These questions required students to apply something they know to a new and different situation. Questions on this level will require students to apply an abstract idea in a concrete situation to solve a problem or to relate it to a prior experience. They will also prompt students to decompose a concept or idea into parts, to show relationships among the parts, and to make sense of its organizational structure.

High-level questions: High-level questions were defined as questions from the synthesis or evaluation levels of Bloom's (1956) Taxonomy. These questions required students to take parts of different topics and put them together to make sense of new ideas, as well as to make informed judgments about the value of ideas or materials. The questions in this level required students to use standards and criteria to support opinions and views for any given purpose.

Improvement of questioning strategies: Improvement of questioning strategies were defined as an improvement of the ability of the teacher to move from the lower levels to the higher levels of Bloom's Taxonomy, as well as improving the ability to engage children in mathematical conversations that can be classified as high-press, as defined by Kazemi and Stipek (2001).

Mathematical pedagogical content knowledge: Mathematical pedagogical content knowledge was defined as the marriage between common mathematics knowledge and knowledge required by mathematics teachers to understand the mathematics deeply enough to make sense of mathematical contexts and determine whether procedures and algorithms that students invent will work, and if the procedure worked for one case, to determine if it will always work.

Summary

This chapter provided a brief history of mathematics reform in the United States, beginning with the national study *A Nation at Risk* (1983) and moving to current national legislation on education, No Child Left Behind (2001/2007). It follows the reform movement in education, generally, and mathematics, specifically, over the past 25 years. Justification was presented for conducting a study that researches the questioning strategies of teachers based on teacher self-evaluation through video review. It recognized the importance of counting the actual number of questions that are typically asked by today's teachers, as well as the significance of identifying the kinds of questions that teachers ask during a typical lesson. In addition, a primary focus of this study was to determine whether teachers who possessed a greater mathematical content knowledge were more likely to ask more questions and also questions that rank higher on Bloom's (1956) Taxonomy of cognitive categories. Lastly, this study determined the ability of teachers to engage students in *high-press* questioning exchanges as determined by Kazemi and Stipek (2001).

Chapter 2 gives a review of related research that is pertinent to the discovery of the aforementioned relationships. It describes the mathematical reform movement in more detail and reports the findings of prior studies in related areas.

CHAPTER 2

LITERATURE REVIEW

Mathematics education has been the object of periodic change for more than 40 years. The momentum for reform in mathematics education began in the early 1980s in response to *back to basics* efforts that resulted from the *new math* of the 1960s and 1970s. Piaget and other developmental psychologists helped shift the focus of mathematics educators from the mathematics content itself to determine how children actually learn mathematics (Van De Walle, 2004).

In 1980, the NCTM described what school mathematics programs should look like in *An Agenda for Action* (NCTM, 1980). The publication focused on the essential need for students to solve problems. NCTM followed up in 1989 with its publication of the *Curriculum and Evaluation Standards for School Mathematics* that described its vision for mathematics teaching and learning in grades K-4, grades 5-8, and grades 9-12 (NCTM, 1989). *Curriculum and Evaluation Standards for School Mathematics* provided major direction for states and schools in developing their curriculum guidelines. No other American mathematics curriculum document has ever had such an enormous effect on school mathematics or any other area of the curriculum (Van De Walle, 2004). In 2000, the NCTM published its *Principles and Standards for School Mathematics* (NCTM, 2000) that added underlying principles for school mathematics and clarified and elaborated on the 1989 standards. Most recently, NCTM (2006) published its *Curriculum*

Focal Points for Prekindergarten through Grade 8 Mathematics. While *Principles and Standards for School Mathematics* remains the comprehensive reference on developing mathematical knowledge across the grades, *Curriculum Focal Points for Prekindergarten Through Grade 8 Mathematics* described an approach to curriculum development that focuses on areas of emphasis within each grade from prekindergarten through eighth grade (NCTM, 2006).

As we move farther into a new century as well as a new millennium, Van De Walle (2004) contended that the vision of high-quality, engaging mathematics instruction for all students in the United States has not been realized, but progress has been made. He stated that the change is visible, albeit slow and incremental.

There are several regularly published large-scale reports that inform the American public about the effectiveness of its schools. These reports also influence political decisions and provide useful data for mathematics education. One of the major reports is the National Assessment of Educational Progress (NAEP). The NAEP is a congressionally mandated program that assesses what students know and can do in various curricular areas. It utilizes a set of criterion-referenced assessments with the results published annually as *The Nation's Report Card*. Much of the NAEP is designed to reflect the effectiveness of current curricula as determined by the achievement of students in grades 4, 8, and 12 (Silver & Kennedy, 2000).

The results of NAEP can be interpreted as both good and bad. The good news is that U. S. students are clearly doing better now than they were in 1973 (Silver & Kennedy, 2000). This is a definite contrast to what some observers say when reporting that our students do not know *good ole' basic mathematics*. The bad news is that even though the performance is increasing steadily, the overall performance of students remains dismal (Van De Walle, 2004).

The largest study of mathematics and science ever conducted was the Third International Mathematics and Science Study (TIMSS). Data were gathered in grades four, eight, and 12 from about 500,000 students and from teachers, as well. The most widely reported results were that U. S. fourth graders were above the average of the TIMSS countries, eighth-graders were below the international average, and 12th graders were significantly below average (U. S. Department of Education, 2003). Another finding of the TIMSS curriculum analysis was that U. S. curricula were unfocused, contained many more topics than most countries' curricula, and involved much more repetition in teaching than found in other countries. In the United States, teachers typically attempt to cover everything in their textbooks and, consequently, rarely teach any topic in depth. The lack of depth in content has required teachers to spend a tremendous amount of class time reviewing and reteaching the topic (Schmidt, McKnight, & Raizen, 1996).

A video study was conducted at the eighth grade level in conjunction with the TIMSS in the United States, Germany, and Japan (U.S. Department of Education, 2003). The results indicate extreme contrasts between instructional strategies in the United States and Japan. Findings of the study showed that the typical goal of a U. S. eighth grade mathematics teacher was to teach students how to complete a procedure. The typical goal of a Japanese teacher was to help students understand mathematical concepts. The study found that, in many ways, Japanese teaching resembled the recommendations of the U. S. reform movement more closely than did American teaching. With the results of studies like this in mind, a closer look into American teaching is warranted.

In 2007, another TIMSS study was conducted-Trends in International Mathematics and Science Study. In the U.S., TIMSS was administered to a random sample that included about 500 schools and 20,000 students in grades four and eight. Fourth-graders from 16 countries and eighth-graders from 20 countries were compared to the results of the 2003 Third International Mathematics and Science Study. Findings from the 2007 study showed that both fourth and eighth-graders from the U.S. improved in mathematics compared to the first study in 1995. Thirty-six countries participated in the 2007 study at grade four and 48 countries at grade eight, however, not all of these countries participated in the prior study, making improvement measurement for them impossible (Gonzales et al., 2008).

There is a growing body of literature that demonstrates a positive association between reformed teaching and student learning. The findings of these studies reflect the importance of the teacher's role in promoting student understanding, of focused professional development in preparation for that role, of complex and challenging tasks, and of fruitful classroom interactions (Cohen, 1994; Henningsen & Stein, 1997; Hiebert et al, 1997; Lappan, 1997; Schoen, Cebulla, Finn, & Fi, 2003; Stigler & Hiebert, 1999; Weglinsky, 2000). The skills that teachers possess, both in their personal understanding of mathematical content and in the way they convey that mathematical content to their students, are critical to the mathematical success of students.

Content Knowledge and Pedagogy

There seems to be a multitude of ways to describe and define the knowledge needed by teachers to teach their content. Ma (1999) sums up teaching in the following way

> One thing is to study whom you are teaching, the other thing is to study the knowledge you are teaching. If you can interweave the two things together nicely, you will succeed… Believe me, it seems to be simple when I talk about it, but when you really do it, it is very complicated, subtle, and takes a lot of time. It is easy to be an elementary school teacher, but it is difficult to be a good elementary school teacher. (p. 136)

In the mid-1980s, Shulman introduced the notion of *pedagogical content knowledge* to refer to the special knowledge that teachers need to actually teach a particular subject (Shulman, 1986, 1987; Wilson, Shulman, & Richert, 1987). In *Adding it Up* (National Research Council, 2001), the National Research Council stated that three kinds of knowledge are crucial for teaching school mathematics: knowledge of mathematics, knowledge of students, and knowledge of instructional practices. The Instructional Triangle (National Research Council, p. 314) illustrates this (Figure 1). Mathematics, students, and teachers are the vertices of the triangle and the arrows portray the instructional practices, including questioning strategies utilized by the teacher.

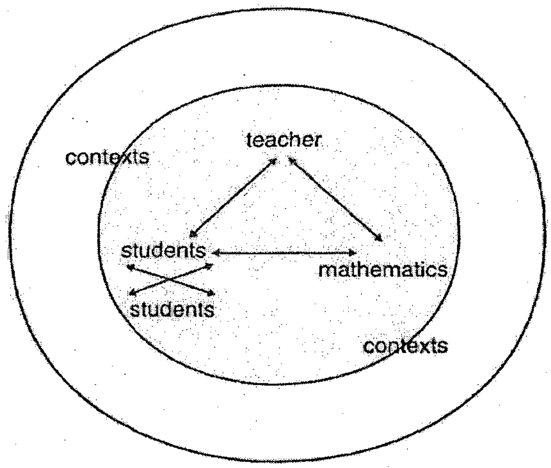

Figure 1: The instructional triangle

Mathematical knowledge is defined by the National Research Council (2001) as knowledge of mathematical facts, concepts, procedures, the relationships among them, and knowledge of mathematics as a discipline, particularly how mathematical knowledge is produced, the nature of discourse in mathematics, and the norms and standards of evidence that guide argument and proof. Knowledge of students and how they learn mathematics includes general knowledge of how various mathematical ideas develop in children over time as well as specific knowledge of how to determine where in a developmental trajectory a child might be. Knowledge of instructional practice includes knowledge of the curriculum, knowledge of tasks and tools for teaching important mathematical ideas, knowledge of how to design and manage classroom discourse, and knowledge of classroom norms that support the development of mathematical proficiency. However, more than knowledge is needed. Teachers need to *know* as well as understand how to *do*. Understanding norms that support productive classroom activity is different from being able to develop and use such norms with a diverse class.

Stienbring (1998) stressed the fact that the distinction between content knowledge and pedagogical knowledge is not independent of the model of the teaching/learning process. Looking linearly at this process, he stated that "Mathematical content knowledge is primarily needed during the first step in this process, whereas pedagogical content knowledge is necessary for the conditions and forms of the transmission of school mathematics" (p. 158). However, if one sees teaching and learning mathematics as an autonomous system, "pedagogical content knowledge does not primarily serve to organize the transmission of mathematical content knowledge" (p. 159). Therefore, he declared that "A new type of professional knowledge for mathematics teachers is needed—a kind of mixture between mathematical content knowledge and pedagogical knowledge" (p. 159).

In addition, teaching mathematics well and asking good questions calls for an increased understanding of the mathematics being taught (Schuster & Anderson, 2005). It is important to encourage teachers to increase their knowledge of mathematics; however, studies show that increasing the quantity of teachers' mathematics coursework is not sufficient (Ball, 2003; Kent, Pligge, & Spence, 2003). Teachers will only improve the quality of their teaching if they learn the mathematics in ways that make a difference for the skill with which they are able to do their work. The ultimate goal is to improve students' learning, not to produce teachers who know more mathematics. Teachers' opportunities to learn must equip them with the mathematical knowledge and skill that will enable them to teach mathematics effectively (Ball). It takes significantly more insight, mathematical skill, and understanding for a teacher to teach a mathematics concept than it does to carry out the mathematical procedure itself. Ball, Hill, & Bass state

> The improvement of mathematics teaching in this country depends on, among other things, the improvement of our understanding of its mathematical nature and demands, and the provision of opportunities for professionals to acquire the appropriate mathematical knowledge and skill to do that work well. (2005, p. 15)

Ball et al. (2005) contended that teachers need what they defined as a *mathematical knowledge for teaching*. This is a different kind of professional knowledge than is demanded by other mathematically intensive fields like physics, engineering, accounting, or carpentry. Hill, Rowan, and Ball (2005) developed tests that measure this mathematical knowledge for teaching, and these tests have been proven to positively predict gains in student achievement. For example, in one analysis of 700 first and third grade teachers (combined student load of over 3,000 students), Hill et al. found that teachers' performance on *knowledge for teaching* items—including both common and specialized content knowledge—significantly predicted student gains on standardized tests. *Specialized content knowledge for teaching* items are those that relate to making sense of mathematical contexts and determining whether procedures and algorithms that students invent will work, and if the procedure worked for one case, to determine if it will always work.

For example, the question may be a two digit by two digit multiplication problem. *Common content knowledge* would be the ability to simply answer the problem correctly; however, *specialized content knowledge for teaching* would require that several different invented methods be scrutinized to determine if the procedure will work for all numbers, some numbers, or never. In this study, the researchers controlled for things such as student socio-economic status (SES), student absence rate, teacher credentials, teacher experience and average length of mathematics lessons. The results were clear: the students taught by teachers who answered more items correctly gained more over the course of one year of instruction (Hill, Rowan et al., 2005).

Further, Hill, Rowan et al. (2005) compared teachers who made average scores on the measure of teacher knowledge with teachers who made very high scores (in the top quartile) and found that the students in the higher-scoring teachers' classes gained as much as the equivalent of an additional three weeks of instruction. In addition, the effect size of the teachers' scores was comparable to the effect size of SES on student gain scores. This suggests that improving teachers' teaching knowledge may be one way to lower the achievement gap of lower SES children.

Hill and Ball (2004) tested the idea of *mathematical knowledge for teaching* in professional development. They wanted to find out if there was a way to prepare teachers for their work by helping them improve their specialized mathematical knowledge for teaching. They found that teachers did learn mathematical knowledge for teaching during professional development sessions and also that the length of the professional development along with the content of the professional development focusing on proof, analysis, exploration, communication, and representations produced greater performance gains.

Additionally, Hill and Ball (2004) studied whether specialized mathematics content knowledge for teaching exists in tandem with common content knowledge—the skills that a mathematically literate adult would possess. They found that it takes knowledge over and above what the common adult possesses to understand the specialized mathematics that is needed to teach children. Therefore, specialized mathematical knowledge for teaching does positively predict gains in student achievement and this knowledge can be improved through particular types of professional development (Hill, Rowan et al., 2005).

Teachers must think from the learner's perspective and consider what it takes for someone to understand a mathematical idea when seeing it for the first time. Dewey (1956) captured this idea with the notion of *psychologizing* the subject matter, seeing the structures of the subject matter as it is learned, not only in its finished logical form. With this in mind, teachers must teach and question students for understanding.

According to brain research done by the Committee of Inquiry into the Teaching of Mathematics (1982, p. 71), "Conceptual structures are richly interconnected bodies of knowledge. It is these which make up the substance of mathematical knowledge stored in long-term memory." Classic and current brain research shows that due to brain makeup, things the brain does not understand are more likely to be forgotten (Levine, 2002; Lovell, 1958).

It is the interconnections that teachers make and their ability to carefully scaffold questions that help children remember concepts. Teaching mathematics with understanding means creating experiences in which these interconnections can be made because, without them, there would be a real danger that questions put in isolation would make the learning process rather piecemeal and incoherent (Marshall, 2006). Simply posing open-ended mathematical problems that require mathematical reasoning is not sufficient to help students learn to reason mathematically. Neither is merely asking students to explain their thinking. Students must learn to use publicly established ideas, methods, and language to make, inspect, validate, improve, and extend mathematical knowledge; and teachers must create and provide resources for and create an environment that encourages and makes possible complex student work (Ball & Bass, 2003).

Levine (2002) stated that higher thinking is the ultimate educational harvest. He contended that teachers should aim at five forms of higher thinking and strive to cultivate them from an early age and throughout the education of a child. Teachers should carefully inspect how they teach to determine whether they are fostering the growth of these "lofty neurodevelopmental functions" (p 192). The five forms of higher thinking as defined by Levine are conceptual thinking, problem-solving, critical thinking, rule-guided thinking, and creative thinking. Lessons incorporating these forms of higher thinking through questioning or learning tasks will address children of many different learning styles, thus enabling them to remember information more easily. Conceptual thinking involves thinking with concrete or abstract concepts. It could also be a verbal or non-verbal process. Problem solving involves knowing there is a problem, previewing outcomes, assessing feasibility, mobilizing resources, eliciting logical thinking, exploring a variety of strategies, getting started, pacing, self-monitoring, and reflecting. Critical thinking involves the student knowing the facts and then comparing his or her point of view with that of another person based on those facts. It also involves weighing evidence, communicating, and knowing when to get outside help. Rule-guided thinking is thinking in terms of *if...then*. Lastly, creative thinking involves divergent thinking, taking a fresh look, suspending self-evaluation, and taking risks.

Darling-Hammond (2000) found that states interested in improving student achievement may be well-advised to attend, at least in part, to the preparation and qualifications of the teachers they recruit and retain in the profession. It stands to reason that student learning should be enhanced by the efforts of teachers who are more knowledgeable in their field and are skillful at teaching it to others.

Chen and Lin (2004) found in a study of eighth-grade students that their conjecturing abilities were obscure and undeveloped. After the students had participated in 15 hours of investigative teaching by teachers skilled in scaffolding questioning techniques, every student at least dared to make conjectures and test their accuracy. In some cases the students tested their conjectures repeatedly. The qualitative aspect of the study showed that students' conjecturing abilities could be developed by way of practicing conjecturing, testing, probing, refuting, and arguing in defense against the conjectures brought up by others or by oneself. This study, like Fraivillig, Murphy, and Fuson's (1999), incorporated Vygotsky's theory of *Zone of Proximal Development* (Vygotsky, 1930/1978) and the concept of scaffolding support (Wood, Bruner, & Ross, 1976).

It is now recognized that the teaching profession constitutes its own large body of knowledge, in sharp contrast to the outdated perspective of teachers as skilled technicians who simply apply bodies of disciplinary knowledge produced by others. This includes knowledge of the content of the disciplines, of students, and of a variety of instruction and assessment strategies (National Commission on Mathematics and Science Teaching for the 21st Century, 2000; NCTM, 2000; National Research Council, 1996). These three key areas of teacher knowledge are continually consulted and integrated by teachers as they make hundreds of professional decisions every day (Loucks-Horsley, Love, Stiles, Mundry, & Hewson, 2003).

Many researchers refer to the knowledge that teachers need to function effectively as pedagogical content knowledge (Ball, 2003; Hill, Rowen et al., 2005; Hill, Schilling, & Ball, 2004; Kahan, Cooper, & Bethea, 2003). Pedagogical content knowledge is defined by Loucks-Horsley et al. (2003) as an understanding of what makes learning specific concepts easy or difficult for learners, awareness of what concepts are more fundamental than others, and knowledge of ways of representing and formulating subject matter to make it accessible to learners. Developing this pedagogical content knowledge in mathematics requires content knowledge in pure mathematics. Teachers with limited mathematical understanding will have very restricted pedagogical content knowledge.

In *Adding it Up*, the National Research Council (2001) describes the importance of pedagogical content knowledge in this way

> Effective teaching—teaching that fosters the development of mathematical proficiency over time—can take a variety of forms, each with its own possibilities and risks. All forms of instruction can best be examined from the perspective of how teachers, students, and content interact in contexts to produce teaching and learning. The effectiveness of mathematics teaching and learning is a function of teachers' knowledge and use of mathematical content, of teachers' attention to and work with students, and of students' engagement in and use of mathematical tasks. Effectiveness depends on enactment, on the mutual and interdependent interaction of the three elements—mathematical content, teacher, students—as instruction unfolds. (p. 9)

Knowledge of the subject matter and students' developmental levels helps teachers ask better questions. Using better questions, teachers facilitate different levels of discourse needed in the classroom, being not only concerned with what students say about the topic but also with why they say it (Loucks-Horsley et al., 2003; NCTM, 2000; National Research Council, 1996).

This idea of adding pedagogical content knowledge to teachers was to complement the general knowledge they already had in a certain area. Pedagogical content knowledge was thought to include familiarity with topics children find interesting or difficult, the representations most useful for teaching an idea, and learners' typical errors and misconceptions. With this labeling, the importance of teachers' understanding more than just the common knowledge of the subject they were teaching (that is, what an educated adult would know of a subject) became evident. This common knowledge would not be sufficient for teaching. For example, scholars showed that what teachers would need to understand about fractions, place value, or slope, for instance, would be substantially different from what would suffice for other adults (Ball, 1988, 1990, 1991; Borko et al., 1992; Leinhardt & Smith, 1985).

Weiss, Pasley, Smith, Banilower, and Heck (2003) described this teaching for understanding

> Teachers also must be skilled in helping students develop an understanding of the content, meaning that they need to know how students typically think about particular concepts, how to determine what a particular student or group of students thinks about those ideas, and how to help students deepen their understanding. (p. 28)

According to Keeley and Rose (2006), when teachers address topics about which they are mathematically confident, they encourage student questions and mathematical conversations, spend less time on unrelated topics, encourage discussions to move in new directions based on student interest, and present topics in a more coherent way—strategies descriptive of standards-based teaching. In contrast, when teachers are presenting content about which they are not as well informed, they often discourage active participation and discussion by students, rely on teacher presentation rather than student engagement, and spend time on tangential issues.

Despite this wealth of research, Hill, Schilling et al. (2004) argue that the mathematical content teachers need to be successful is not yet mapped precisely. The previous research that has been done in this area has been primarily single-teacher case studies, expert-novice comparisons, cross-national comparisons, and studies of new teachers. These kinds of studies were critical to get the process started; however, they lack the power to propose and test hypotheses regarding the organization, composition, and characteristics of content knowledge for teaching.

Shulman (1986) proposed three categories of subject-matter knowledge for teaching: content knowledge, pedagogical content knowledge, and curriculum knowledge. Content knowledge refers to the "amount and organization of knowledge per se in the mind of teachers" (p. 9). This included both facts and concepts in a domain and also the reason the facts and concepts are true and how knowledge is generated and structured in the discipline. Pedagogical content knowledge, according to Shulman (1986) and Wilson, Shulman, and Richert (1987), goes beyond knowledge of subject matter to the dimension of subject matter knowledge for teaching. This is the category that has become of central interest to researchers and educators, because it represents content ideas as well as an understanding of what makes learning a topic difficult or easy for students (Hill, Schilling et al., 2004). Lastly, curriculum knowledge involves awareness of how topics are arranged both within a school year and over longer periods of time, and ways of using curriculum resources, such as textbooks, to organize a program of study for students. Shulman's theory included one additional category called general pedagogical knowledge. This included classroom management, knowledge of learners and their characteristics, knowledge of educational contexts (e.g., school district policies), and knowledge of educational ends, purposes, and values.

Working from a psychological/cognitive perspective, Leinhardt and Smith (1985) proposed a different organization of teacher knowledge: lesson structure knowledge and subject matter knowledge. Lesson structure knowledge includes planning and running a lesson smoothly and providing clear explanations. Subject matter knowledge includes concepts, algorithmic operations, the connections among different algorithmic procedures, the subsets of the number systems, understanding classes of student errors, and curriculum presentation. Other ways of dividing and discussing the knowledge teachers need to teach have been advanced as well. Grossman (1990) reorganized Shulman's categories into four and extended them slightly. Ball (1990) further elaborated and described the differences between teachers' ability to execute an operation and their ability to represent that operation accurately for students, clearly demarcating two dimensions in teachers' content knowledge. Based on this study, Ball proposed a distinction between knowledge *of* mathematics and knowledge *about* mathematics, corresponding roughly to knowledge of concepts, ideas, and procedures and how they work, on the one hand, and knowledge about *doing mathematics* on the other (e.g., how one decides that a claim is true, a solution is complete, or a representation is accurate).

According to Hill, Schilling et al. (2004), posing these potential categories of content knowledge for teaching has created three contributions to the development of theory about this knowledge:

- To refocus researchers' attention on the centrality of subject matter and subject matter knowledge in teaching
- To draw attention back to disciplines and their structures as a basis for theorizing about what teachers should know
- To focus attention on what expert teachers know about content and how they use or report using this knowledge of subject matter in their teaching

With Stienbring's (1998) research in mind, Margolinas, Coulange, and Bessot (2005) defined *didactic knowledge* as any knowledge related to the mathematical knowledge to be taught. It is important to know the *what* and the *why* for the content to be taught effectively. Additionally, Choi, Land, and Turgeon (2005) discovered that effective questioning required a certain level of domain knowledge (or content knowledge) and metacognitive skills.

Questioning

With the new vision that the mathematical reform movement has provided, teachers are encouraged and charged with eliciting mathematical discussion among students, and this discussion is often initiated in the form of a question or questions. Teachers are to emphasize students' different ways of thinking and not only accept but encourage multiple solution strategies (Jacobs & Ambrose, 2003; NCTM, 1989, 2000;

National Research Council, Mathematics Learning Study Committee, 2001). This new vision is requiring teachers to redefine their roles as inquirers into children's thinking. Teachers should no longer follow a scripted lesson plan but rather elicit and respond to students' ideas to help them construct their own understandings of mathematics. During this questioning dialogue, teachers must be able to assess where students' understandings lie, scaffold their thinking, and push them to the next level of understanding through their questioning strategies. Teachers cannot simply ask questions in the order they are listed on a plan but must ask questions as they make sense of the students' thinking (Kloss, 1988; Newman, Griffin, & Cole, 1989; Vygotsky, 1978).

Fraivillig et al. (1999) conducted a study that investigated the instructional strategies used by teachers that were found to advance children's mathematical thinking. They found three separable but overlapping components that composed these teaching strategies: eliciting children's solution methods (eliciting), supporting children's conceptual understanding (supporting), and extending children's mathematical thinking (extending). These three related teaching components and the particular classroom climate in which they occurred formed a framework that they used to describe observed examples of successful mathematics teaching. They named this framework Advancing Children's Thinking (ACT).

In each of the three components of the ACT framework (eliciting, supporting, and extending), teachers' questioning strategies play a key role. Teachers elicit a variety of solution methods by probing students, through questioning, for better mathematical descriptions and by using challenging follow-up questions to their lessons. They support students' conceptual understanding by providing assisted practice in the student's zone of proximal development (Sowder & Schappelle, 2002). The Zone of Proximal Development as defined by Vygotsky (1930/1978) is "the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance, or in collaboration with more capable peers" (p. 86).

More commonly, the Zone of Proximal Development is thought of as the gap between a learner's current or actual development level, determined by independent problem-solving, and his or her emerging or potential level of development. Teachers extend students' thinking by pushing them to attempt alternative solution methods. The three components of the ACT framework are all addressed in various ways by the ability of the teacher to scaffold and ask high level questions (Sowder & Schappelle, 2002).

Knowing what questions to ask and how to actively involve and challenge all students is one of the most complex aspects of mathematics reform (Dugdale, Matthews, & Guerrero, 2004). "Questions may be one of the most powerful technologies invented by humans. Even though they require no batteries and need not be plugged into the wall, they are tools which help us make up our minds, solve problems, and make decisions" (McKenzie as quoted in Schuster & Anderson, 2005, p. 1). Solving and discussing mathematical problems is also an essential part of doing and learning mathematics. Most teachers agree that this mathematical discussion is important; however, questioning students in ways that will elicit this discussion, including articulating, justifying, and debating ideas, can be very challenging (O'Connor, 2001). Teachers must plan and structure opportunities for students to share their thinking, compare strategies, and consider ideas of others (Turner, Junk, & Empson, 2007).

According to Sullivan and Lilburn (2002), it is vital that teachers attend to improving their questioning strategies, and use these strategies not only to find out what students understand but also to help students articulate how they arrived at those understandings. Dantonio and Beisenherz (2001) contended that the development of productive questions can help focus learning on the process of thinking while attending to the study of content.

There has been widespread recognition of the importance of generating quality questions for instructional purposes. Cummings (1994, p. 462) writes, "Always the beautiful answer who asks a more beautiful question." Throughout the years, studies have proved that teachers are not asking good questions. As far back as 1912, Stephens studied classroom practice and found that two thirds of the questions asked in a typical classroom required only reciting memorized text. Daines (2001) discovered that 93% of the questions asked by elementary and secondary teachers were at the literal level of comprehension and 88% of students' answers were also at the lowest level of cognitive skills. Daines concluded then that the "constant model of asking literal questions and repeating students' answers to low-order questions seems to connote to students that teachers expect them to perform at the factual and recall level of thinking" (p. 373).

An almost identical conclusion was reached by Barnes (1983) in the university setting. She found that an overwhelming percentage of questions asked by college professors, regardless of institution, class or level, were on the lowest cognitive level, a level she refers to as *cognitive memory*. In 1984, Gall found that pleas for teachers to use higher-level questions were not being answered because teachers' curricula were primarily textbook driven and the text neither promoted nor supported such questioning by the teacher.

Later, Lord and Baviskar (2007) conducted research at the college level that yielded very similar results. Their study concentrated on the questions that are asked on science exams. One finding from their work was that students concentrate their studies on terms and definitions and spend little time on application and analysis. The recommendation from this study was that instructors should teach in a manner that is parallel to the way they test and the examinations they prepare for students should contain more questions around the mid and upper levels of Bloom's Taxonomy (1956). In response to the research by Lord and Baviskar, highly esteemed academic societies in the field of science, such as the National Association of Biology Teachers and the National Science Teachers Association, are encouraging a modification in the way instructors evaluate students: a change from evaluating factual content knowledge to evaluating understanding.

Wolf (1987) came to the same conclusion in a study conducted at the secondary level and added that teachers do not attempt to become better questioners because it is not valued by their supervisors. Teachers realize that skillful question asking is an art that must be constant, consistent, and practiced unconsciously and continuously. Planning for something of this intensity takes much time and dedication. Wolf found that teachers were not willing to devote the time and effort to something that was not valued by their superiors.

Sullivan and Lilburn (2002) defined *good* questions as having three main features: (a) they require more than remembering facts and procedures; (b) students have the ability to learn by attempting to answer the question, and the teacher has the ability to learn about the student from his/her attempt to answer; and (c) there are multiple solutions and multiple strategies to a good question. Although the authors do not specifically reference Bloom's (1956) Taxonomy of cognitive skills, they stated that good questions require students to comprehend the task, apply the concepts and appropriate skills, and to analyze and synthesize major concepts involved, each one representing levels of Bloom's Taxonomy. In general, good questions have the potential to make children more aware of what they know and do not know. Students can become aware of where their understanding is incomplete. Good questions also prompt children to be creative and think critically (Sullivan & Lilburn).

Educational researchers have shown that activation of prior knowledge is critical to learning of all types. It has also been proven through research that background knowledge influences what we perceive. Teachers can use cues and questions as one technique to activate students' prior knowledge (Marzano, Pickering, & Pollock, 2001). Research in classroom behavior indicated that cueing and questioning might account for up to 80% of what occurs in classrooms on any given day (Davis & Tinsley, 1967; Fillippone, 1998; Marzano, Pickering, & Pollock, 2001). Marzano et al. (2001) also found that teachers, on average, ask more questions than they think they do.

There are many research studies that indicate that higher-level questions—questions that require students to analyze information—produce more gains in learning than lower-level questions that only require students to recall or recognize information. Unfortunately, most of the questions teachers ask tend to be lower order in nature (Barnes, 1983; Cotton, 2008; Davis & Tinsley, 1967; Fillippone, 1998; Guszak, 1967;

Huitt, 2004; Mueller, 1973; Weiss et al., 2003; Wilson & Kenney, 2003; Wolf, 1987). Questioning has generally been thought of as something that teachers do after the students have been engaged in a learning experience. However, questions can effectively be used before the lesson to establish a *mental set* for students to use in processing the learning experience. The use of higher-level questions in both of these instances produced deeper levels of learning (Hamaker, 1986; Osman & Hannafin, 1994; Pressley et al, 1992; Pressley, Symons, McDaniel, Snyder, & Turnure, 1988; Pressley, Tenenbaum, McDaniel, & Wood, 1990). Cotton found that in most cases above the primary grades, a combination of high- and low-level questions is superior to the exclusive use of one or the other. Also, increasing the use of higher cognitive questions to at least 20% produces superior learning gains for students above the primary grades, particularly for secondary students.

According to Bright and Joiner (2005), questioning is one of the most useful classroom assessment techniques. Questions should help reveal the specifics of students' thinking to the teacher, to the student being questioned, and to other students in the class. The careful selection of activities and tasks can help accomplish this goal, but it is more often through questioning during the summary and debriefing of students' solutions that specific elements of their thinking can be revealed.

Teachers' understanding of mathematics content is reflected in the task selection and design of questions. Researchers stated, "Observations, which include assessment tasks along with the criteria for evaluating students' responses, must be carefully designed to elicit the knowledge and cognitive processes that are most important for competence in the domain" (Pellegrino, Chudowsky, & Glaser, 2001, p 7). The choice of tasks and the design of questions are greatly influenced by the teacher's view of what is important mathematics to learn.

Weiss et al. (2003) observed a national sample of 364 mathematics and science lessons in grades K-12. Their conclusions about *typical* mathematics instruction suggested that a teacher's perception of what mathematics is important to learn is related to the kinds of questions asked. Together, these form one way to distinguish effective from less effective instruction. They found that in many of these lessons the mathematics and science were presented to the students as *static* bodies of knowledge, focusing on vocabulary and algorithms. The observers found the teachers doing all the thinking throughout the lesson; there was no investigative spirit present in the classroom. In general, the researchers found that the teacher had knowledge that he or she was attempting to transmit to his or her students. According to the researchers, asking *better* questions has implications for the overall quality of instruction: "The vision of high-quality instruction should emphasize attention to appropriate questioning and helping students make sense of the mathematics/science concepts they are studying" (p. 104-105).

Unfortunately, Weiss et al. (2003) also found that the most common instructional pattern is "low-level, fill-in-the-blank questions, asked in rapid-fire, staccato fashion, with an emphasis on getting the right answer and moving on, rather than helping students make sense of the concepts" (p. 65). A similar conclusion was found by Wilson and Kenney (2003). In general "teachers' questioning is dominated by recall questions" (p. 55). With few exceptions instruction seemed to be oriented much more toward covering the textbook and getting students to say the right things rather than helping them make sense of the underlying mathematical ideas. Overall, the main finding of their survey was that "questioning is among the weakest elements of mathematics and science instruction, with only 16 percent of lessons nationally incorporating questioning that is likely to move student understanding forward" (p. 65). According to Wilson and Kenney, if the teacher limits questions to a narrow band of procedural questions, the answers given may not be sufficient for the teacher to make informed inferences about the breadth or depth of students' understanding. That is, the teacher may take a series of correct answers by a student as evidence of understanding, when in fact those answers are very limited evidence of the student's ability to give the correct answers and tell the teacher very little about the level of the student's understanding.

Wiess et al. (2003) conducted a study of K-12 mathematics and science classrooms. They determined that the kinds of questions teachers ask are key in determining the extent to which lessons are likely to help students learn important mathematics concepts. Teachers can use questioning to monitor student understanding of new ideas and to encourage students to think more deeply, but this kind of effective questioning is rare in American classrooms. The researchers commonly saw questioning that was unlikely to deepen students' understanding, including teachers asking a series of questions too rapidly and asking questions focused only on a correct answer without checks for complete understanding. In fact, only 16% of the lessons contained high-level questions, and at least 66% of the lessons contained *inadequate questioning.*

Weiss et al. (2003) summarized by stating that classes with effective classroom assessment, including questioning, can be described as having greater intellectual rigor. They contended that

> Fewer than 1 in 5 mathematics and science lessons are strong in intellectual rigor; include teacher questioning that is likely to enhance student conceptual understanding; and provide sense-making appropriate for the needs of the students and the purposes of the lesson. (p.103)

The researchers recommended that teachers be given opportunities to analyze a variety of lessons in relation to key elements of high-quality instruction, particularly teacher questioning and sense-making focused on conceptual understanding. Teachers should start with group discussions of videos of other teachers' practice and move toward examining their own practice. Finally, Weiss et al. came to the same conclusion as many other researchers: teacher content knowledge is not sufficient preparation for high-quality instruction.

Black and Wiliam (1998) examined 250 research studies on classroom assessment and found that formative assessment, including questioning, not only improved learning but that the achievement gains were among the largest ever reported for educational interventions. Wilson and Kenney (2003) concluded that if mathematics teachers were to focus their efforts on classroom assessment that is primarily formative in nature, students' learning gains would be impressive. These efforts included gathering data through classroom questioning and discourse, using a variety of assessment tasks, and attending primarily to what students know and understand.

In addition, teachers need to understand and implement instructional practices that foster self-regulation among and within their students (Butler, 2002; De Corte, Verschaffel, & Eynde, 2000; Pape, Bell, & Yetkin, 2003). Pape et al. conducted a study of urban seventh-grade mathematics students, and several principles emerged as crucial to instructional practices that support middle school students' mathematical thinking, one being that teachers educe classroom discourse and multiple representations. This discourse required students not only to explain their procedures but also to justify their reasoning, which falls into the higher levels of Bloom's (1956) Taxonomy.

Ilaria (2002) conducted a study on questioning strategies designed to address the increased call for communication that is required in a student-centered classroom. He stated that the purpose for questioning is to help students explore their ideas during the communication process. With this in mind, teachers need guidelines for questions that engage students in mathematical thinking. However, Ilaria found that teachers cannot use prepared questions or prescribed strategies. Teacher questions must be based on the responses received from the student so the teacher can continue the conversation and engage the student in mathematical thinking. Glenn (2001) stated that communication and questioning are a part of a larger equation for effective teaching. According to Reynolds and Muijs (1999), effective teachers tend to ask more process questions, searching for explanations. But sadly, the majority of questions found in the study were product questions, asking only for a single response by students.

Axiak (2004) declared that the quality of teachers' questioning is undoubtedly one of the crucial factors affecting the quality of students' learning. Using student teachers, he conducted a study whose focus was limited to the use of questioning in evaluating student thinking in a one-on-one interview situation. He found that the questions asked by this group were not directed toward what the students already knew, but instead, the student teachers tended to focus questions on what they thought the students should know. Only one teacher in the study recognized that getting the answer to a question incorrect is part of the learning process and that the focus was not on getting the answers right or wrong, but rather engaging in thinking about the mathematics behind the question.

Daines (2001) conducted a study that demonstrated that both preservice teachers and classroom teachers need more practice asking higher-order thinking questions in a classroom setting. The study consistently showed that teachers were spending the majority of their time asking questions at the literal level. Not only do they need to practice asking high-level questions, but it is crucial that another teacher, instructional specialist, or coach help by providing feedback and advice on a regular basis.

Teaching involves a complex cycle of planning, acting, observing, and reflecting in a highly dynamic atmosphere that changes from second to second (Loucks-Horsley et al., 2003). Teachers must process information and make decisions on a variety of levels simultaneously. In doing so, they must draw on their ability to apply knowledge about students, content, curriculum, instruction, assessment, and schools and communities. To be successful in a multifaceted environment like the one described, teachers need opportunities to develop their pedagogical content knowledge though critical reflection of their own classroom practice. Professional development strategies that employ such intense reflection have been shown to develop teachers' content knowledge and sophisticated pedagogical reasoning skills and also to increase student achievement (Heller, Kaskowitz, Daehler, & Shinohara, 2001).

According to Loucks-Horsely et al. (2003), practicing any profession is a complex and uncertain endeavor that requires an expert knowledge base peculiar to that profession. This is because professionals are constantly being called on to make decisions in unique and complex circumstances and without *absolute* knowledge. Past experience and a base of expert knowledge do not provide professionals with a set of fixed rules to follow but only with heuristics that can guide professional judgment and decision making. To make decisions that are informed rather than reactive, a crucial characteristic of professional practice is reflection on past and current actions to inform future decisions.

Further, Sullivan and Lilburn (2002) found that teachers of mathematics are not as good as teachers of other subject areas in asking open questions. Mathematics teachers tended to ask closed questions. The authors use *open* and *closed* as one way to categorize questions. They define *closed* questions as those that can be successfully answered with just one word, number, or phrase that is given from memory. *Open* questions are those that require a student to do more than recall known facts. These questions have the potential to stimulate thinking and reasoning. The authors believe that it is vital that more attention be paid to the questioning strategies of mathematics teachers in order to emphasize problem solving, application, and the development of a variety of thinking skills. U.S. educational goals expect students to think, to learn, to analyze, to criticize, and to be able to solve unfamiliar problems. It follows that the ability of teachers to ask good questions should be a part of the instructional repertoire for all mathematics teachers.

Use of Video

In conjunction with the Third International Mathematics and Science Study (TIMSS), Stigler and Heibert (1999) conducted teacher professional development sessions through video recording. Through this study, Stigler and Hiebert found that teaching is a cultural activity. They determined that teachers learn how to teach indirectly, through years of participation in classroom life, and that they were largely unaware of some of the most widespread attributes of teaching in their own culture. The fact that teaching is a cultural activity explains why it has been so resistant to change; however, recognizing the cultural nature of teaching provides new ideas and insights into what educators need to do in order to improve it. Stigler and Heibert also found that many American students receive poor-quality mathematics teaching. After having analyzed hundreds of lessons on video they reported that none of the American lessons could be rated as containing high-quality mathematics.

In a later reflection on the video segment of the TIMSS study, Stigler and Hiebert (2000) found that many American teachers believed they were changing the way they teach, when in fact, the video review showed that they had retained their traditional practices. In this study, one indicator showed that U. S. teachers usually just stated mathematical concepts rather than developing them. A second indicator revealed no instances of working through proofs or reasoning deductively in the U. S. lessons. The third indicator revealed that almost all U.S. students' time during seat work (96%) was devoted to practicing procedures.

Further, Stigler and Hiebert (2000) analyzed the lessons of the teachers who had reported through self-reflection that their lessons were in line with the ideals of the mathematics reform movement. Unfortunately, the researchers found that the content those teachers were teaching continued to be low-level and unengaging, but they did tend to make students work in groups, used more time in student seat work and less time for teacher lectures, and used textbooks less than the average U.S. teacher. All of these were defined as *marginal* improvements by the authors because the content continued to be low-level. In one example, a teacher put a group of students together simply to try to determine the name of a geometric shape. When one of the students in the group remembered the name of the shape, he or she told the others and the task was complete. There was no interaction or questioning by the teacher. In general, the study revealed that some teachers did make an attempt to change features of their instruction, but very few appeared to shift their goals toward deeper mathematical understanding.

Stigler and Hiebert (2000) evaluated the videos lesson-by-lesson. They defined the lesson as the place where all the relevant factors of teaching are woven together, factors such as goals for students' learning, attention to students' thinking, analyses of curriculum and pedagogy, and assessment. They considered the individual classroom lesson as the smallest unit that preserves the system of teaching. The lesson is where everything must come together; in essence, all the interactions among all the individual features of teaching occur in one classroom lesson. A corollary is that lessons capture the system of teaching and lessons afford teachers the opportunity of working toward improving teaching through careful planning. This planning includes learning to ask higher-order questions and anticipating student responses.

The idea of teachers being reflective practitioners has been around in the realm of education for many years (Dewey, 1933; Nikolic, 2002). However, the incorporation of teacher video for reflective purposes has only become more common as video technology has become more widely affordable and available to teachers. Teacher video has been used successfully in support of different approaches to teacher training and professional development. Such an approach engages the teacher as a critical inquirer, not only of his or her own practice, but also in the entire process of teaching. Implications for the use of video technology in teacher professional development are substantial (McCurry, 2000).

Nikolic (2002) found the use of video for self-reflection to be a significant component of teachers' daily work, presenting a model of systemic self-evaluation to help teachers generate classroom solutions through self-study, thus leading them to more complex forms of classroom inquiry. The researcher contended that self-evaluation has become more widely used for two reasons: self-evaluation generates opportunities for meaningful professional growth and positive change in teacher behavior, and, by utilizing a variety of reflective teaching techniques, teachers are better able to interpret their own and their students' behavior. Nielsen (1990) recommended that teachers embrace a more reflective assessment of their teaching as early as possible, beginning with their first student teaching experiences.

Pape (2004) conducted a video study of the problem-solving skills of middle school students. He found that the instructional practice of teachers requiring students to evaluate the strategies of fellow students and other representations of mathematics made the students better problem solvers. As part of the classroom discourse, the students were not only required to listen to the strategies of other students, but also to comment on those strategies from a mathematical standpoint giving not only mathematical procedures but also mathematical justifications, which would be defined as *high press* interactions by Kazemi and Stipek (2001) and rank high on Bloom's (1956) Taxonomy. *High-press* interactions are described as questions that engage students in a mathematical explanation that consists of more than just a description of a procedure. These questions require students to understand relationships among multiple strategies as well as explore contradictions in solutions and the pursuit of alternative strategies.

Pape (2004) recommended that further research be done to examine instructional practices that hold promise for changing the ways in which students approach the understanding of mathematics. This study is an investigation of those instructional practices (i.e., questioning techniques of teachers).

Professional Development

In general, to deliver high-quality lessons, teachers and those who support them need to know both mathematics and how children learn mathematics. The ability to teach truly shows an understanding of understanding. Teaching for understanding is not easy especially when teachers themselves were not taught for understanding but were brought up relying on rote memorization. Teachers need professional support that really fosters understanding mathematics (Marshall, 2006).

The research community in mathematics education generally agrees on the importance of "enabling teachers to reflect on their practice from a cognitive perspective" (Artzt & Armour-Thomas, 1999, p. 211). Margolinas, Coulange, and Bessot (2005) conducted a study on a special part of teachers' knowledge that they called *didactic knowledge*, with a concentration on the level of observation of students' mathematical activity when interacting with a problem. The goal of the study was to deepen the understanding of the phenomenon of teachers learning from classroom experience. They found that the conditions for this reflection were not satisfied in the ordinary practice of teaching. They reached a conclusion similar to Ponte, Matos, Guimaraes, Leal, & Canavarro (1994)

> Quite significantly, the views and attitudes that underwent the most significant changes had to do with issues that were specifically addressed in the training activities and meetings. On the other hand, the views and attitudes that proved to be more resilient were related to some hidden cultural and professional dimensions which had not been addressed on those occasions. This suggests that significant change may be brought about by external influences when teachers interact in groups with the potential for strong internal dynamics. (Margolinas, Coulange, & Bessot, 2005, p. 357)

In order to change teaching practices, Cwikla (2004) found that teachers need sustained and on-going, rather than short-term, professional development to help them understand new ideas and allow them time to change their practices. Professional development must also focus teachers' thinking and learning on understanding students' thinking. Also, teachers need training on how to question students based on what the students already know. Ball, Hill, and Bass (2005) contended that little improvement is possible without direct attention to the practice of teaching.

Since 1990, Ball et al. (2005) have consistently reported that the mathematical knowledge of many teachers is dismayingly thin. They believe that strong standards and quality curriculum are important; however, no curriculum teaches itself, and standards do not operate independently of professionals' use of them. They concluded, "How well teachers know mathematics is central to their capacity to use instructional materials wisely, to assess students' progress, and to make sound judgments about presentation, emphasis, and sequencing" (p. 14). The authors stated that it is not surprising that U.S. teachers lack sound mathematical understanding and skill; after all, most teachers—and most other adults in this country—are graduates of the very systems that needed to be improved. In fact, many teachers are not among the best graduates of the system and many are even working with less than minimal qualifications in the poorer areas of the U.S.

Rosenberg, Heck, and Banilower (2005) studied the relationships among professional development, college-level content preparation, and traditional teaching practices. The results of the study indicated that teachers were equally likely to engage in traditional teaching practices regardless of how many hours of professional development they attended. Interestingly, teachers with strong college-level content preparation were significantly more likely to engage in traditional practices than teachers with less content preparation. In the same study, the researchers also determined the relationships among professional development, college-level content preparation, and investigative teaching practices. The result indicates that teachers with more hours of professional development tended to score higher on this composite (indicating that they were more likely to use investigative teaching practices), but the relationship began to level off with high amounts of professional development. Also, teachers with strong college-level content preparation tended to have higher scores on this composite than teachers with less content preparation.

Lane (2003) found that students taught by teachers who participated in on-going staff development focused on understanding mathematics concepts, learning about and practicing problem solving strategies, honing questioning skills, and learning new mathematics vocabulary scored higher than students taught by teachers who did not participate in the staff development. Also, the results demonstrate that what teachers know and are able to do makes a difference in the mathematics that children learn.

Teachers must possess solid knowledge of the content they teach, but they must also go a step beyond only understanding how to work problems for themselves; they must understand how children make sense of mathematics and be able to lead them, through facilitation of classroom discourse (e.g., teacher questioning, teacher/student interviewing, student/student mathematical conversation) to a sound understanding of mathematics, including mathematical connections between theory and practice (Hill, Rowan, & Ball, 2005; Ma, 1999; National Research Council, 2001). Much of the classroom discourse can be improved when teachers are efficient and skilled questioners (Dugdale, Matthews, & Guerrero, 2004; Jacobs & Ambrose, 2003). One way to evaluate teachers' questioning techniques and attempt to improve them is through the use of self-reflection via video technology (Nikolic, 2002).

Summary

Mathematics reform has been the object of discussion at the national level for over 40 years. One of the leading topics in the reform movement has been on the improvement of mathematics instruction in the U.S. American students have been outperformed by students of other countries on international tests and have been found to be inferior in international studies such as the Third International Math and Science Study and the most current notable international study, Trends in International Math and Science Study. As a consequence of the low performance of the students in the U.S., a closer look has been taken at the content knowledge and pedagogical knowledge of teachers. Many of the studies cited found that teaching mathematics effectively requires an in-depth understanding of mathematics content as well as an understanding of how children learn mathematics. In particular, it is important to focus on the questioning strategies utilized by teachers because the ability of teachers to ask higher-level questions promotes higher-level thinking by students.

This study investigated questions related to teachers' ability to ask high-level questions of their students and the relationship of their ability to ask high-level questions and their own personal mathematical content knowledge. Chapter 3 describes the study sample, data collection, and instrumentation.

CHAPTER 3

METHODS

The purposes of this study were (a) to determine whether an intense two-week professional development session with middle-school teachers, along with follow-up classroom visits, video review of lessons (with feedback), and two six-hour follow-up sessions each semester would improve the questioning strategies of teachers and promote higher level questioning based on Bloom's Taxonomy of cognitive categories; (b) to determine whether, through reflection and instructor feedback, teachers gained the ability to involve students in high press questioning situations (Kazemi & Stipek, 2001); (c) to investigate whether the professional development resulted in an increase in teacher content knowledge; and (d) to determine whether there was a relationship between teacher pedagogical content knowledge and the ability of the teacher to ask better questions.

Research Design

The study's design was an interrupted time series quasi-experimental design. Quasi-experimental designs are designs of necessity rather than choice and are employed when randomization of individuals is not possible. The function of any quasi-experimental design is the same as an experimental one; namely, to test the existence of a causal relationship between two or more variables. Essential to either experimental or quasi-experimental designs is the ability to establish a comparison base. For quasi-experimental designs the two most common methods of doing so are the use of either nonequivalent comparison groups or an interrupted time series (Johnson & Kuby, 2000).

An interrupted time series is a design that is marked by multiple observations of the experimental units before, during, and after an intervention is introduced. Generally, data analysts look for changes in the response variable that occur during or after the intervention. Such changes may be in mean response for quantitative variables or relative frequencies for categorical variables (Johnson & Kuby, 2000). This study is categorized as in interrupted time series because there is only one group of teachers in the study and they are being evaluated multiple times throughout the course of the project.

While a quasi-experimental design is weaker than a true experimental design, it is not without merit. Such a design may provide a mechanism for chipping away at the uncertainty surrounding the existence of a particular causal relationship and have an advantage over randomized experiments of being relatively easy for the non technical research client or consumer to understand (Hedrick et al., 1993).

Sample

Participants in this study were middle school teachers from 10 public school districts in southwest Arkansas. Following selection of partnering schools that met the criteria established for project funding, project staff recruited throughout south Arkansas and secured 19 participants. This study focused on the third year of a three-year project, with 14 participants being new to the project in the third year. Of the 19 participants, 4 had high school mathematics teacher certification and 15 had elementary or middle school certification. Two teachers had just graduated and not yet taught in a school, however, but each of these did have contracts for the coming year. No teachers in the project had advanced degrees. Only 4 teachers attended alone from their school; all others attended the institute as a school district team. Two teachers included in the project were assigned as part-time math coaches for their district, responsible for assisting and supporting other mathematics teachers with resources, data analysis, instructional strategies, and classroom support. Experience levels ranged from zero years experience to twenty-seven . One participant was unable to complete the requirements for the project and was released. These teachers were employed in schools with fourth through eighth grade levels and a variety of school demographics (high poverty to average income) and achievement levels (ranging from schools in year three of school improvement to higher achieving schools). Schools classified as being in *school improvement* are schools that were unable to make adequate yearly progress (AYP) as defined by the Arkansas Department of Education and the No Child Left Behind Act of 2001 (Arkansas Department of Education, 2006). Schools that were unable to make AYP for three consecutive years were denoted as *year 3 school improvement* schools. Teachers involved in this project were not paid a stipend, however, they were given, free of charge, 6 hours of mathematics graduate credit from Southern Arkansas University.

Instrumentation

The instruments used in this study for the video reflection included three forms: a video reflection form, an instructor feedback form, and Bloom's Taxonomy form. The members of the instructor team developed these forms and each one is available in Appendix A, B, and C, respectively. The video reflection form was used for the participants to articulate their personal thoughts and feelings about the quality of the lesson. This form was filled out and turned in with each video submitted. The form required teachers to reflect over the three-part lesson format, including the launch, explore, and summary. The teacher also made comments about the content of the lesson and his or her ability to actively engage the students. The form was also useful for the participants who wished to ask specific questions to the instructor reading his or her reflection. It also required teachers to reflect on questions they had not asked in the lesson, but retrospectively wished they had asked.

The instructor feedback form was used by the instructors to communicate information to the participants concerning the lesson. This form looked very similar to the video reflection form that the teacher submitted but this form was completed by the instructor and returned to the teacher as constructive feedback on his or her lesson. The instructor watched the teacher's video and commented on the three-part lesson format, suggesting ideas or changes when necessary or appropriate. The form also gave the instructor a place to comment on the teacher's questioning and ability to scaffold student thinking. Additionally, there was a space for general lesson comments and/or concerns.

The Bloom's Taxonomy form was used by the participants to record the questions asked during the lesson and to code the questions based on Bloom's Taxonomy of cognitive categories in order to see how many questions the teacher actually asked at each level. The Bloom's Taxonomy form was created as a Microsoft Excel document that enabled teachers to quickly see column totals. It was created to extend as many pages as necessary for the teachers. In addition, teachers were required to obtain parental permission to video record each student. A sample letter was given to each teacher and the teacher was required to personalize the letter with his or her own school information. The sample letter is provided in Appendix D.

Participants took the Content Knowledge for Teaching Mathematics (CKT-M) Measure for Middle School (2004-Form A) (Hill, Ball, Schilling, & Bass, 2005) in July, 2006. Using the same instrument, participants took a post-test in April of 2007. This measure, developed by Learning Mathematics for Teaching/Study of Instructional Improvement at the University of Michigan, had a test-retest reliability of r=0.89. It was constructed so that a teacher with average mathematics knowledge would receive a score equivalent to the mean score (SD = 1). This instrument is designed to measure how a group of teachers' knowledge develops over time, not to make highly accurate measurements about individual's mathematical knowledge. Pre-testing and post-testing with the same instrument is considered a valid measure of growth, providing at least four months have elapsed between the testing dates and no discussion of the items has taken place (Hill, Ball, Schilling, & Bass, 2005). All scores were reported in standard deviations from the mean. Participants were told they would be required to take a pre-test and a post-test, however, they did not know whether the post-test would be the exact same test or whether it would be a different form of the test. Released items from the elementary version of this test can be found in Appendix E.

Other tests of this nature were available but were not chosen for the purposes of this study like the Diagnostic Mathematics Assessments for Middle School Teachers (Bush, 2005) developed in Louisville, KY and the Knowledge of Algebra for Teaching (Floden & McCrory, 2005) developed in East Lansing, MI.

Treatment

Participants participated in an intense two-week professional development session that lasted 6 hours per day for ten days. The sessions included instruction in mathematics, pedagogy, and theory. The sessions engaged teachers in hands-on mathematics activities designed to deepen their mathematics content as well as to broaden their experiences in pedagogical strategies found to be effective with students (Lappan, et al., 1998). Teachers learned to use video cameras and practiced the process of video recording their lessons and coding their questions. Research was provided about effective questioning strategies and teachers participated in sample high-press questioning exchanges (Kazemi & Stipek, 2001). Effective lesson planning was incorporated and practice time for developing and understanding the launch, explore, summary model (Lappan et al.) was provided.

At the end of the two weeks, participants were given video cameras and DVDs to record lessons for reflection purposes. The participants were required to video record lessons approximately once per month (according to a set schedule). They were then asked to watch the DVD of their lesson and to record each question that was asked during the lesson on an electronic form created in Microsoft Excel and developed by members of the instructional team. Each teacher was asked to record his or her questions and then rank the questions according to Bloom's (1956) Taxonomy as low level, medium level, or high level. The teachers were required to use color highlighting to denote the questions that fall into each part of the three-part lesson: launch, explore, and summary. The launch was defined as the beginning part of the lesson, lasting only a few minutes, that attracts the attention of the students and gets them personally interested in what they are preparing to learn. The explore section of the lesson was the portion of the lesson in which the students were actively engaged in a mathematics activity, game, or problem to be solved. It is usually done in cooperative groups or with partners. The summary section, the most important part of the lesson, is found at the end. This is the time when teachers question the students about how they thought about and solved the problem, what strategies they used, what they have discovered, what they have learned, and how their work relates to the work of the other groups in the class. The formal definition of mathematical terms should come during the summary of the lesson, a time in which students have created a personal relationship with the topic (Lappan, et al., 1998).

The participants were engaged in the following five activities in relation to each video submitted: list and count total number of questions; rank each question on Bloom's Taxonomy of cognitive categories; show divisions within the lesson (launch, explore, summary); write an analysis/reflection of the lesson, including questioning techniques; and indicate high-press question exchanges.

Data Collection

Prior to any data collection participant consent forms were signed by each teacher. This form is found in Appendix F. A Human Subjects Application was filled out and submitted to each university, Southern Arkansas University (Appendix G) and Louisiana Tech University (Appendix H). The Human Use Committee at each university approved the study (Appendix I) and data collection began.

Teachers were given the Content Knowledge for Teaching Mathematics (Hill et al., 2005) Form A as a pre-test in July of 2006 and were given the same test as post-test in April of 2007 that measured pedagogical content knowledge.

Additional data include the DVDs of the teachers' lessons that were watched and coded by the teacher and the researcher and the Bloom's Taxonomy Microsoft Excel spreadsheet. Participants used the video reflection form to reflect on the lesson. The form included a space for the participants to discuss each part of the lesson as well as their reflections on their questioning strategies. In addition, participants were asked to denote any question/answer exchange that could be classified as *high-press*. The videos were delivered to the instructors, and the forms were submitted electronically. After the instructor viewed the video he or she made comments concerning the participants' reflections that included feedback on the lesson, both mathematically and pedagogically, and the form was then returned to the participant electronically.

Null Hypotheses

$H_0$1: There will be no significant difference in the number of high-level questions asked in the first video and the number of high-level questions asked in the last video as defined by Bloom's (1956) Taxonomy.

$H_0$2: There will be no significant difference in the proportion of high-press questioning exchanges in the first video and the proportion of high-press questioning exchanges in the last video.

$H_0$3: There will be no significant difference in the pre-test score on the CKT-M and the post-test score on the CKT-M.

$H_0$4: There will be no relationship between the participants' pedagogical mathematics content knowledge score as measured by the CKT-M and his or her ability to ask students higher-level questions as determined by Bloom's (1956) Taxonomy.

Analysis of the Data

To test null hypothesis 1, the researcher used a *chi-square* test to determine if there was a significant difference in the number of high-level questions asked in the first video and the number asked in the last video. Justification for use of the chi-square test was provided in a written discussion of use of chi-square tests in a text by Cronk (2002). Cronk described this nonparametric test as useful when the corresponding parametric procedures are inappropriate. The observed number of cases in the chi-square test was the number of high-level questions asked video A and video B. The expected number of cases was found by adding the total number of high-level questions asked in video A and video B divided by 2. A significant chi-square test ($p < .05$) indicates that the data vary from the expected values. A test that is not significant ($p > .05$) indicates that the data are consistent with the expected or chance values.

To test null hypothesis 2, a review of each video was done in order to determine the ability of participants to engage students in high-press questioning exchanges. A two-proportion z-test was performed to test hypothesis 2. According to Johnson and Kuby (2000), a two-proportion z-test is used to test differences in proportions. This study used the two-proportion z-test to determine if there was a significant difference in the proportion of high-press questioning exchanges in the first and the last video.

To test null hypothesis 3 a paired t-test was conducted to determine whether the participants showed a significant difference in their pre- and post-test scores of content knowledge on the CKT-M. Johnson and Kuby (2000) stated that a paired t-test is the appropriate test to use when comparing the means of two sets, in this case the pre-test and the post-test.

To test hypothesis 4 a Spearman Rank Order correlation was conducted to determine whether there was a relationship between participants' content knowledge and his or her ability to ask students higher-level questions as determined by Bloom's Taxonomy. Participants' content knowledge post-test scores on the CKT-M were correlated with the frequency of higher-level questions asked in the final video. This correlation was tested for statistical significance. The Spearman Rank Order correlation coefficient was discussed by Gravetter & Wallnau (2009) as a widely used statistic for describing the linear relationship between two variables. The range of this statistic is from -1.00 to +1.00 with -1.00 indicating a perfect inverse relationship—the strongest possible inverse relationship. A score of +1.00 indicates the strongest possible direct relationship. Scores approaching 0.00 indicate the tendency toward no relationship (Gravetter & Wallnau, 2001). The Spearman Rank Order correlation was used because of small sample size and non-normal distribution of data. All hypothesis tests used the 5% level of statistical significance.

Summary

Participants in this study were middle school mathematics teachers. They participated in an intense two-week summer institute that focused on mathematical content knowledge as well as mathematical pedagogical content knowledge. After returning to school in the fall they participated in a series of self-reflections on lessons they taught and captured to DVD. A member of the instructor team gave them feedback on their lessons. The goal of the participants was to ask more high-level questions in their lessons and fewer low-level questions, as well as to ask high-press questions and elicit and facilitate high-press mathematical conversations with children. The quantitative data were analyzed using the Minitab and SPSS statistical software. Chapter 4 describes the results of the study.

CHAPTER 4

RESULTS

This chapter presents the results of the pre- and post-test data and video analysis data described in Chapter 3. The study purported to evaluate the relationship of the questioning strategies of middle school teachers to the mathematics pedagogical content knowledge of the teachers. Also, statistical testing was done to determine whether or not teachers' reflection on their lessons, along with feedback from a member of the instructor team of the partnering higher education institute (Southern Arkansas University) increased the ability of the teacher to ask higher-level questions according to Bloom's Taxonomy. All hypothesis tests used the 5% level of statistical significance.

Hypothesis 1

H_0 1: There will be no significant difference in the number of high-level questions asked in the first video and the number of high-level questions asked in the last video as defined by Bloom's (1956) Taxonomy.

Teachers video recorded their lessons and scripted each question asked onto the Bloom's Taxonomy form. The questions were then classified according to Bloom's (1956) categories of cognitive levels. The specific number of questions in each category is shown in Table 1. The column for synthesis and evaluation of Video A and Video B is shaded because this is the category of questions compared for this study.

Table 1

Video Data from Lessons A and B

	Video A			Video B		
Participant	Know/ Comp No.	App/ Analysis No.	Syn/ Eval No.	Know/ Comp No.	App/ Analysis No.	Syn/ Eval No.
1	22	3	1	28	5	1
2	39	8	1	25	21	15
3	14	1	1	20	2	1
4	30	23	3	34	28	8
5	49	12	4	85	40	14
6	26	8	0	10	4	2
7	66	37	24	36	45	6
8	19	12	6	36	6	6
9	18	0	0	22	1	0
10	62	4	0	34	9	0
11	19	3	1	23	79	0
12	12	3	0	23	2	1
13	20	6	1	28	4	2
14	26	11	5	22	3	0
15	50	14	4	45	5	0
16	20	9	4	26	10	6
17	49	15	1	53	17	0

18	36	19	2	18	26	9
Total	577	188	58	568	307	71

Note. Bold participant number indicates participants with secondary certification.

The hypothesis was rejected, $\chi 2(1, N = 17) = 4.85, p < .05$. When the two videos are compared, there is a significant difference in the number of high-level questions asked. [Data from participant 7 was omitted because it was considered an outlier.]

Even though no statistical comparison was done on the number of low-level questions and the number of middle-level questions, one can examine the table and make some general statements about the questioning levels of the teachers involved in this study. There were about the same number of low-level questions asked in Video A as asked in Video B. There was an increase of over 1/3 in middle-level questions asked in Video A as compared to Video B.

Hypothesis 2

H_0 2: There will be no significant difference in the proportion of high-press questioning exchanges in the first video and the proportion of high-press questioning exchanges in the last video.

The Bloom's Taxonomy Form was also used to track the high-press questioning exchanges. This worked well because the questions were already scripted on this form. The actual number of low-press and high-press exchanges for each teacher in Lesson A and Lesson B are listed in Table 2. Even though the later lesson only had ten accounts of high-press exchanges, this was still a significant change considering there were zero accounts of high-press exchanges in the first lesson.

Table 2

Number of High-Press Exchanges in Lesson A compared to Lesson B

Participant	Lesson A	Lesson B
	# of high press exchanges	# of high press exchanges
1	0	1
2	0	1
3	0	0
4	0	0
5	0	2
6	0	0
7	0	2
8	0	1
9	0	0
10	0	0
11	0	1
12	0	0
13	0	0
14	0	0
15	0	0
16	0	1
17	0	1
18	0	0

Note. Bold indicates participants with secondary certification.

The null hypothesis for the second hypothesis was rejected, $Z = -2.627, p = .014$, two-tailed. There is a significantly greater proportion of high-press exchanges found in Video B.

Hypothesis 3

$H_0 3$: There will be no significant difference in the pre- test score on the CKT-M and the post-test score on the CKT-M.

Table 3 shows the pre-test and post-test score on the CKT-M, along with individual teacher change in column 4.

Table 3

Pre-Test and Post-Test Results on CKT-M

Participant	Pre-test (z-score)	Post-test (z-score)	Change (SD units)
1	-0.234	-0.234	0.000
2	0.643	0.936	0.292
3	-0.526	-0.088	0.439
4	0.205	0.936	**0.731**
5	1.228	2.398	**1.170**
6	-1.257	-1.404	-0.146
7	-0.965	0.058	**1.023**
8	-0.965	-0.819	0.146
9	-0.673	0.497	**1.170**
10	0.058	0.205	0.146
11	-0.673	-0.673	0.000
12	-0.234	0.058	0.292
13	1.228	1.374	0.146
14	-1.550	-0.526	**1.023**
15	0.936	1.667	**0.731**
16	1.959	2.544	**0.585**
17	1.228	1.959	**0.731**
18	-0.380	1.228	**1.608**

Note. Bold type in the "change" column indicates significant improvement for individual teachers at 5%. Bold participant number indicates participants with secondary certification.

The third null hypothesis was rejected. Participants scores on the first administration of the CKT-M (M = 0.002, SD = 0.999322) were significantly lower $t(17) = 4.81$, $p < .05$ (one-tailed), $d = 0.560$.

Table 3 indicates that two participants (1 and 11) scored exactly the same on the pre- and post-test. Only one teacher (6) decreased during the interim between the pre- and post-test. Fifteen teachers had an increase in their post-test scores as compared to their pre-test scores, and of those fifteen, nine teachers demonstrated significant improvement. All secondary certified teachers scored above average on both the pre-and post-test, and interestingly 3 of the 4 secondary teachers showed significant gains. Further, the effect size is 0.560 and an effect size greater than 0.5 is considered moderate and significant. This effect size of 0.560 is roughly equivalent to a 4-item improvement on this 30-item test (Hill, Ball, Schilling, & Bass, 2005).

Hypothesis 4

$H_0 4$: There will be no relationship between the participant's pedagogical mathematics content knowledge score as measured by the CKT-M and his or her ability to ask students higher-level questions as determined by Bloom's (1956) Taxonomy.

There was failure to reject hypothesis 4. Sample coefficient of correlation is $r = .242$ which gives a p-value of $p = .333$. With $r = .242$ the relationship is considered to be small, but because $p > .05$ there is not a significant correlation between these two variables. This indicates that a small relationship exists, but that in 1/3 of the cases this relationship would happen by chance. Therefore, there was no significant relationship found between a teacher's mathematics pedagogical content knowledge as measured by the CKT-M and the teacher's ability to ask high-level questions, according to Bloom's (1956) Taxonomy.

Table 4 is a summary of each hypothesis, the test statistic used, the p-value found, and the significance.

Table 4

Summary of Hypotheses and Results

Hypothesis	Test statistic	*P*-value	Significance
Hypothesis 1: There will not be a significant difference in the number of high level questions asked in the first video to the number asked in the last video.	$\chi 2 = 4.85$	$p < .05$	significant
Hypothesis 2: There will not be a significant difference in the proportion of high-press questioning exchanges in the first video to the proportion of high-press questioning exchanges in the last video.	Z=-2.627	p=.014	significant
Hypothesis 3: There will be no significant difference in the pre-test score on the CKT-M and the post-test score.	t=4.81	$p < .05$	significant
Hypothesis 4: There will be no relationship between the participant's pedagogical content knowledge score and their ability to ask students higher-level questions as determined by Bloom's (1956) Taxonomy.	r = ..242	p = .333	Not significant

CHAPTER 5

DISCUSSION AND CONCLUSIONS

This chapter contains the findings and recommendations from the study. There are five sections included in this chapter: Summary of the Study, Findings and Limitations, Conclusions, Recommendations and Summary.

Summary of the Study

This study was developed to investigate the questioning strategies of teachers and to determine whether there is a relationship between the content knowledge of teachers and their ability to question students at the higher levels of Bloom's (1956) Taxonomy. It also tested content knowledge of teachers. The type of information used in this study was data gathered from video recording and analysis and pre-test and post-test data.

The research questions that were addressed in this study were:

1. Will video review and reflection of teachers' lessons, along with feedback from instructors viewing the videos, increase the number of high-level questions asked by middle school teachers, as measured by Bloom's (1956) Taxonomy?
2. Will video review and reflection of teacher's lessons, along with feedback from instructors, encourage teachers to ask more high-press questions or elicit more high-press questioning situations as defined by Kazemi and Stipek (2001)?

3. Will teachers show a significant difference in content knowledge between the pre-test and post-test as measured by the Content Knowledge for Teaching Mathematics test (Hill, Ball, Schilling, & Bass, 2005)?

4. Is there a relationship between mathematical pedagogical content knowledge as measured by the CKT-M and the ability of the teacher to ask higher-level questions as defined by Bloom (1956)?

The review of literature in this study begins with an overview of the changes in mathematics education reform traced back more than 40 years. It summarizes publications and positions of the National Council of Teachers of Mathematics and other major national and international mathematics studies over the years. The review focuses on areas in content knowledge and pedagogy, questioning, use of video, and professional development.

The sample from this study was drawn from a population including middle school teachers working at schools in southwest Arkansas. The sample was one of convenience because the teachers were all participants in a Math and Science Partnership (MSP) grant funded by No Child Left Behind (NCLB, 2001) monies and enrolled in two graduate classes at Southern Arkansas University. The teachers represented 10 public school districts and included 19 participants, with one dropping out during the course of the project. Of the 19 participants, 4 were secondary certified and 15 were elementary certified.

The research design used was an interrupted time series quasi-experimental design. This design was one of necessity and not one of choice. It was selected because random assignment of teachers to pre-test groups was not possible. The study incorporated the use of an interrupted time-series because multiple observations were made of the experimental units. Such a design provided the researcher a mechanism for investigating the possible existence of a causal relationship, albeit with less statistical power than a random-assignment design.

Findings and Limitations

Direction for this study was provided by positing the following null hypotheses:

H_0 1: There will be no significant difference in the number of high-level questions asked in the first video and the number of high-level questions asked in the last video as defined by Bloom's (1956) Taxonomy.

H_0 2: There will be no significant difference in the proportion of high-press questioning exchanges in the first video and the proportion of high-press questioning exchanges in the last video.

H_0 3: There will be no significant difference in the pre-test score on the CKT-M and the post-test score on the CKT-M.

$H_0$4: There will be no relationship between the participants' pedagogical mathematics content knowledge score as measured by the CKT-M and his or her ability to ask students higher-level questions as determined by Bloom's (1956) Taxonomy.

The first hypothesis determined whether or not teachers were able to ask better questions according to Bloom's (1956) Taxonomy after reviewing videos of themselves teaching a lesson. The *chi-square* test results reflected that the teachers in this study did show a significant difference in the number of high-level questions asked in the later video. The process of watching the video and actually ranking each question that was asked made the teachers more aware of the type of questions they were asking during a lesson. Teachers reported not having spent much time in the past planning questions to ask their students. The study's focus on this particular aspect of the lesson made teachers more aware of the type of questions they asked their students. The fact that teachers asked more higher-level questions tended to result in more time asking questions and less time spent lecturing or showing students how to solve problems.

Keep in mind in reference to hypothesis one that the researcher did not consider student responses, only teacher questions. There may have been situations in which a low-level question actually elicited a very elegant and high-level response from students. Situations like this were not taken into consideration for the purposes of this study. In addition, there may have been very well formulated high-level questions that were not answered by students or high-level questions that students were not able to answer correctly or that did not elicit high-level thinking. These situations were not considered in the study either. Additionally, the study was based on lessons teachers recorded in the fall (first video) and different lessons recorded in the spring of the same school year (last video). It is difficult to determine whether the recorded lessons were characteristic of the teachers' daily instruction or whether the teachers' actions and planning were different on recording days. This problem could be addressed in the future by randomly recording teachers' lessons. Teacher lesson plans could also be collected throughout the school year to provide additional data describing the teacher's instructional strategies, including possible key questions for students.

Another reminder to consider in reference to hypothesis one is that there is a very subjective component in the process of coding the questions that were asked on the video.

Even with very strong descriptors there is still an element of human interpretation in the classification of the teachers' questions.

The second hypothesis compared the proportion of high-press exchanges in the first video to the proportion of high-press exchanges in the last video. The two-proportion *z*-test revealed a significant difference between the proportions of high-press exchanges in the two videos. No participants in the study reported having heard of *high-press questioning* before the summer institute and none had any high-press exchanges in their first video, although some participants did have instances of high-level questioning. After reflecting over the videos, participants were encouraged to plan situations in their lessons that would elicit high-press questioning exchanges. Every teacher entered the study having some background knowledge of Bloom's (1956) Taxonomy, but no teacher had any prior experiences with Kazemi and Stipek's (2001) method of promoting conceptual thinking using high-press questioning.

The study's results indicated that there were more high-press exchanges in the later video. With zero occurrences of high-press exchanges in the first video a small increase can cause significance, however; one must keep in mind that there were still very few occurrences of high-press exchanges in the latter video. One critical component of high-press questioning exchange is the response of the students. Students must go beyond descriptions or summaries of steps to solve problems and link their problem-solving strategies to mathematical reasons. The answer a student gives to a teacher in a high-press exchange requires both explanation and justification. Often a student must understand the relationship among a variety of strategies to solve a particular problem, not just his or her own typical strategy. Therefore, a limitation to the study could be that the teachers did not video the same math class each time. They may not have recorded math classes that had similar classroom environments that were conducive to discourse of this nature. Since teachers were not restricted in their choice of which class to video each time, variables including time of day, number of students, behavior of students, age and grade of students, or class topic could have effected the results. In addition, the ability level or prior classroom experiences of the students could make a difference as to whether they were able to successfully engage in high-press types of exchanges.

Teachers may hesitate in using opportunities for high-press questioning due to a number of reasons. These include the additional difficulty involved in planning as well as the fear of student/parent complaints.

A further consideration could be that the length of the lesson was not controlled. Teachers may not have been able to engage students in this type of interaction due to time constraints. The researcher used the first and the last lesson for comparison. The last lesson was assigned late in the spring. This scheduling could have been restricting to the teachers, requiring them to video record during a week specified by their districts for testing, test review, assemblies, extra-curricular conflicts or other school-based curricular or non-curricular issues. These issues could affect the lesson the teacher selected for taping and the outcome of the lesson.

The third hypothesis tested the change in mathematics pedagogical content knowledge that the teachers displayed between the first day of the summer institute and the last meeting in the late spring of the following year. Results of a paired *t*-test revealed a significant increase in the mathematics pedagogical content knowledge of the collection of participants between testing periods. In addition, half of the individual participants showed significant improvement in the test.

Keep in mind that the teachers attended an intense two-week training during the summer. Teachers can only absorb a limited amount of learning at a time and there was not much time to process, practice, or internalize what they learned. The participants were not post-tested until late in the spring after only two content sessions per semester. In addition, teachers were given the same form of the test for both the pre-testing and post-testing administrations. Even though the teachers did not know that they would be taking the same form of the test again, there could have been some retention of the questions asked on the summer administration of the exam.

The fourth and final hypothesis tested was to determine whether there was a relationship between the participants' mathematics pedagogical content knowledge score as measured by the CKT-M and their ability to ask students higher-level questions as determined by Bloom's (1956) Taxonomy. The results of the Pearson product-moment correlation indicated a moderate relationship between a teachers' pedagogical content knowledge as measured by the CKT-M and the teacher's ability to ask high-level questions to students, however, the outcome was not significant.

The result of no significant relationship between teachers' mathematics pedagogical content knowledge and their ability to ask high-level questions implies an interesting finding. Bloom's Taxonomy of cogitative categories is not content specific. This could indicate that Bloom's (1956) Taxonomy is not the best way to judge a math teacher's questioning strategies. Perhaps a better way to study the questioning strategies of math teachers is to focus on the teacher's ability to engage students in high-press exchanges as defined by Kazemi and Stipek (2001).

One disadvantage of using Bloom's (1956) Taxonomy in this situation is that the teacher could have asked a high-level question as defined in this study but not have enough content knowledge to determine whether the student answered the question correctly. This is similar to the situation considered in the first hypothesis and is a result of not considering student responses. In general, experts believe that teachers having a higher understanding of mathematics content will be more effective teachers (Schuster & Anderson, 2005); however, increasing the mathematics content is not enough (Ball, 2003; Kent, Pligge, & Spence, 2003). This is displayed in hypothesis four with the result that there is not a relationship between mathematics pedagogical content and questioning-asking when setting the bar for teachers based on Bloom's Taxonomy.

An overall limitation of the present study was that it was conducted with a very small sample of teachers and school districts represented. Because of this, it is difficult to apply the findings of this study to teachers in general. Also, it is likely that the teachers in the study participated in other professional development opportunities throughout the year. Therefore, it is difficult to attribute their gains entirely to this treatment.

An additional consideration to ponder in this study is that the teachers chose the lessons that they would use for reflection and submission. Each teacher did this according to a monthly schedule. Even though classroom visits were made to each teacher, it is still possible that the lessons that were submitted were not characteristic of what the teacher does in class on a normal basis. Inaccuracy in the coding of questions affecting the results of this study could also be a limitation. Because much of the study is based on the recording and coding of the questions that the teachers asked, combined with the small sample of teachers, a few inconsistencies in the coding of questions could make a large difference in the results.

Conclusions

One may conclude from this study that teachers' questioning strategies can be changed through self-reflection using video recording and feedback. In this project, a significant increase was found in the number of high-level questions teachers asked in the last video, as compared to the number of high-level questions they asked in the first video, before reflection and feedback from instructors. It may be concluded that this intervention could help other teachers become more effective questioners also. It is important to increase the high-level questioning of teachers because many studies have shown that, currently, teachers questions reside at the lowest level of Bloom's (1956) Taxonomy (Barnes, 1983; Daines, 2001; Gall, 1984; Kenney, 2003; Weiss et al., 2003; & Wolf, 1987). Further, other studies show that the use of higher-level questions produced deeper levels of student learning (Hamaker, 1986; Osman & Hannafin, 1994; Pressley et al., 1992; Pressley, Symons, McDaniel, Snyder, & Turnure, 1988; and Pressley, Tenebaum, McDaniel, & Wood, 1990).

Another conclusion that can be drawn from this study is that video reflection on a teachers' lesson can help them begin to engage students in high-press exchanges. This study shows that teachers can become more comfortable with this type of questioning with on-going professional development and practice. The self-reflection component helps teachers look for opportunities in the lesson that high-press exchanges would fit and thus to plan for these exchanges in future lessons. The results of this study are promising based on Wilson & Kenney's (2003) finding that questioning is one of the weakest elements of mathematics and science instruction. Wilson & Kenney determined that low-level questioning by the teacher gives the teacher insufficient evidence of student understanding. It is also encouraging based on the results of Weiss et al. (2003) that fewer than one in five math and science lessons are strong in intellectual rigor and include teacher questioning that is likely to enhance student conceptual understanding. Teachers must plan and structure opportunities for students to share their thinking, compare strategies, and consider ideas of others (Turner, Junk, & Empson, 2007). Questioning sequences that enlist these opportunities are typical in high-press exchanges as defined by Kazemi and Stipek (2001). Kazemi and Stipek determined that sustained high-press exchanges focused student attention on concepts rather than procedures. A focus on concept rather than procedure is strongly encouraged by the National Council of Teachers of Mathematics (1980, 1989, 1991, 1995, 2000, & 2006).

Results on the CKT-M show that intense mathematical professional development during summer months can increase the mathematics pedagogical content knowledge of teachers. Therefore, one can conclude that teachers who immerse themselves in this type of in-service may be able to increase their mathematics pedagogical content knowledge significantly. Research shows that having sufficient content knowledge is needed in order for teachers to teach well and ask good questions (Schuster & Anderson, 2005). Results from this study are promising because they support the findings of Hill, Rowan, et al. (2005) that specialized mathematical content knowledge does positively predict gains in student achievement.

The results from the test on the last hypothesis indicate that a teacher's pedagogical mathematics content has no effect on ability to ask high-level questions, according to Bloom's (1956) Taxonomy. There was a medium positive correlation found between the participants' scores on the CKT-M and the number of high-level questions asked in the last video which implies that having more pedagogical content knowledge might boost teacher's willingness to ask higher-level questions. However, the correlation was not significant so scoring higher on a mathematics pedagogical content knowledge test does not indicate an ability to ask higher-level questions.

Implications for Practice

Results from hypothesis one showed that video reflection by teachers can most definitely impact questioning strategies as defined by Bloom's Taxonomy (1956). The probability of this happening by chance is less than one time in 1,000. This result implies that professional developers interested in increasing the level of questioning by teachers can utilize a treatment such as this one to enable teachers to reflect on their questioning strategies. One reason this practice works is because it causes teachers to pay attention to what they are actually saying in class. The process of scripting each question gave them the opportunity to not only find out what is actually being said in their classrooms, but also to reflect on other aspects of their lessons (e.g., wait time, which was not considered in this study).

Similarly, results from hypothesis two implies that video taping and reflection by teachers can help to increase the occurrences of high-press exchanges that teachers elicit in their lessons. This will help mold the socio-mathematical norms that teachers expect students to follow in their classrooms. For example, the superficial practice of discussing different strategies is a social norm, but comparing the mathematical concepts underlying different strategies is a socio-mathematical norm as described by Kazemi and Stipek (2001). This result is promising because high-press questioning creates deeper mathematical understanding in students, which in turn, could increase achievement.

The findings from both hypothesis one and hypothesis two could support the process of video recording and teacher reflection in schools. This taping process would be relatively easy to implement in local and regional professional learning communities. Video recording and reflection could easily be incorporated into the professional growth plans of teachers and supported by local or regional mathematics specialists, math coaches, or administrators. If desired, the video recordings could be converted into action research projects at individual school buildings or districts. The video recordings could be scripted, discussed, and converted into case studies for future reference.

The result for hypothesis three implies to professional developers that immersion in intense, concentrated, mathematical activities over an extended time can improve teachers' pedagogical content knowledge. This finding is promising to those providing teacher training and staff development and is consistent with research from Joyce and Showers (2002) that sustained, on-going, professional development is most effective.

Hypothesis four resulted in no significant relationship between the pedagogical content knowledge of teachers as measured by the CKT-M and the ability of teachers to ask high-level questions as defined by Bloom's Taxonomy (1956); however, there are still worthy implications from this finding. One implication is that Boom's Taxonomy may not be the best questioning model to base professional development of mathematics teachers. Since Bloom's Taxonomy is not content specific perhaps it would be more appealing to choose some other model, such as low-press versus high-press questioning as defined by Kazemi and Stipek (2001) or eliciting, supporting, and extending as described by Sowder and Schappelle (2002).

Recommendations for Further Research

Considering the status of student achievement scores in the United States compared to other countries it is obvious that our students could benefit from having teachers more prepared to engage their students in higher-level thinking (U.S Department of Education, 2003; Gonzales, et al., 2008). Summer institutes like Southern Arkansas University's are one step closer to achieving that goal.

One suggestion for future research is to repeat this study in a more controlled, laboratory environment with student teachers. A more controlled environment could produce more reliable results because fewer extraneous variables would be present. It would be beneficial to have access to a control group of teachers who did not participate in the coding of questions, self-reflections, and the in-service provided to determine what, if any, differences are shown in their questioning strategies between the lessons taught at the beginning and the end of the school year. Also, specifications could be made such as requiring teachers to record the same class for each video to alleviate some of the differences created by comparing two unlike groups of students each time. Also, lesson times should be monitored to some degree. The teacher should submit lessons for comparison that are about the same in length. It is difficult to compare a short 20-25 minute lesson with a 90-minute activity. Lesson format must also be considered.

Concerning pre-testing and post-testing of participants, the researcher recommends post-testing teachers at the end of the summer institute and including additional, more regularly scheduled content sessions and post-tests during the course of the project.

The most significant recommendation for future study is to compare the relationship of high-press exchanges to the participants' score on the mathematics pedagogical content knowledge post-test. Because ranking a teacher's questioning on Bloom's (1956) Taxonomy is largely related to the verbs that the teacher has chosen to use in the lesson, it is possible that a teacher could ask a high-level question that did not elicit higher-order thinking. Also, a teacher could possibly engage students in high-press exchanges using questions that do not rank high according to Bloom's Taxonomy as well. In the future, it would be interesting to determine if any correlations exist between the teacher's content knowledge and their ability to engage students in high-press questioning exchanges.

Additional research is recommended that includes teacher certification as one variable. Investigating relationships involving content certification as well as grade band certification would add an interesting dimension to this study.

In general, the most obvious overall recommendation for further study is to conduct research that determines the relationship between teacher questioning and student achievement. This was not possible in the present study because student achievement data were not available. In addition, the researcher recommends considering student responses, teacher wait time, and other classifications of questions.

Summary

The purpose of this study was to determine whether an intense two-week professional development program for middle-school mathematics teachers, along with follow-up classroom visits, video review of lessons (with feedback), and two six-hour follow-up sessions each semester would improve teacher questioning strategies and promote higher level questioning based on Bloom's (1956) Taxonomy of cognitive categories. A second purpose of the study was to determine whether, through reflection and instructor feedback, teachers would gain the ability to involve students in high-press questioning situations. The third purpose of the study was to investigate whether professional development would result in an increase in teacher content knowledge. The fourth and final purpose was to determine if there was a relationship between teachers' pedagogical content knowledge and their ability to ask better questions.

The researcher used four statistical tests including *chi-square*, *z*-test, *t*-test, and Pearson correlation. The population for this study was a group of 18 middle school mathematics teachers from southwest Arkansas. The instruments used for the study included several forms created by the instructor team as well as the CKT-M pre- test and post-test (Hill, Ball, Schilling, & Bass, 2005). Prior to gathering data, human use forms and participant consent forms were completed from both Southern Arkansas University and Louisiana Tech University. In addition, each teacher was required to have parental consent for each student in any class that participated in the video recording. All such forms were completed and filed.

Results showed significance in the first three of the four hypothesis, with no relationship found between the teachers' pedagogical content knowledge on the post-test (CKT-M) and teachers' use of high-level questions as defined by Bloom's Taxonomy (1956). However, statistical significance was found between the number of high-level questions asked in the first video and the last video and the number of high-press exchanges that teachers asked in the first and last video. Additionally, there was a statistically significant increase in the pre-test and post-test scores that measured teachers' pedagogical content knowledge.

Additional research is recommended in the area of teacher questioning. The most critical addition for future study is to add a component that links teacher questioning to student achievement. This could be done by collecting achievement data for each teachers' class and comparing the students' scores before and after the treatment. Another recommendation for future study includes recruiting a larger sample of teachers and a decrease in the number of extraneous variables.

Important findings were discovered in this study that have a direct effect on ways to enhance teaching and learning. A worthwhile professional development opportunity for teachers could be to institute the practice of video recording teachers' lessons periodically and reviewing them to develop actual case studies for reflection. Transcripts from those cases could then be used by other teachers or staff developers to strengthen teaching and learning in their own classes and schools.

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APPENDIX A

VIDEO REFLECTION FORM

SAU NCLB MATHEMATICS OF THE MIDDLE SCHOOL
2006-2007

VIDEO COVER SHEET for TEACHERS

TEACHER:	SCHOOL:
GRADE LEVEL:	DATE (S) OF LESSON (S):
TOPIC OF LESSON (S):	
Teacher comments about the math content:	
Teacher comments about student engagement:	
Comments about instructional strategies: Launch, Explore, and Summarize Write a brief description on what you have PLANNED for the three parts of the lesson. If this does not go as planned you can make note of that later in the comment section.	

Launch	Explore	Summarize

Comments about questioning strategies:

Describe in this space how you felt about your questions and your ability to scaffold the questions. List any questions that you wish you had asked after reflecting on the video. Were there times when a student was headed in a particular direction that you wish you had explored further? If so, discuss this in the space provided here.

General Comments about the lesson:

How did it go? Include comments on how well the lesson fit with the plans that you had, or if you made changes from your plans, describe your thinking on why you did this. Where you satisfied with the lesson or are there things you wish you had done differently? Will you do this the same next time? If not, describe how you will make changes and why.

APPENDIX B

INSTRUCTOR FEEDBACK FORM

SAU NCLB MATHEMATICS OF THE MIDDLE SCHOOL
2006-2007

LESSON FEEDBACK FROM COURSE INSTRUCTOR

TEACHER:	SCHOOL:	
GRADE LEVEL:	DATE (S) OF LESSON (S):	
TOPIC OF LESSON (S):		
Comments about the math content:		
Comments about student engagement:		
Comments about instructional strategies: Launch, Explore, and Summarize:		
Launch	Explore	Summarize

Comments about questioning strategies:

Cont. on back if necessary….

APPENDIX C

BLOOM'S TAXONOMY RECORDING FORM

NO CHILD LEFT BEHIND
THE MATHEMATICS OF THE MIDDLE SCHOOL

TEACHER:	SCHOOL:
GRADE LEVEL:	DATE:
LESSON TITLE:	BLOOMS

Content Questions Asked:	Knowledge Comprehension	Application Analysis	Synthesis Evaluation	Other (Classroom Mgmt., etc.)
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				
11				
12				
13				
14				
15				
16				
17				
18				
19				
20				
21				
22				
23				
24				
25				
TOTALS:	0	0	0	0

APPENDIX D

SAMPLE LETTER FOR PARENTAL PERMISSION

TO VIDEO STUDENTS

September X, 200X

Dear Parent,

As your child's teacher, I am enrolled in two graduate courses at SAU in Magnolia. The courses are designed to strengthen teachers' math knowledge and techniques. As part of the course, the teacher's mathematics class is video taped in the classroom at our school. The tapes will be primarily of the teacher teaching and asking questions, but may occasionally capture your child's face, voice, or movement on tape.

I would like to have your permission to utilize the videos for study with my graduate class and instructors at SAU. The video clips and/or images will only be used for teacher instruction and/or training and for materials related to the course.

Sincerely,

teacher name

I give permission for my child to be included in video and/or pictures captured as part of the taping of my child's teacher. I understand that the video will be recording randomly during math instruction.

____

parent signature

APPENDIX E

RELEASED ITEMS FROM THE CKT-M

LEARNING MATHEMATICS FOR TEACHING

MATHEMATICAL KNOWLEDGE FOR TEACHING (MKT) MEASURES

MATHEMATICS RELEASED ITEMS 2008

University of Michigan, Ann Arbor
610 E. University #1600
Ann Arbor, MI 48109-1259
(734) 647-5233
www.sitemaker.umich.edu/lmt

 Measures development supported by NSF grants REC-9979873, REC- 0207649, EHR-0233456 & EHR 0335411, and by a subcontract to CPRE on Department of Education (DOE), Office of Educational Research and Improvement (OERI) award #R308A960003.

Study of Instructional Improvement

lmt learning mathematics for teaching project

December 26, 2008

Dear Colleague:

Thank you for your interest in our survey items measuring mathematical knowledge for teaching. To orient you to the items and their potential use, we explain their development, intent, and design in this letter.

The effort to design survey items measuring teachers' knowledge for teaching mathematics grew out of the unique needs of the *Study of Instructional Improvement* (SII). SII is investigating the design and enactment of three leading whole school reforms and these reforms' effects on students' academic and social performance. As part of this research, lead investigators realized a need not only for measures which represent school and classroom processes (e.g., school norms, resources, teachers' instructional methods) but also teachers' facility in using disciplinary knowledge in the context of classroom teaching. Having such measures will allow SII to investigate the effects of teachers' knowledge on student achievement, and understand how such knowledge affects program implementation. While many potential methods for exploring and measuring teachers' content knowledge exist (i.e., interviews, observations, structured tasks), we elected to focus our efforts on developing survey measures because of the large number of teachers (over 5000) participating in SII.

Beginning in 1999, we undertook the development of such survey measures. Using theory, research, the study of curriculum materials and student work, and our experience, we wrote items we believe represent some of the competencies teachers use in teaching elementary mathematics – representing numbers, interpreting unusual student answers or algorithms, anticipating student difficulties with material. With the assistance of the University of California Office of the President[1], we piloted these items with K-6 teachers engaged in mathematics professional development. This work developed into a sister project to SII, *Learning Mathematics for Teaching* (LMT). With funding from the National Science Foundation, LMT has taken over instrument development from SII, developing and piloting geometry and middle school items.

We have publicly released a small set of items from our projects' efforts to write and pilot survey measures. We believe these items can be useful in many different contexts: as open-ended prompts which allow for the exploration of teachers' reasoning about mathematics and student thinking; as materials for professional development or teacher education; as exemplars of the kinds of mathematics teachers must know to teach. We encourage their use in such contexts. <u>However, this particular set of items is, as a group, NOT appropriate for use as an overall measure, or scale, representing teacher knowledge.</u> In other words, one <u>cannot</u> calculate a teacher score that reliably indicates either level of content knowledge or growth over time.

We ask users to keep in mind that these items represent steps in the process of developing measures. In many cases, we released items that failed, statistically speaking, in our piloting; in these cases, items may contain small mathematical ambiguities or other imperfections. If

---

[1] Elizabeth Stage, Patrick Callahan, Rena Dorph, principals.

you have comments or ideas about these items, please feel free to contact one of us by email at the addresses below.

These items are the result of years of thought and development, including both qualitative investigations of the content teachers use to teach elementary mathematics, and quantitative field trials with large numbers of survey items and participating teachers. Because of the intellectual effort put into these items by SII investigators, we ask that *all* users of these items satisfy the following requirements:

1) Please request permission from SII for any use of these items. To do so, contact Geoffrey Phelps at gphelps@umich.edu. Include a brief description of how you plan to use the items, and if applicable, what written products might result.

2) In any publications, grant proposals, or other written work which results from use of these items, please cite the development efforts which took place at SII by referencing this document:

> Hill, H.C., Schilling, S.G., & Ball, D.L. (2004) Developing measures of teachers' mathematics knowledge for teaching. Elementary School Journal 105, 11-30.

3) Refrain from using these items in multiple choice format to evaluate teacher content knowledge in any way (e.g., by calculating number correct for any individual teacher, or gauging growth over time). Use in professional development, as open-ended prompts, or as examples of the kinds of knowledge teachers might need to know is permissible.

You can also check the SII website (http://www.sii.soe.umich.edu/) or LMT website (http://www.sitemaker.umich.edu/lmt) for more information about this effort.

Below, we present three types of released item – elementary content knowledge, elementary knowledge of students and content, and middle school content knowledge. Again, thank you for your interest in these items.

Sincerely,

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Dean, School of Education
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Content Knowledge for Teaching Mathematics Measures (MKT measures)
Released Items, 2008
ELEMENTARY CONTENT KNOWLEDGE ITEMS

1. Ms. Dominguez was working with a new textbook and she noticed that it gave more attention to the number 0 than her old book. She came across a page that asked students to determine if a few statements about 0 were true or false. Intrigued, she showed them to her sister who is also a teacher, and asked her what she thought.

Which statement(s) should the sisters select as being true? (Mark YES, NO, or I'M NOT SURE for each item below.)

	Yes	No	I'm not sure
a) 0 is an even number.	1	2	3
b) 0 is not really a number. It is a placeholder in writing big numbers.	1	2	3
c) The number 8 can be written as 008.	1	2	3

2. Imagine that you are working with your class on multiplying large numbers. Among your students' papers, you notice that some have displayed their work in the following ways:

Student A	Student B	Student C
35	35	35
x25	x25	x25
125	175	25
+75	+700	150
875	875	100
		+600
		875

Which of these students would you judge to be using a method that could be used to multiply any two whole numbers?

	Method would work for all whole numbers	Method would NOT work for all whole numbers	I'm not sure
a) Method A	1	2	3
b) Method B	1	2	3
c) Method C	1	2	3

3. Ms. Harris was working with her class on divisibility rules. She told her class that a number is divisible by 4 if and only if the last two digits of the number are divisible by 4. One of her students asked her why the rule for 4 worked. She asked the other students if they could come up with a reason, and several possible reasons were proposed. Which of the following statements comes closest to explaining the reason for the divisibility rule for 4? (Mark ONE answer.)

a) Four is an even number, and odd numbers are not divisible by even numbers.

b) The number 100 is divisible by 4 (and also 1000, 10,000, etc.).

c) Every other even number is divisible by 4, for example, 24 and 28 but not 26.

d) It only works when the sum of the last two digits is an even number.

4. Ms. Chambreaux's students are working on the following problem:

Is 371 a prime number?

As she walks around the room looking at their papers, she sees many different ways to solve this problem. Which solution method is correct? (Mark ONE answer.)

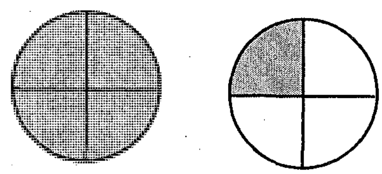
a) Check to see whether 371 is divisible by 2, 3, 4, 5, 6, 7, 8, or 9.

b) Break 371 into 3 and 71; they are both prime, so 371 must also be prime.

c) Check to see whether 371 is divisible by any prime number less than 20.

d) Break 371 into 37 and 1; they are both prime, so 371 must also be prime.

5. Mrs. Johnson thinks it is important to vary the whole when she teaches fractions. For example, she might use five dollars to be the whole, or ten students, or a single rectangle. On one particular day, she uses as the whole a picture of two pizzas. What fraction of the two pizzas is she illustrating below? (Mark ONE answer.)



a) 5/4

b) 5/3

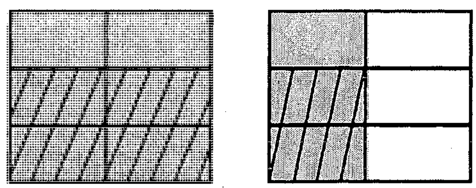
c) 5/8

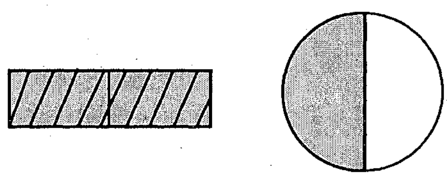
d) 1/4

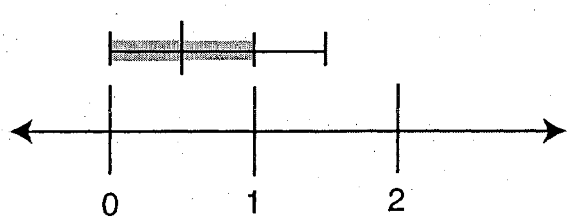
6. At a professional development workshop, teachers were learning about different ways to represent multiplication of fractions problems. The leader also helped them to become aware of examples that do <u>not</u> represent multiplication of fractions appropriately.

Which model below <u>cannot</u> be used to show that $1\frac{1}{2} \times \frac{2}{3} = 1$? (Mark ONE answer.)

A) 

B) 

C) 

D) 

7. Which of the following story problems could be used to illustrate $1\frac{1}{4}$ divided by $\frac{1}{2}$? (Mark YES, NO, or I'M NOT SURE for each possibility.)

	Yes	No	I'm not sure
a) You want to split $1\frac{1}{4}$ pies evenly between two families. How much should each family get?	1	2	3
b) You have $1.25 and may soon double your money. How much money would you end up with?	1	2	3
c) You are making some homemade taffy and the recipe calls for $1\frac{1}{4}$ cups of butter. How many sticks of butter (each stick = $\frac{1}{2}$ cup) will you need?	1	2	3

8. As Mr. Callahan was reviewing his students' work from the day's lesson on multiplication, he noticed that Todd had invented an algorithm that was different from the one taught in class. Todd's work looked like this:

$$\begin{array}{r} 983 \\ \underline{\times\ 6} \\ 488 \\ +\underline{5410} \\ 5898 \end{array}$$

What is Todd doing here? (Mark ONE answer.)

a) Todd is regrouping ("carrying") tens and ones, but his work does not record the regrouping.

b) Todd is using the traditional multiplication algorithm but working from left to right.

c) Todd has developed a method for keeping track of place value in the answer that is different from the conventional algorithm.

d) Todd is not doing anything systematic. He just got lucky – what he has done here will not work in most cases.

ELEMENTARY KNOWLEDGE OF STUDENTS AND CONTENT ITEMS

9. Mr. Garrett's students were working on strategies for finding the answers to multiplication problems. Which of the following strategies would you expect to see some elementary school students using to find the answer to 8 x 8? (Mark YES, NO, or I'M NOT SURE for each strategy.)

	Yes	No	I'm not sure
a) They might multiply 8 x 4 = 32 and then double that by doing 32 x 2 = 64.	1	2	3
b) They might multiply 10 x 10 = 100 and then subtract 36 to get 64.	1	2	3
c) They might multiply 8 x 10 = 80 and then subtract 8 x 2 from 80: 80 – 16 = 64.	1	2	3
d) They might multiply 8 x 5 = 40 and then count up by 8's: 48, 56, 64.	1	2	3

10. Students in Mr. Hayes' class have been working on putting decimals in order. Three students — Andy, Clara, and Keisha — presented 1.1, 12, 48, 102, 31.3, .676 as decimals ordered from least to greatest. What error are these students making? (Mark ONE answer.)

a) They are ignoring place value.

b) They are ignoring the decimal point.

c) They are guessing.

d) They have forgotten their numbers between 0 and 1.

e) They are making all of the above errors.

11. You are working individually with Bonny, and you ask her to count out 23 checkers, which she does successfully. You then ask her to show you how many checkers are represented by the 3 in 23, and she counts out 3 checkers. Then you ask her to show you how many checkers are represented by the 2 in 23, and she counts out 2 checkers. What problem is Bonny having here? (Mark ONE answer.)

a) Bonny doesn't know how large 23 is.

b) Bonny thinks that 2 and 20 are the same.

c) Bonny doesn't understand the meaning of the places in the numeral 23.

d) All of the above.

12. Mrs. Jackson is getting ready for the state assessment, and is planning mini-lessons for students focused on particular difficulties that they are having with adding columns of numbers. To target her instruction more effectively, she wants to work with groups of students who are making the same kind of error, so she looks at a recent quiz to see what they tend to do. She sees the following three student mistakes:

I) $\begin{array}{r} \scriptstyle 1 \\ 38 \\ 49 \\ +\ 65 \\ \hline 142 \end{array}$ II) $\begin{array}{r} \scriptstyle 1 \\ 45 \\ 37 \\ +\ 29 \\ \hline 101 \end{array}$ III) $\begin{array}{r} \scriptstyle 1 \\ 32 \\ 14 \\ +\ 19 \\ \hline 64 \end{array}$

Which have the same kind of error? (Mark ONE answer.)

a) I and II

b) I and III

c) II and III

d) I, II, and III

13. Ms. Walker's class was working on finding patterns on the 100's chart. A student, LaShantee, noticed an interesting pattern. She said that if you draw a plus sign like the one shown below, the sum of the numbers in the vertical line of the plus sign equals the sum of the numbers in the horizontal line of the plus sign (i.e., 22 + 32 + 42 = 31 + 32 + 33). Which of the following student explanations shows sufficient understanding of <u>why this is true</u> for all similar plus signs? (Mark YES, NO or I'M NOT SURE for each one.)

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

	Yes	No	I'm not sure
a) The average of the three vertical numbers equals the average of the three horizontal numbers.	1	2	3
b) Both pieces of the plus sign add up to 96.	1	2	3
c) No matter where the plus sign is, both pieces of the plus sign add up to three times the middle number.	1	2	3
d) The vertical numbers are 10 less and 10 more than the middle number.	1	2	3

14. Mrs. Jackson is getting ready for the state assessment, and is planning mini-lessons for students around particular difficulties that they are having with subtracting from large whole numbers. To target her instruction more effectively, she wants to work with groups of students who are making the same kind of error, so she looks at a recent quiz to see what they tend to do. She sees the following three student mistakes:

I

$$\begin{array}{r} {\scriptstyle 4\ 12} \\ \cancel{5}0\cancel{2} \\ -\ 6 \\ \hline 406 \end{array}$$

II

$$\begin{array}{r} {\scriptstyle 4\quad 15} \\ 3\cancel{5}00\cancel{5} \\ -\quad 6 \\ \hline 34009 \end{array}$$

III

$$\begin{array}{r} {\scriptstyle 6\,9\,8\,15} \\ \cancel{7}\cancel{0}\cancel{0}\cancel{5} \\ -\ 7 \\ \hline 6988 \end{array}$$

Which have the same kind of error? (Mark ONE answer.)

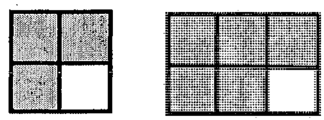
a) I and II

b) I and III

c) II and III

d) I, II, and III

15. Takeem's teacher asks him to make a drawing to compare $\frac{3}{4}$ and $\frac{5}{6}$. He draws the following:



and claims that $\frac{3}{4}$ and $\frac{5}{6}$ are the same amount. What is the <u>most likely</u> explanation for Takeem's answer? (Mark ONE answer.)

a) Takeem is noticing that each figure leaves one square unshaded.

b) Takeem has not yet learned the procedure for finding common denominators.

c) Takeem is adding 2 to both the numerator and denominator of $\frac{3}{4}$, and he sees that that equals $\frac{5}{6}$.

d) All of the above are equally likely.

16. A number is called "abundant" if the sum of its proper factors exceeds the number. For example, 12 is abundant because $1 + 2 + 3 + 4 + 6 > 12$. On a homework assignment, a student incorrectly recorded that the numbers 9 and 25 were abundant. What are the most likely reason(s) for this student's confusion? (Mark YES, NO or I'M NOT SURE for each.)

	Yes	No	I'm not sure
a) The student may be adding incorrectly.	1	2	3
b) The student may be reversing the definition, thinking that a number is "abundant" if the number exceeds the sum of its proper factors.	1	2	3
c) The student may be including the number itself in the list of factors, confusing proper factors with factors.	1	2	3
d) The student may think that "abundant" is another name for square numbers.	1	2	3

MIDDLE SCHOOL CONTENT KNOWLEDGE ITEMS

17. Students sometimes remember only part of a rule. They might say, for instance, "two negatives make a positive." For each operation listed, decide whether the statement "two negatives make a positive" sometimes works, always works, or never works. (Mark SOMETIMES, ALWAYS, NEVER, or I'M NOT SURE)

	Sometimes works	Always works	Never works	I'm not sure
a) Addition	1	2	3	4
b) Subtraction	1	2	3	4
c) Multiplication	1	2	3	4
d) Division	1	2	3	4

18. Mrs. Smith is looking through her textbook for problems and solution methods that draw on the distributive property as their primary justification. Which of these familiar situations could she use to demonstrate the distributive property of multiplication over addition [i.e., a (b + c) = ab + ac]? (Mark APPLIES, DOES NOT APPLY, or I'M NOT SURE for each.)

	Applies	Does not apply	I'm not sure
a) Adding $\frac{3}{4}+\frac{5}{4}$	1	2	3
b) Solving $2x - 5 = 8$ for x	1	2	3
c) Combining like terms in the expression $3x^2 + 4y + 2x^2 - 6y$	1	2	3
d) Adding 34 + 25 using this method: $\begin{array}{r} 34 \\ \underline{+25} \\ 59 \end{array}$	1	2	3

19. Students in Mr. Carson's class were learning to verify the equivalence of expressions. He asked his class to explain why the expressions a – (b + c) and a – b – c are equivalent. Some of the answers given by students are listed below.

Which of the following statements comes <u>closest</u> to explaining why a – (b + c) and a – b – c are equivalent? (Mark ONE answer.)

a) They're the same because we know that a – (b + c) doesn't equal a – b + c, so it must equal a – b – c.

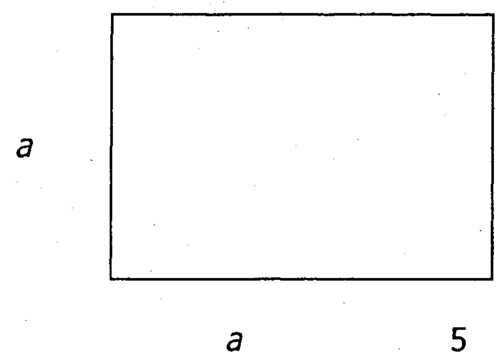
b) They're equivalent because if you substitute in numbers, like a=10, b=2, and c=5, then you get 3 for both expressions.

c) They're equal because of the associative property. We know that a – (b + c) equals (a – b) – c which equals a – b – c.

d) They're equivalent because what you do to one side you must always do to the other.

e) They're the same because of the distributive property. Multiplying (b + c) by –1 produces –b – c.

20. Ms. Whitley was surprised when her students wrote many different expressions to represent the area of the figure below. She wanted to make sure that she did not mark as incorrect any that were actually right. For each of the following expressions, decide whether the expression correctly represents or does not correctly represent the area of the figure. (Mark REPRESENTS, DOES NOT REPRESENT, or I'M NOT SURE for each.)



		Correctly represents	Does not correctly represent	I'm not sure
a)	$a^2 + 5$	1	2	3
b)	$(a + 5)^2$	1	2	3
c)	$a^2 + 5a$	1	2	3
d)	$(a + 5)a$	1	2	3
e)	$2a + 5$	1	2	3
f)	$4a + 10$	1	2	3

21. Ms. Hurlburt was teaching a lesson on solving problems with an inequality in them. She assigned the following problem.

$$-x < 9$$

Marcie solved this problem by reversing the inequality sign when dividing by -1, so that $x > -9$. Another student asked why one reverses the inequality when dividing by a negative number; Ms. Hurlburt asked the other students to explain. Which student gave the best explanation of why this method works? (Mark ONE answer.)

a) Because the opposite of x is less than 9.

b) Because to solve this, you add a positive x to both sides of the inequality.

c) Because $-x < 9$ cannot be graphed on a number line, we divide by the negative sign and reverse the inequality.

d) Because this method is a shortcut for moving both the x and 9 across the inequality. This gives the same answer as Marcie's, but in different form: $-9 < x$.

TOPIC-SPECIFIC ITEMS (2006-2008 PILOTS)

22. At the close of a lesson on reflection symmetry in polygons, Ms. White gave her students several problems to do. She collected their answers and read through them after class. For the problem below, several of her students answered that the figure has two lines of symmetry and several answered that it has four.

How many lines of symmetry does this figure have?

Which of the following is the most likely reason for these incorrect answers? (Circle ONE answer.)

a) Students were not taught the definition of reflection symmetry.

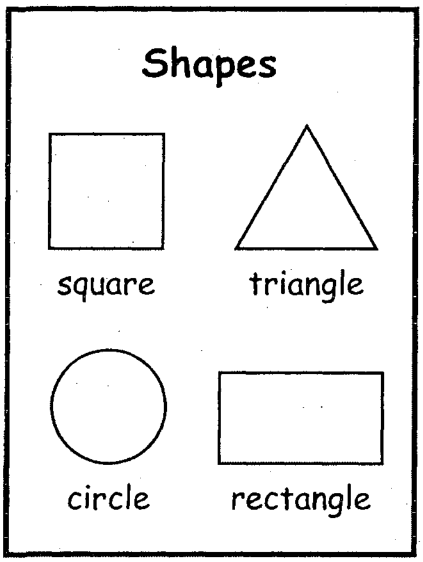
b) Students were not taught the definition of a parallelogram.

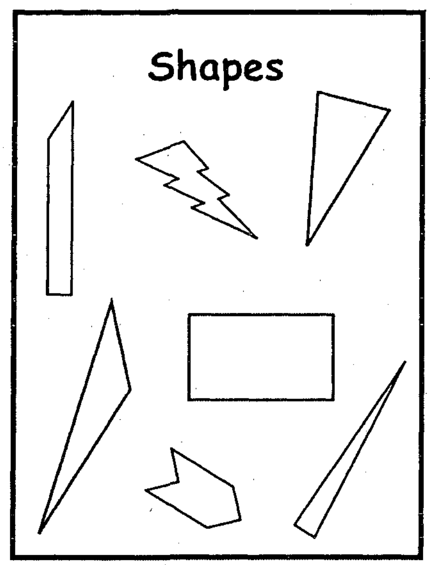
c) Students confused lines of symmetry with edges of the polygon.

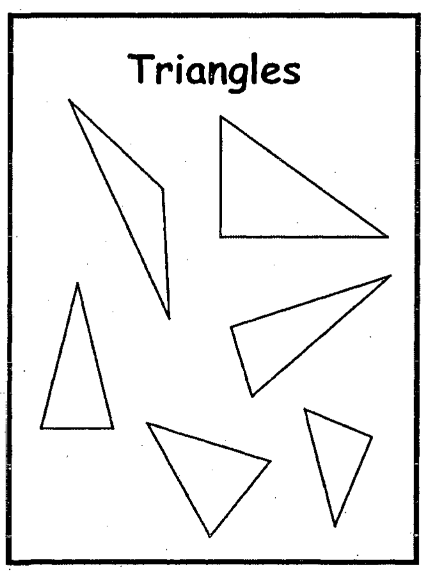
d) Students confused lines of symmetry with rotating half the figure onto the other half.

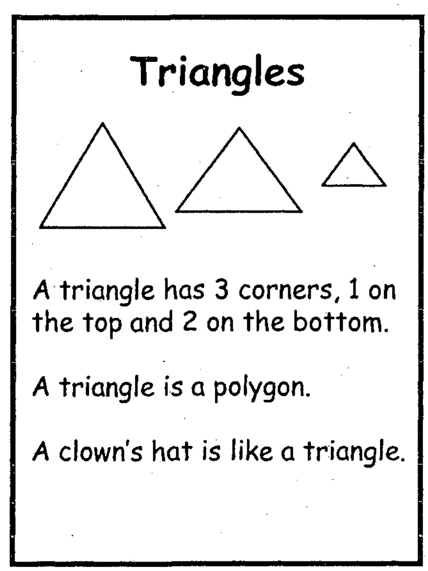
23. Ms. Miller wants her students to write or find a definition for triangle, and then improve their definition by testing it on different shapes. To help them, she wants to give them some shapes they can use to test their definition.

She goes to the store to look for a visual aid to help with this lesson. Which of the following is <u>most</u> likely to help students improve their definitions? (Circle ONE answer.)

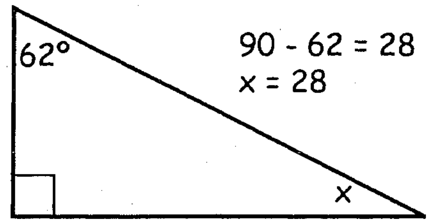
a)


b)


c)


d)


24. Ms. Donaldson's class was working on an assignment where they had to find the measures of unknown angles in triangles. One student consistently found the measures of unknown angles in right triangles by subtracting the known angle from 90. For example:



$90 - 62 = 28$
$x = 28$

Ms. Donaldson was concerned that this student might run into difficulty when trying to find the measures of unknown angles in more general triangles. Which of the following questions would be best to ask the student in order to help clarify this issue? (Circle ONE answer.)

a) "What do you get when you add 90 + 62 + 28?"

b) "Why does subtracting 62 from 90 give you the measure of the unknown angle?"

c) "How could you find the missing angle in an isosceles triangle?"

d) "How did you know that this was a right triangle?"

e) "What if this angle measured 17° instead of 62°?"

25. As an early introduction to mathematical proof, Ms. Cobb wants to engage her students in deductive reasoning. She wants to use an activity about the sum of the angles of a triangle, but her students have not yet learned the alternate interior angle theorem. They do, however, know that a right angle is 90 degrees and that a point is surrounded by 360 degrees. Which of the following activities would best fit her purpose? (Circle ONE answer.)

a) Have students draw a triangle and a line parallel to its base through the opposite vertex. From there, have them reason about the angles of the triangle and the angles the triangle makes with the parallel line.

b) Have the students use rectangles with diagonals to reason about the sum of the acute angles in a right triangle.

c) Have students use protractors to measure the angles in several different triangles and from there reason about the sum of the angles of a triangle.

d) Have students cut out a triangle then tear off the three corners and assemble them, and from there reason about the sum of the angles of a triangle.

26. Mrs. Davies' class has learned how to tessellate the plane with any triangle. She knows that students often have a hard time seeing that any quadrilateral can tessellate the plane as well. She wants to plan a lesson that will help her students develop intuitions for how to tessellate the plane with any quadrilateral.

Which of the following activities would best serve her purpose? (Circle ONE answer.)

a) Have students cut along the diagonal of various quadrilaterals to show that each can be broken into two triangles, which students know will tessellate.

b) Provide students with multiple copies of a non-convex kite and have them explore which transformations lead to a tessellation of the plane.

c) Provide students with pattern blocks so that they can explore which of the pattern block shapes tessellate the plane.

d) These activities would serve her purpose equally well.

27. Ms. Abdul is preparing a unit to introduce her students to proportional reasoning. She is considering three versions of a problem that are the same except for the numbers used. Which version of the Mr. Short and Mr. Tall problem below is likely to be the most challenging for students? (Circle ONE answer.)

a) A picture depicts Mr. Short's height as 4 paper clips and as 6 buttons. The height of Mr. Tall (not shown) is given as 6 paper clips. How many buttons in height is Mr. Tall?

b) A picture depicts Mr. Short's height as 4 paper clips and as 7 buttons. The height of Mr. Tall (not shown) is given as 5 paper clips. How many buttons in height is Mr. Tall?

c) A picture depicts Mr. Short's height as 2 paper clips and as 9 buttons. The height of Mr. Tall (not shown) is given as 5 paper clips. How many buttons in height is Mr. Tall?

d) All three of the problems are equally challenging.

28. Mr. Garrison's students were comparing different rectangles and decided to find the ratio of height to width. They wondered, though, if it would matter whether they measured the rectangles using inches or measured the rectangles using centimeters.

As the class discussed the issue, Mr. Garrison decided to give them other examples to consider. For each situation below, decide whether it is an example for which different ways of measuring produce the same ratio or a different ratio. (Circle PRODUCES SAME RATIO, PRODUCES DIFFERENT RATIO, or I'M NOT SURE for each.)

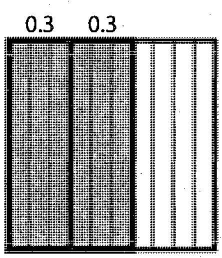
	Produces same ratio	Produces different ratio	I'm not sure
a) The ratio of two people's heights, measured in (1) feet, or (2) meters.	1	2	3
b) The noontime temperatures yesterday and today, measured in (1) Fahrenheit, or (2) Centigrade.	1	2	3
c) The speeds of two airplanes, measured in (1) feet per second, or (2) miles per hour.	1	2	3
d) The growths of two bank accounts, measured in (1) annual percentage increase, or (2) end-of-year balance minus beginning-of-year balance.	1	2	3

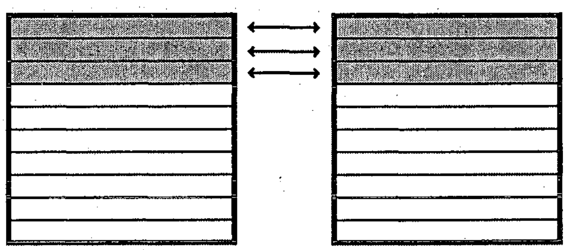
29. Ms. Austen was planning a lesson on decimal multiplication. She wanted to connect multiplication of decimals to her students' understanding of multiplication as repeated addition. She planned on reviewing the following definition with her class:

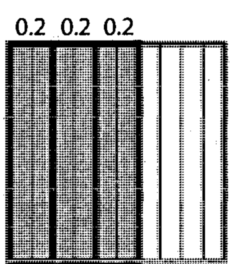
> The repeated addition interpretation of multiplication defines $a \times b$ as b added together a times, or a groups of b.

After reviewing this definition of repeated addition, she planned to ask her students to represent the problem 0.3 x 2 using the repeated addition interpretation of multiplication.

Which of the following representations <u>best</u> illustrates the repeated addition definition of
0.3 x 2? (Circle ONE answer.)

a) 

b) 

c) 

d) These representations illustrate the repeated addition definition of 0.3 x 2 equally well.

e) Multiplication of decimals cannot be represented using a repeated addition interpretation of multiplication.

30. Mr. Shephard is using his textbook to plan a lesson on converting fractions to decimals by finding an equivalent fraction. The textbook provides the following two examples:

Convert $\frac{2}{5}$ to a decimal: $\frac{2}{5}=\frac{4}{10}=0.4$

Convert $\frac{23}{50}$ to a decimal: $\frac{23}{50}=\frac{46}{100}=0.46$

Mr. Shephard wants to have some other examples ready in case his students need additional practice in using this method. Which of the following lists of examples would be best to use for this purpose? (Circle ONE answer.)

a) $\frac{1}{4}$ $\frac{8}{16}$ $\frac{8}{20}$ $\frac{4}{5}$ $\frac{1}{2}$

b) $\frac{1}{20}$ $\frac{7}{8}$ $\frac{12}{15}$ $\frac{3}{40}$ $\frac{5}{16}$

c) $\frac{3}{4}$ $\frac{2}{3}$ $\frac{7}{20}$ $\frac{2}{7}$ $\frac{11}{30}$

d) All of the lists would work equally well.

31. Ms. James' class was investigating patterns in whole-number addition. Her students noticed that whenever they added an even number and an odd number the sum was an odd number. Ms. James asked her students to explain why this claim is true for all whole numbers.

After giving the class time to work, she asked Susan to present her explanation:

> *I can split the even number into two equal groups, and I can split the odd number into two equal groups with one left over. When I add them together I get an odd number, which means I can split the sum into two equal groups with one left over.*

Which of the following best characterizes Susan's explanation? (Circle ONE answer.)

a) It provides a general and efficient basis for the claim.

b) It is correct, but it would be more efficient to examine the units digit of the sum to see if it is 1, 3, 5, 7, or 9.

c) It only shows that the claim is true for one example, rather than establishing that it is true in general.

d) It assumes what it is trying to show, rather than establishing why the sum is odd.

32. To introduce the idea of grouping by tens and ones with young learners, which of the following materials or tools would be most appropriate? (Circle ONE answer.)

a) A number line

b) Plastic counting chips

c) Pennies and dimes

d) Straws and rubber bands

e) Any of these would be equally appropriate for introducing the idea of grouping by tens and ones.

33. Mr. Foster's class is learning to compare and order fractions. While his students know how to compare fractions using common denominators, Mr. Foster also wants them to develop a variety of other intuitive methods.

Which of the following lists of fractions would be best for helping students learn to develop several different strategies for comparing fractions? (Circle ONE answer.)

a) $\frac{1}{4} \quad \frac{1}{20} \quad \frac{1}{19} \quad \frac{1}{2} \quad \frac{1}{10}$

b) $\frac{4}{13} \quad \frac{3}{11} \quad \frac{6}{20} \quad \frac{1}{3} \quad \frac{2}{5}$

c) $\frac{5}{6} \quad \frac{3}{8} \quad \frac{2}{3} \quad \frac{3}{7} \quad \frac{1}{12}$

d) Any of these would work equally well for this purpose.

34. Ms. Brockton assigned the following problem to her students:

How many 4s are there in 3?

When her students struggled to find a solution, she decided to use a sequence of examples to help them understand how to solve this problem. Which of the following sequences of examples would be best to use to help her students understand how to solve the original problem? (Circle ONE answer.)

a) How many:
    4s in 6?
    4s in 5?
    4s in 4?
    4s in 3?

b) How many:
    4s in 8?
    4s in 6?
    4s in 1?
    4s in 3?

c) How many:
    4s in 1?
    4s in 2?
    4s in 4?
    4s in 3?

d) How many:
    4s in 12?
    4s in 8?
    4s in 4?
    4s in 3?

35. Ms. Williams plans to give the following problem to her class:

> Baker Joe is making apple tarts. If he uses $\frac{3}{4}$ of an apple for each tart, how many tarts can he make with 15 apples?

Because it has been a while since the class has worked with fractions, she decides to prepare her students by first giving them a simpler version of this same type of problem. Which of the following would be most useful for preparing the class to work on this problem? (Circle ONE answer.)

I. Baker Ted is making pumpkin pies. He has 8 pumpkins in his basket. If he uses $\frac{1}{4}$ of his pumpkins per pie, how many pumpkins does he use in each pie?

II. Baker Ted is making pumpkin pies. If he uses $\frac{1}{4}$ of a pumpkin for each pie, how many pies can he make with 9 pumpkins?

III. Baker Ted is making pumpkin pies. If he uses $\frac{3}{4}$ of a pumpkin for each pie, how many pies can he make with 10 pumpkins?

a) I only

b) II only

c) III only

d) II and III only

e) I, II, and III

APPENDIX F

PARTICIPANT CONSENT FORM

SOUTHERN ARKANSAS UNIVERSITY
INFORMED CONSENT DOCUMENT

The Department/School of Science and Technology, Center for Teaching Excellence in Math and Science supports the practice of protection for human subjects participating in research and related activities. The following information is provided so that you can decide whether you wish to participate in the present study. You should be aware that even if you agree to participate, you are free to withdraw at any time, and that if you do withdraw from the study, you will not be subjected to reprimand or any other form of reproach.

1. Procedures to be followed in the study, as well as identification of any procedures, which are experimental.

Teachers video one lesson per month and log and rank their questions, as well as write a reflection of the lesson. The instructors provide feedback and classroom visits. The teachers attend a two week institute on mathematical content the summer before the video reflection begins. The teachers also take a pre and post test to determine difference in pedagogical content knowledge.

2. Description of any attendant discomforts or other forms of risk involved for subjects taking part in the study. None

3. Description of benefits to be expected from the study or research.
   Increased content knowledge of middle school math teachers, increased ability to engage students in higher-order questioning exchanges

4. Appropriate alternative procedures that would be advantageous for the subject.
   none

"I have read the above statement and have been fully advised of the procedures to be used in this project. I have been given sufficient opportunity to ask any questions I had concerning the procedures and possible risks involved and I assume them voluntarily. I likewise understand that I can withdraw from the study at any time without being subjected to reproach."

____________________________ ____________________________

Subject Date

APPENDIX G

HUMAN SUBJECTS APPLICATION SOUTHERN

ARKANSAS UNIVERSITY

SOUTHERN ARKANSAS UNIVERSITY

APPLICATION FOR APPROVAL TO USE HUMAN SUBJECTS

This should be attached to the project proposal or description and submitted to the Institutional Review Board for Treatment of Human Subjects

1. Name of principal Investigator(s) or Responsible Individuals:________________Lynne Nielsen____

2. Departmental Affiliation:___Science and Tech Center for Teaching Excellence in Math and Science
3. Title of Project:_______No Child Left Behind Middle School Math III
   ____

4. Funding Agency (If applicable):______none___________________________________

5. Project Purposes (s):

This is a dissertation that will study. **The purposes of this study are: (a) to determine whether an intense two-week professional development with middle school teachers, along with follow-up classroom visits, video review of lessons (with feedback), and two six-hour follow-up sessions each semester will serve to improve the questioning strategies of teachers and promote higher level questioning based on Bloom's Taxonomy of cognitive categories; (b) to determine whether, through reflection and instructor feedback, teachers gain the ability to involve students in "high press" questioning situations (Kazemi and Stipek, 2001); (c) to investigate whether the professional development resulted in an increase in teacher content knowledge; (d) to determine whether there is a relationship in teacher pedagogical content knowledge and the ability of the teacher to ask better questions**

6. Describe the proposed subjects: (age, sex, race, or other special characteristics, such as students in a specific class, etc.)

The subjects of this study are the students enrolled in Math for the Middle School during the fall 2006 and spring 2007 semesters. They are area middle school teachers.

7. Describe how the subjects are to be selected:

The subjects were from a convenience sample, they are the students enrolled in the Math for the Middle School classes.

8. Describe the proposed procedures in the project. Any proposed experimental activities that are included in evaluation, research, development, demonstration instruction, study, treatments, debriefing, questionnaires, and similar projects must be described here if they are not clearly outlined in the project proposal or description. (Use additional page if necessary.)

Attached as chp. 1 and chp 3 (These are draft versions)

9. Will questionnaires, tests, or related research instruments, not explained in question #8 be used? _________Yes _________No (If yes, attach a copy to this application.)

10. Will electrical or mechanical devices be used? _________Yes _____X____No (If yes, attach a detailed description of the device(s).)

11. Are the risks to human subjects outweighed by the benefits of the research?
_____X____Yes, the risks to human subjects are outweighed by the benefits of the research.
_________No, the risks to human subjects are not outweighed by the benefits of the research.
On what page of the project description is this information outlined? _________
If not provided in the project description, such information should be outlined here.

12. Are there any possible emergencies, which might arise in utilization of human subjects in this project? _________Yes _____X____No On what page of the project description are these emergencies discussed? __________ Further detail maybe provided here.

13. What provisions will you take for keeping research data private?

The data will be kept secure in my office, no names will be used in the reports, data is not analyzed at the student level.

APPENDIX H

HUMAN SUBJECTS APPLICATION

LOUISIANA TECH UNIVERSITY

Human Subjects Application
Louisiana Tech University

Do you plan to publish this study? ☐YES ☐NO
Will this study be published by a national organization? ☐YES ☐NO
COMMENTS: **This is a dissertation proposal, I don't know if it will be published or not. It is not being written for publication purposes, however; it would be great if a publication does come out of it.**

STUDY/PROJECT INFORMATION FOR HUMAN SUBJECTS COMMITTEE

Describe your study/project in detail for the Human Subjects Committee. Please include the following information.

TITLE: The effects of video reflection by teachers on questioning strategies in middle school mathematics classes and the relationship of pedagogical content knowledge to the ability of teachers to ask higher-level questions (this is a draft title)

PROJECT DIRECTOR(S): Lynne Nielsen, student; Dr. Lawrence Leonard, committee chair
EMAIL: lynne.nielsen@arkansas.gov
PHONE: 870.299.0832

DEPARTMENT(S): Southern Arkansas University, Center for Teaching Excellence in Mathematics and Science Teaching; Graduate studies, Louisiana Tech University, doctoral dissertation, Dr. Lawrence Leonard, major professor

PURPOSE OF STUDY/PROJECT:

This is a dissertation that will study. The purposes of this study are: (a) to determine whether an intense two-week professional development with middle school teachers, along with follow-up classroom visits, video review of lessons (with feedback), and two six-hour follow-up sessions each semester will serve to improve the questioning strategies of teachers and promote higher level questioning based on Bloom's Taxonomy of cognitive categories; (b) to determine whether, through reflection and instructor feedback, teachers gain the ability to involve students in "high-press" questioning situations (Kazemi and Stipek, 2001); (c) to investigate whether the professional development resulted in an increase in teacher content knowledge; (d) to determine whether there is a relationship in teacher pedagogical content knowledge and the ability of the teacher to ask better questions.

SUBJECTS: Middle school teachers from southwest Arkansas enrolled in NCLB summer math institute, summer of 2006, fall 2006 and spring 2007

PROCEDURE:
Teachers video one lesson per month and log and rank their questions, as well as write a reflection of the lesson. The instructors provide feedback and classroom visits. The teachers attend a two week institute on mathematical content the summer before the video reflection begins. The teachers also take a pre- and post-test to determine difference in pedagogical content knowledge.

INSTRUMENTS AND MEASURES TO INSURE PROTECTION OF CONFIDENTIALITY, ANONYMITY:

Content Knowledge for Teaching Mathematics (CKT-M) Measure for Middle School (2004-Form A); Video reflection form, Instructor feedback form, question log

RISKS/ALTERNATIVE TREATMENTS: none

BENEFITS/COMPENSATION: Increased pedagogical content knowledge of teachers, increased ability to ask higher-level questions in middle school mathematics classes

SAFEGUARDS OF PHYSICAL AND EMOTIONAL WELL-BEING: all data will be kept secure, no names will be used in reporting, data will not be analyzed at the student level

Note: Use the Human Subjects Consent form to briefly summarize information about the study/project to participants and obtain their permission to participate.

APPENDIX I

HUMAN SUBJECTS APPROVAL LETTER

MEMORANDUM

TO: Dr. Corbet Lamkin, Southern Ark. University;
Dr. Lawrence Leonard, Committee Chair; Jonathan Friedmann, Brandon Mik,
Paul O'Meallie, and Daniel Ray

FROM: Barbara Talbot, University Research

SUBJECT: HUMAN USE COMMITTEE REVIEW

DATE: April 15, 2009

In order to facilitate your project, an EXPEDITED REVIEW has been done for your proposed study entitled:

"The Effects of Video Reflection by Teachers on Questioning Strategies in Middle School Mathematics Classes and the Relationship of Pedagogical Content Knowledge to the Ability of Teachers to ask Higher-Level Questions (Draft Title)"

\# HUC-390

The proposed study's revised procedures were found to provide reasonable and adequate safeguards against possible risks involving human subjects. The information to be collected may be personal in nature or implication. Therefore, diligent care needs to be taken to protect the privacy of the participants and to assure that the data are kept confidential. Informed consent is a critical part of the research process. The subjects must be informed that their participation is voluntary. It is important that consent materials be presented in a language understandable to every participant. If you have participants in your study whose first language is not English, be sure that informed consent materials are adequately explained or translated. Since your reviewed project appears to do no damage to the participants, the Human Use Committee grants approval of the involvement of human subjects as outlined.

Projects should be renewed annually. ***This approval was finalized on May 16, 2007 and this project will need to receive a continuation review by the IRB if the project, including data analysis, continues beyond May 16, 2008***. Any discrepancies in procedure or changes that have been made including approved changes should be noted in the review application. Projects involving NIH funds require annual education training to be documented. For more information regarding this, contact the Office of University Research.

You are requested to maintain written records of your procedures, data collected, and subjects involved. These records will need to be available upon request during the conduct of the study and retained by the university for three years after the conclusion of the study. If changes occur in recruiting of subjects, informed consent process or in your research protocol, or if unanticipated problems should arise it is the Researchers responsibility to notify the Office of Research or IRB in writing. The project should be discontinued until modifications can be reviewed and approved.

If you have any questions, please contact Dr. Mary Livingston at 257-4315.