The Mathematical Modeling of Ballet

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Abstract

This project aims to analyze the connections between ballet and mathematics. Specifically, this project focuses on analyzing the three-dimensional surfaces created as a dancer performs ballet choreography. The primary goal is to use a Vicon motion capture system in conjunction with MATLAB to model the three-dimensional lines and surfaces created by a dancer's legs as she performs specific ballet movements. The movements used for this experiment were a pique turn and a rond de jambe. The data was collected using sensors to create objects in Vicon to record the position of the ankle, knee, and hip of the working leg while the dancer performed specific ballet movements. The position data was first analyzed in Excel, where specific criteria were used to eliminate data points that were incorrect due to errors while recording. Then the position data was exported to MATLAB for analysis. The data points were used to create fitted polynomials through the ankle, knee, and hip. Next, line segments were constructed between the ankle and knee and between the knee and hip. The ultimate goal is to find a closed form equation to mathematically describe the surfaces created by the dancers legs for each move and to find a general model for future experiments.

Keywords: ballet, mathematical modeling

1 Introduction and Motivation

Ballet and mathematics may not seem to share any common ground, but perhaps a closer look would reveal some surprising ways in which mathematics can be found in ballet. This project aims to explore one way in which these two fields are related. Specifically, this study

[1] Thanks to Dr. Miguel Gates for the use of his lab, and thanks to everyone who helped record data and take pictures in the lab.

examines the surfaces that a ballerina creates with her legs in three-dimensional space as she performs ballet choreography.

This research was motivated by the observation that ballet dancers move in three-dimensional space. As a dancer performs choreography, each of her legs acts as two line segments joined at the knee. One line segment can be constructed from the hip to the knee, and another line segment can be constructed from the knee to the ankle. By examining fundamental ballet movements and focusing on the working leg, this research aims to see if and how each movement can be modeled using common surfaces in mathematics. Specifically, the surfaces created by ballet movements will be examined through the lens of multivariable calculus and common three-dimensional surfaces.

This paper will summarize this research by first looking at other research relating dance and mathematics and analyzing the specific movements studied here. The methods for recording the data used in this project will then be detailed. The analysis of the data and the models constructed from it will then be presented. Finally, the ramifications and possible future work resulting from this research will be explored.

2 Related Literature and Background Studies

The most common comparison between dance and mathematics is the geometry involved in dance. Some have used this interplay between the two in an innovative way to teach elementary-aged children common geometry concepts, such as angles. One way this is accomplished is through examining the angle a dancer's feet make in first position, with her heels together and toes turned outward. Another way angles can be seen in dance is through examining a dancer's legs in bent positions and finding the angle of the knee. Additionally, groups of dancers can be utilized to demonstrate other geometrical concepts, such as translations and symmetries [2]. As a whole, dance provides teachers with interactive ways to teach students mathematical concepts, especially for students who are kinaesthetic learners [10].

The two ballet movements examined in this research are a rond de jambe and a pique turn. Rond de jambe literally translates to "round of the leg" in French [7]. This movement is begun in first position. The dancer then moves her working leg forward into tendu, or a stretched position, with the foot pointed. The dancer then traces an arc on the floor with her foot as she moves it around to a back tendu. The dancer then pulls her leg back to first position to complete the movement [3]. This movement is pictured in Figure 1.

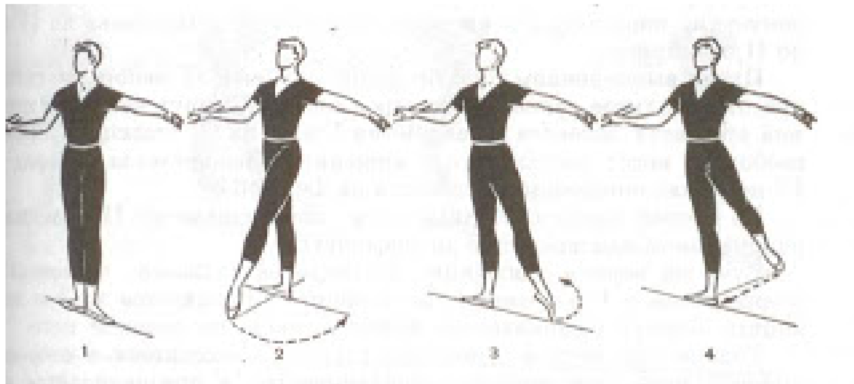

Figure 1: Pictured here is the movement of the working leg for a ballet dancer performing a rond de jambe. Perlowski, Leann. "Ballet Term of the Week - Rond De Jambe." A Time To Dance, 12 Apr. 2010, *atimetodanceblog.blogspot.com/2010/04/ballet-term-of-week-rond-de-jambe.html*.

The second movement analyzed here is a pique turn. The word pique literally translate to "pricking" in French. In particular, this research examined the right leg of a left pique turn. To execute this movement, the dancer begins by stepping to her left onto relev as her right leg is quickly snatched into retir. The momentum of this quick pulling in of the leg allows the dancer to complete a full revolution before lowering her right leg [5]. This movement is pictured in Figure 2.

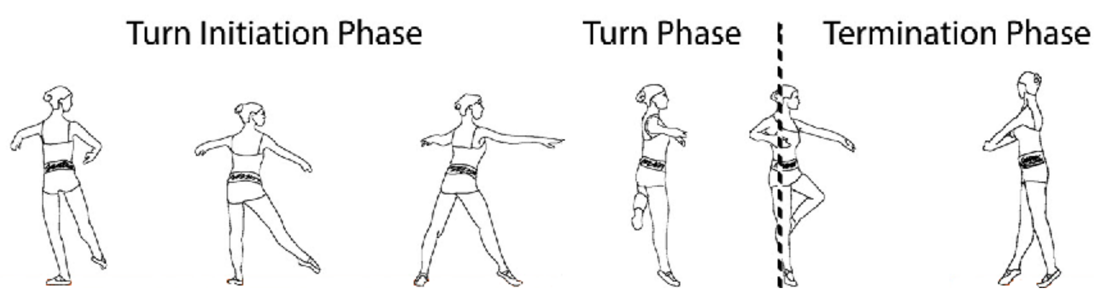

Figure 2: Pictured here is the movement of a ballet dancer performing a pique turn. Zaferiou, Antonia M., et al. "Modification of Impulse Generation during Piqu Turns with Increased Rotational Demands." *Human Movement Science*, vol. 47, 2016, pp. 220230., doi:10.1016/j.humov.2016.03.012.

3 Methods

The methods used to gather and interpret data in this project can be broken into four major categories: recording data, cleaning and editing data, graphically depicting data, and mathematically modeling data. These methods are described in detail in the following section.

3.1 Recording Data

The data for this project was recorded using a Vicon motion capture system. Motion capture is defined by Vicon as "the process of recording the movement of objects or people" [12]. The motion capture lab is located at Louisiana Tech University in Nethken Hall room 156. The lab consists of seven cameras spaced around the room near the ceiling. Small reflective markers are used to track a particular object. The cameras relay the positions of the markers in the room to a computer, which displays the positions of each marker with respect to a grid centered in the room. In this system, three or more markers can be used to create a permanent object by arranging them in a unique configuration. Then whenever the system detects a configuration of markers unique to an object, the collection of markers is displayed on the grid as a single unit.

For this particular project, a total of thirteen markers were used to form four unique objects. The dancer was fitted with a pair of leggings that had markers attached to them for four critical areas of the dancer's working leg: the ankle, directly below the knee, directly above the knee, and the hip. The markers were placed so that the centroid of each object would be an accurate representation of that part of the dancer's leg. Pictured in Figures 3 and 4 are the dancer wearing the leggings and an image of the depiction of the markers in Vicon.

Figure 3: Presented here is a picture of the dancer wearing the leggings equipped with the thirteen markers. Three markers are on the hip, six markers are around the knee, and four markers are around the ankle.

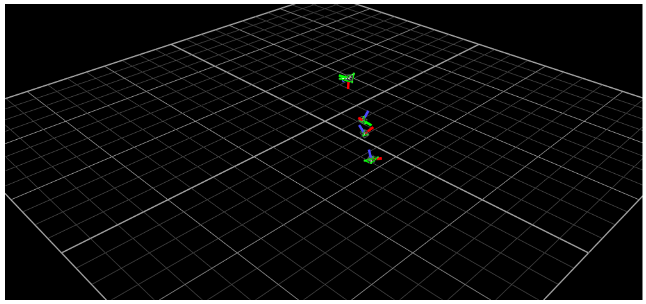

Figure 4: Presented here is the depiction in Vicon of the previous image of the dancer in the leggings.

To actually record the data, a student ran a code in MATLAB while the dancer performed a ballet move. This code imported the positions of the four objects from the Vicon system to MATLAB. The data was transferred into large three-column matrices containing the x, y, and z positions for each marker at each time sample. Each object's positions were sent to a separate matrix. For each object, the centroid was then calculated from the individual marker locations for each sample to create a new matrix for the ankle, below the knee, above the knee, and the hip. Finally, the above the knee centroid matrix and the below the knee centroid matrix were averaged to find a single centroid matrix for the knee.

3.2 Cleaning and Editing Data

After the data was displayed in matrices in MATLAB, it was exported to Excel for cleaning and editing. The first step of cleaning the data was to remove the data points recorded before the dancer began moving and after the dancer stopped moving. To do so, graphs of the x, y, and z positions for each object were examined to find where the objects' locations were unchanging. The same beginning and ending data points were removed from the collection of samples for each object.

Next the data was examined for blips, samples where the recorded location was inaccurate for some reason. Occasionally, while recording data, the location of a particular marker would become blocked from the view of the cameras for a very small set of samples. These samples would have locations a significant distance from the other samples near them. To create an accurate depiction of the dancer's movement, these samples were removed from the set of data points for each object.

Once the data was edited and cleaned, Excel was used to graph each of the x, y, and z locations with respect to time for each object. A trendline of the data was used to determine what degree polynomial, up to sixth, was required to achieve an R^2 value greater than or very close to 0.9. This edited collection of data was then exported back to MATLAB for further analysis.

3.3 Graphically Depicting Data

After the data was exported back to MATLAB, the function *polyfit* was used to find an approximate n^{th} degree polynomial, as determined previously in Excel, for the x, y, and z positions of each object. Once these nine equations were determined, the curves for the ankle, knee, and hip were plotted using the *plot*3 function, giving three curves in three-dimensional space. To create a graphical approximation of an actual surface, for each sample, a straight line was constructed between the ankle and knee and another straight line was constructed between the knee and hip. These lines represent the dancer's leg in between the object positions. These lines were also plotted in MATLAB using the *plot*3 function. The graphical representation was then used to determine which mathematically defined surface would best approximate the surface created by the dancer's leg.

3.4 Mathematically Modeling Data

Once the surface for a particular ballet movement was graphically constructed in MATLAB, a defined mathematical surface was used to approximate the surface created by the dancer's leg. The approximation was also plotted in MATLAB and compared to the original surface for accuracy. The specific models and techniques for measuring accuracy for each movement will be discussed in further detail below.

4 Data and Models

For this particular project, two specific ballet movements were studied: a rond de jambe and a pique turn. The data and models for these two movements are explained further below. Before each set of data was modeled, it was scaled to the unit of meters and shifted to be centered at the origin.

4.1 Rond de Jambe

The simpler of the two movements is the rond de jambe. This movement involves a dancer moving a straight leg in a circular motion without ever picking her foot up off the floor. The theoretical motion of the leg is depicted in Figure 5. Here, the horizontal line represents the x-axis.

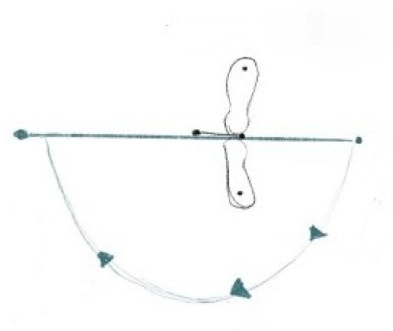

Figure 5: Presented here is the theoretical shape that a dancer's foot creates when she performs a rond de jambe. King, David. "NOTES ON ROND DE JAMBE..." A Ballet Education, 2 Apr. 2018, *aballeteducation.com/2017/03/22/notes-on-ron-de-jambe/.*

Two runs were recording of the rond de jambe. First, one run was analyzed in-depth to construct a mathematical model. Then, the other run was used to test the mathematical model. The analysis of the first set of data is as follows.

Initially, a polynomial was created using *polyfit* for the x, y, and z coordinates for the ankle, knee, and hip, resulting in nine polynomials. Then a straight line was constructed from the ankle to the knee and another from the knee to the hip. Each of these curves was parameterized with respect to time. This resulted in the following graph.

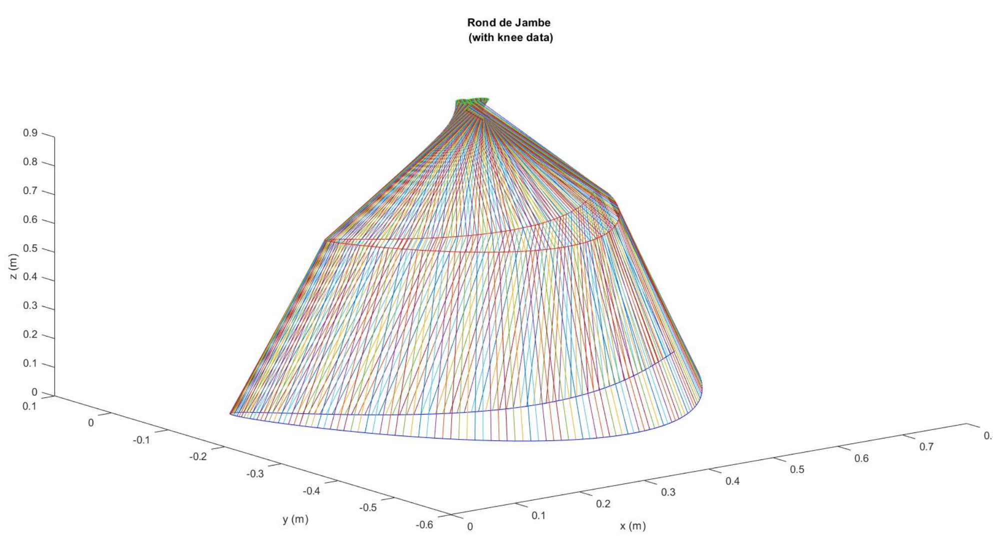

Figure 6: Presented here is a graphical depiction of the data for the first run of the rond de jambe.

Due to the placement of the markers, this resulted in a "dented" cone. For this movement, since the dancer's leg was straight for the duration of the movement, only the hip and ankle coordinates were analyzed. A straight line was constructed from the ankle to the hip for each set of data points in the sample. Graphically, this movement resulted in what appeared to be half of a cone. Removing the knee data resulted in the following graph.

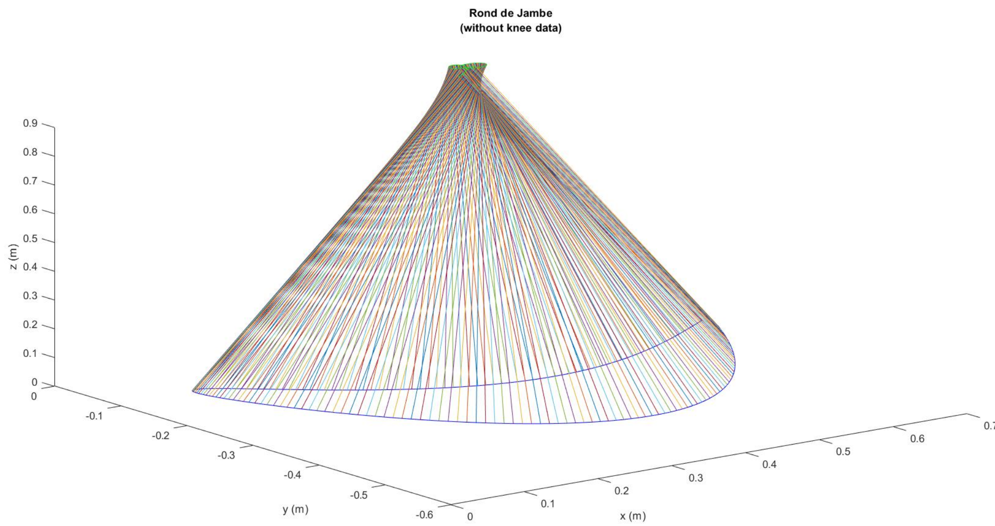

Figure 7: Presented here is a graphical depiction of the data for the first run of the rond de jambe with the knee data removed.

Two separate cone equations were then used to approximate the surface created by the rond de jambe. The formula used is as follows.

$$\frac{z^2}{c^2} = \frac{x^2}{a^2} + \frac{y^2}{b^2} + c \tag{1}$$

In this equation, x,y,and z are variables represented the typical coordinates in $\mathbb{R}^3$. The constant c represents the height of the cone. The constants a and b represent the major and minor axis radii. For both models, c was found by averaging the z-values for the hip centroid.

The first model assumed that there was an unchanging radius throughout the movement. In this case, for the equation above, $a = b$. This radius was found by taking the average distance of the ankle from the origin in the xy- plane using the typical Euclidean distance formula.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \tag{2}$$

This model yielded the following results.

$$\frac{z^2}{0.824230906^2} = \frac{x^2 + y^2}{0.32311325^2} + 0.824230906 \tag{3}$$

This model gives the following graphical representation.

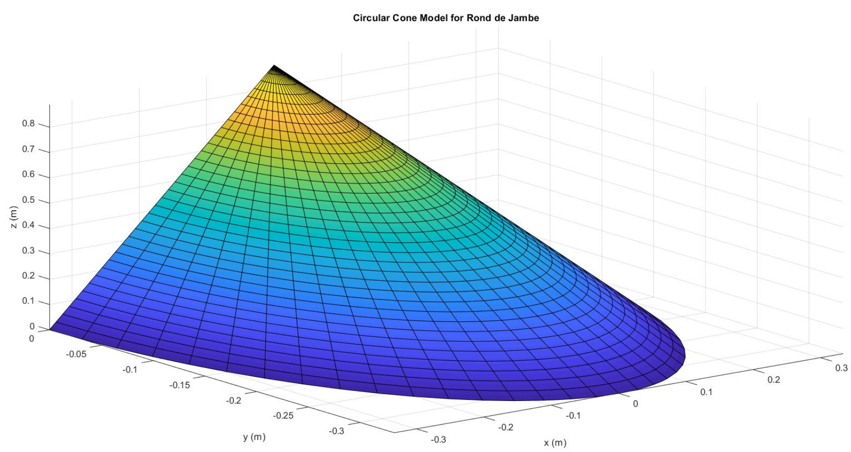

Figure 8: Presented here is the graphical depiction of the circular cone model of a rond de jambe.

The second model assumed the cone was elliptic and had a major and minor axis. The axis lengths for found by taking the maximum x and y values of the shifted data. This model yielded the following results.

$$\frac{z^2}{0.824230906^2} = \frac{x^2}{0.294929683^2} + \frac{y^2}{0.362706573^2} + 0.824230906 \tag{4}$$

This model gives the following graphical representation.

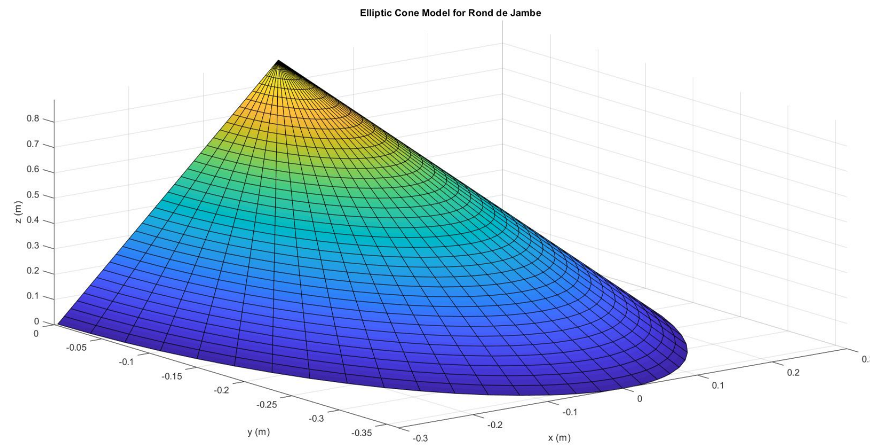

Figure 9: Presented here is the graphical depiction of the elliptic cone model of a rond de jambe.

After these two models were developed, the accuracy of each was analyzed by finding the average error in the xy-plane. The trace of each cone in the xy-plane can be found by setting $z = 0$. Each x-value from the dataset is substituted into the model to find the theoretical y-value. Then the distances between the theoretical and actual y-values is averaged for the entire dataset. The trace for the circular cone model is presented below.

$$y = -\sqrt{0.32311325^2 - x^2} \tag{5}$$

This model yielded an average error of 4.9 cm. The trace for the elliptic cone model is presented below.

$$y = -\sqrt{0.362706573^2 - \frac{0.362706573^2}{0.294929683^2}x^2} \tag{6}$$

This model yielded an average error of 3.3 cm.

After completing the analysis for the first set of data for the rond de jambe, the models were tested on the second set of data. For this data, the traces were found directly and tested to see if a circular cone model or elliptic cone model would most accurately represent the data. The trace for the circular cone model is given below.

$$y = -\sqrt{0.33046047^2 - x^2} \tag{7}$$

This model yielded an average error of 2.7 cm. The trace for the elliptic cone model is given below.

$$y = -\sqrt{0.348502749^2 - \frac{0.348502749^2}{0.295853809^2}x^2} \tag{8}$$

This model yielded an average error of 1.7 cm.

From these models and their respective average errors, the elliptic cone model seems to be the more accurate model for a rond de jambe.

4.2 Pique Turn

The second ballet movement examined in this research is a pique turn. This movement is much more complex than the first movement presented. In this project, the data for this movement was recorded and depicted graphically, but the in-depth analysis is left for future work on this project. Unlike the rond de jambe, there is not a simple theoretical shape for the pique turn. However, it is expected to be somewhat piecewise conical.

First, polynomials were found using *polyfit* for the ankle, knee, and hip. These curves gave the following graphical depiction.

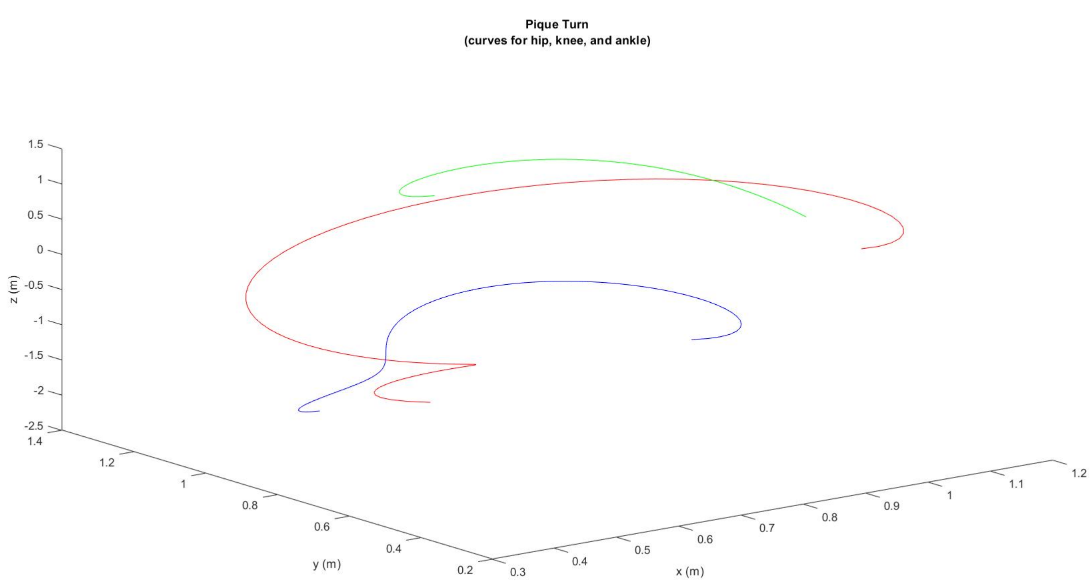

Figure 10: Depicted here are the curves for the hip, knee, and ankle for the pique turn.

Next, the line segment constructions from the ankle to the knee and from the knee to the hip were added to give the idea of a surface in $\mathbb{R}^3$. This is given below.

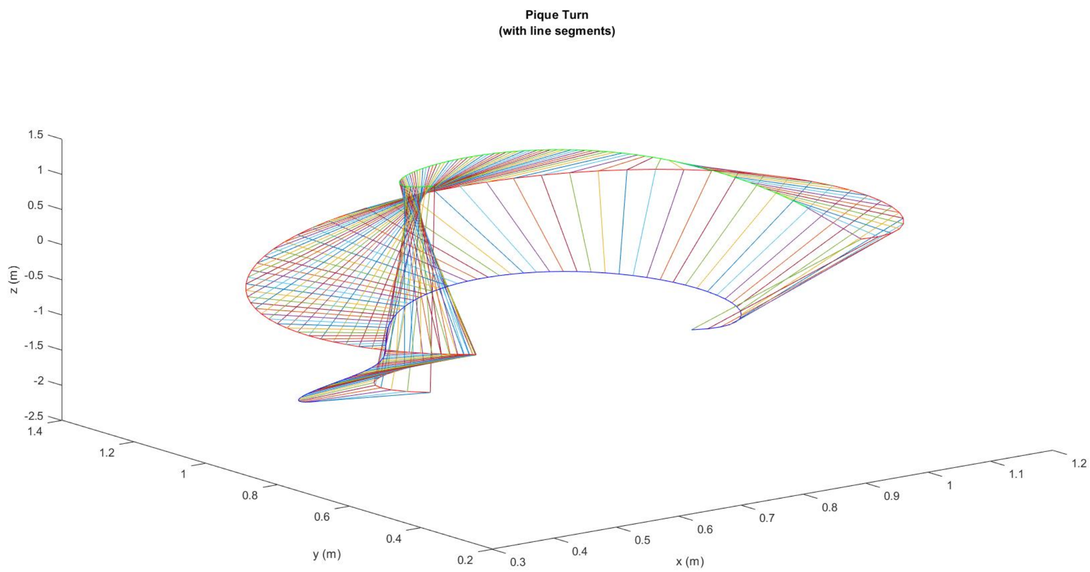

Figure 11: Depicted here are the curves for the hip, knee, and ankle along with line segments from the hip to knee and from the knee to ankle for the pique turn.

The future goals for this research include further analysis of a potential model for the surface created by the pique turn.

5 Conclusion

This project aimed to analyze ballet and how it translates to mathematics. The main area of study was the rond de jambe and which mathematical surface most accurately represent the surface created by a dancer as she performs this movement. After recording location data, two models were tested, a circular cone and an elliptic cone.

In summary, it has been shown that it is possible to accurately represent ballet movements in terms of mathematical surfaces. The surface created by a dancer as she performs a rond de jambe can be represented in terms of an elliptic cone with a fairly small average error. Perhaps more importantly, this research has shown that even areas of study that seemingly have nothing in common with mathematics can still be viewed and studied through the lens of mathematics.

Moving forward, the future goals of this project include, first and foremost, analyzing potential surfaces that could be used to model a pique turn. After this, this research aims to explore other fundamental movements of ballet and the mathematical surfaces that best represent them. This research could also be expanded by testing the models given here on different dancers. The ultimate goal is simply to continue to explore ways in which ballet can be analyzed through mathematics.

6 References

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7 Appendix

Presented here are the codes from MATLAB used in this research project.

Dancer2.m
Author: Dr. Miguel Gates
This code was used to record data in MATLAB that was obtained in Vicon.

RondRunFour.m
Author: Kendall Gibson
This file contains the data used to test the already determined models of the rond de jambe.

RondRunFive.m
Author: Kendall Gibson
This file contains the data used to determine the models for the rond de jambe along with code to compute the polynomials to represent the data and graphically depict both the original data and the two models.

PiqueRunFour.m
Author: Kendall Gibson
This file contains the data used to graphically depict the pique turn.